Dependence Structure and Extreme Comovements in International Equity and Bond Markets

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INTRODUCTION

Equity returns are more dependent in bear markets than in bull markets

- **Longin and Solnik (2001), Ang and Bekaert (2002), Das and Uppal (2003) report this fact for international equity markets.**
- **In a domestic context, Ang and Chen (2002) and Patton (2004) find a similar behavior for domestic equity portfolios.**

Economic importance of this asymmetric dependence for portfolio allocation.

- Patton (2004) shows that knowledge of asymmetric dependence leads to gains that are economically significant.

- Ang and Bekaert (2002) in a regime switching (RS) setup find that the cost of ignoring the difference between regimes of high and low dependence increases in presence of a risk-free asset.

- Das and Uppal (2003) find a small loss when a conservative agent ignores the simultaneous jumps in international markets, but a large cost for more aggressive agents.

INTRODUCTION

The usual tool to investigate this asymmetric dependence is Exceedance Correlation of Longin and Solnik (2001)

$$Ex\_Corr(X,Y;v_1,v_2)=\begin{cases} Corr\left(X,Y\,|X \leq v_1, Y \leq v_2\right) \text{ for } v_1 \leq 0,\ v_2 \leq 0 \\ Corr\left(X,Y\,|X \geq v_1, Y \geq v_2\right) \text{ for } v_1 > 0,\ v_2 > 0 \end{cases}$$

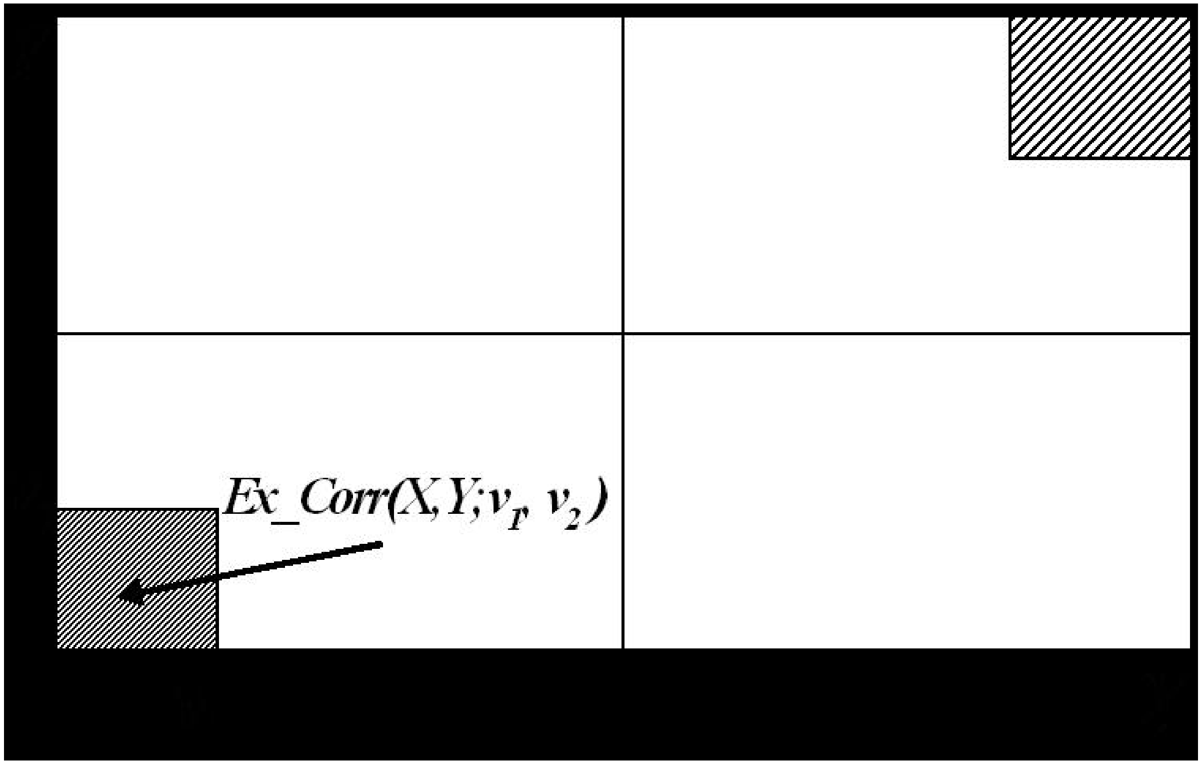
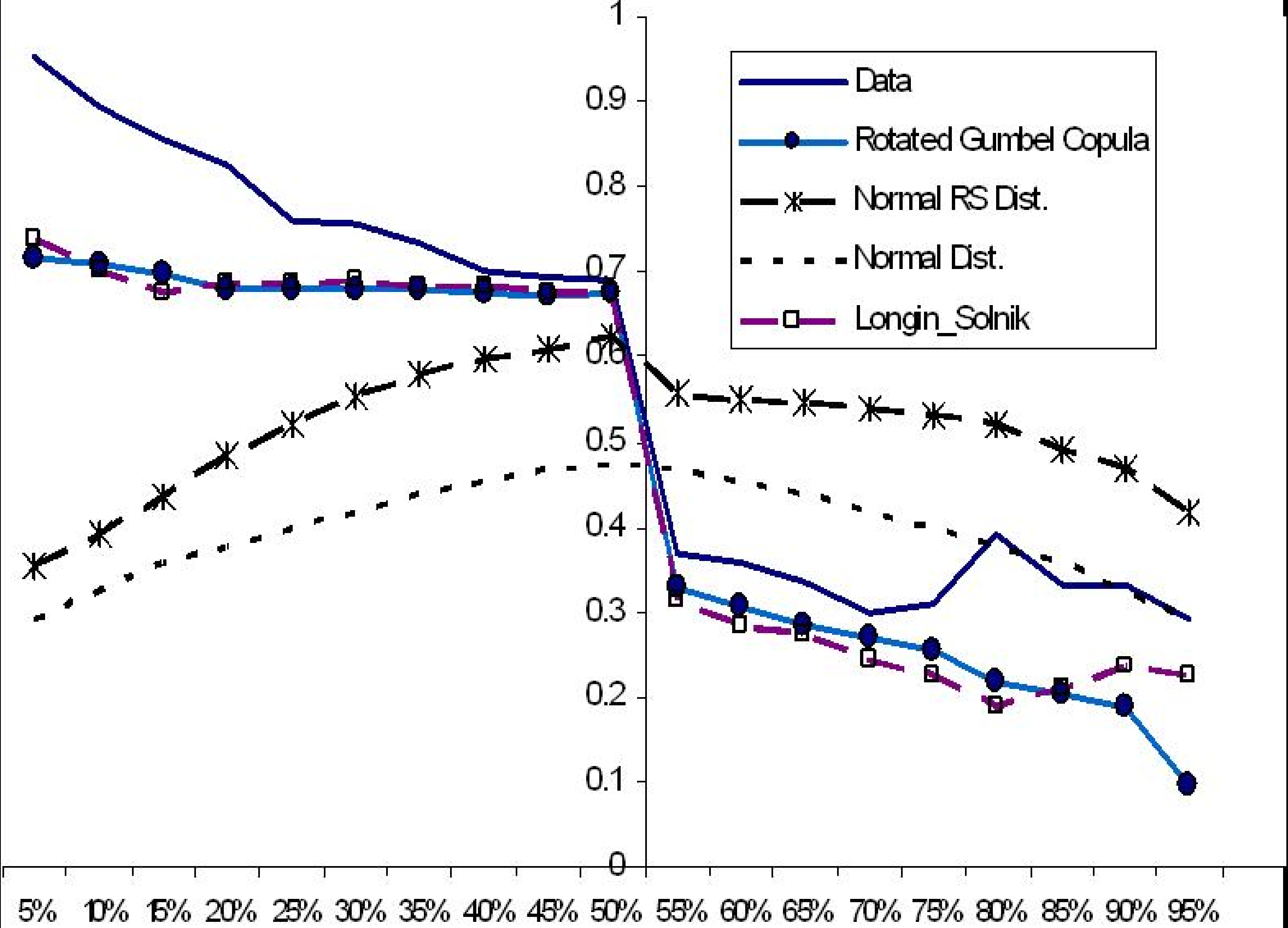


Figure 1: Exceedance Correlation



Data
Rotated Gumbel Copula
Normal RS Dist.
Normal Dist.
Longin_Solnik
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5%
10%
15%
20%
25%
30%
35%
40%
45%
50%
55%
60%
65%
70%
75%
80%
85%
90%
95%

INTRODUCTION

Which models can capture this dependence asymmetry?

- We show analytically that some classical GARCH and RS models with Gaussian innovations cannot adequately capture this dependence asymmetry.

- We construct a model which specifies well this dependence asymmetry and clearly distinguishes it from marginal asymmetry.

- We apply this model to international bond and equity markets to investigate their dependence structure.

Outline of Presentation

1. Stylized Facts
2. Exceedance correlation vs tail dependence
3. Asymmetric dependence modeling and classical models
4. A Model of international bond and equity markets
5. Empirical evidence
6. Conclusion

1. Stylized facts

- **Fact 1: There exists asymmetry in exceedance correlation: large negative returns are more correlated than large positive returns.**

Longin and Solnik (2001) , Ang and Chen (2002)

- **Fact 2: Asymptotically, exceedance correlation is zero for very large positive returns and strictly positive for very large negative returns.**

Longin and Solnik (2001) use Extreme Value Theory (EVT)

1. Stylized facts

- EVT just considers the tails of the distribution (all GPD); it does not allow to determine if a certain data-generating distribution can produce this asymmetry.

- Exceedance correlation is very difficult to compute even in a simple model and is affected by marginal characteristics. Therefore it is not a right measure to assess asymmetric dependence and determine which model can produce it.

- We need a more adapted extreme dependence measure.

2. Exceedance correlation vs Tail dependence function

Tail Dependence Function

$$\tau^L(\alpha) = \Pr\left[F_X(X) \leq \alpha \,|\, F_Y(Y) \leq \alpha\right]$$

$$\tau^U(\alpha) = \Pr\left[F_X(X) \geq 1 - \alpha \,|\, F_Y(Y) \geq 1 - \alpha\right]$$

Tail Dependence Coefficient (TDC)

$$\tau^L = \lim_{\alpha \to 0} \tau^L(\alpha), \text{ and } \ \tau^U = \lim_{\alpha \to 0} \tau^U(\alpha)$$

Remark: For (X, Y) Normal, we have $\tau^L = \tau^U = 0$ (Tail-independence)

2. Exceedance correlation vs Tail dependence function

Fact 2': **Upper extreme returns are tail-independent, while lower extreme returns are tail-dependent.**

i.e.$\tau^U = 0$ and $\tau^L > 0$

Argument: In the context of EVT with a logistic function used by Longin and Solnik (2001), asymptotic correlation and TDC are zero at the same time.

Asymptotic correlation is $\rho_a = 1 - \alpha^2$

while TDC is $\tau = 2 - 2^\alpha$

3. Asymmetric dependence modeling and problems with some classical models

Proposition 2.1:

- **Any GARCH model with constant mean and symmetric conditional distributions has a symmetric unconditional distribution and hence has symmetric TDCs.**

- **If the conditional distribution of a RS model has zero TDC, then the unconditional distribution also has zero TDC.**

The key point is the fact that GARCH and RS unconditional distributions can be seen as mixtures of symmetric TDC distributions.

3. Asymmetric dependence modeling and problems with some classical models

Remark

A RS model in first and second moments can capture finite distance asymmetry as in Ang and Chen (2002) and Ang and Bekaert (2002).

However this asymmetry is not separable from skewness in marginal distributions.

3. Asymmetric dependence modeling and problems with some classical models

Issues for modeling

How to separate marginal asymmetries from asymmetry in dependence?

How to take into account not only asymmetries at finite distance but also in asymptotic dependence?

4. A model of international bond and equity markets

Disentangle marginal distributions from dependence with Copula.

Copula (Definition)
(also called dependence function)

$$F\left(x_1, \cdots, x_n\right) = C\left(F_1\left(x_1\right), \cdots, F_n\left(x_n\right)\right)$$

where F, F_i, and C are cumulative distribution functions.

From Sklar (1959) Theorem C exists and is unique when all F_i are continuous.

4. A model of international bond and equity markets

Copula (Definition)

$$f(x_1, \cdots, x_n) = \underbrace{\prod_{i=1}^{n} f_i(x_i)}_{\text{Marginal Dist.}} \underbrace{c(F_1(x_1), \cdots, F_n(x_n))}_{\text{Dependence function}}$$

with $c(u_1, \cdots, u_n) = \dfrac{\partial^n}{\partial u_1 \cdots \partial u_n} C(u_1, \cdots, u_n)$

f, f_i, and c are density functions

By writing it in this form we understand why copula completely disentangles marginal distributions from the dependence structure.

4. A model of international bond and equity markets

Reparameterization

$$f(x_1, \cdots, x_n; \delta, \theta) = \prod_{i=1}^{n} f_i(x_i; \delta_i) \times c(u_1, \cdots, u_n; \theta)$$

$u_i = F_i(x_i; \delta_i)$, for $i = 1, \cdots, n$

$\delta = (\delta_1, \cdots, \delta_n)$ are the parameters of marginal distributions

θ contains all parameters of copula

4. A model of international bond and equity markets

- Two Countries.

- Each country: one bond index & one equity index.

	$Equity$	$Bond$
$Country\ A$	x_1	x_2
$Country\ B$	x_3	x_4

4. A model of international bond and equity markets

Specification of marginal distributions

$$x_{i,t} = \mu_i + \lambda_i \sigma_{i,t}^2 + \sigma_{i,t} z_{i,t}, \qquad z_{i,t} \rightsquigarrow N(0,1)$$

$$\sigma_{i,t}^2 = \omega_i + \beta_i \sigma_{i,t-1}^2 + \alpha_i \left(z_{i,t-1} - \gamma_i \sigma_{i,t-1} \right)^2$$

So, the vector of parameters is

$$\delta = (\delta_1, \cdots, \delta_4)$$

with $\delta_i = (\mu_i, \lambda_i, \omega_i, \beta_i, \alpha_i, \gamma_i,)$

4. A model of international bond and equity markets

Dependence Structure Specification

$$\begin{aligned} C\left(u_{1,t},\cdots,u_{4,t};\rho^N,\rho^A\,|s_t\right) = \;& s_t C_N\left(u_{1,t},\cdots,u_{4,t};\rho^N\right) + \\ & (1-s_t)\,C_A\left(u_{1,t},\cdots,u_{4,t};\rho^A\right) \end{aligned}$$

where $u_{i,t} = F_{i,t}\left(x_{i,t};\delta_i\right)$, with $F_{i,t}$ the conditional cdf of $x_{i,t}$ and s_t is a Markov Chain which takes value 0 or 1.

C_N is the normal copula defined as
$C_N\left(u_1,\cdots,u_4;\rho^N\right) = \Phi_\rho\left(\Phi^{-1}\left(u_1\right),\cdots,\Phi^{-1}\left(u_4\right)\right)$

and C_A is an asymmetric copula.

4. A model of international bond and equity markets

Multivariate copula construction problem (n larger than 2)

No problem for constructing bivariate copula.

But, for n larger than 2, the problem of constructing copulas with given bivariate margins is, as mentioned by Nelson (1999, p. 86) "... perhaps the most important open question today concerning copulas...".

Multivariate copulas impose same dependence among all pairs of marginal distributions.

4. A model of international bond and equity markets

How to construct a 4-variate dependence structure for our application?

More specifically, we want to build a 4-variate copula with :

(i) tail independence for upper returns and tail dependence for lower returns; and
(ii) different levels of dependence for different pairs.

The existing families of copulas solve only one of these two problems.

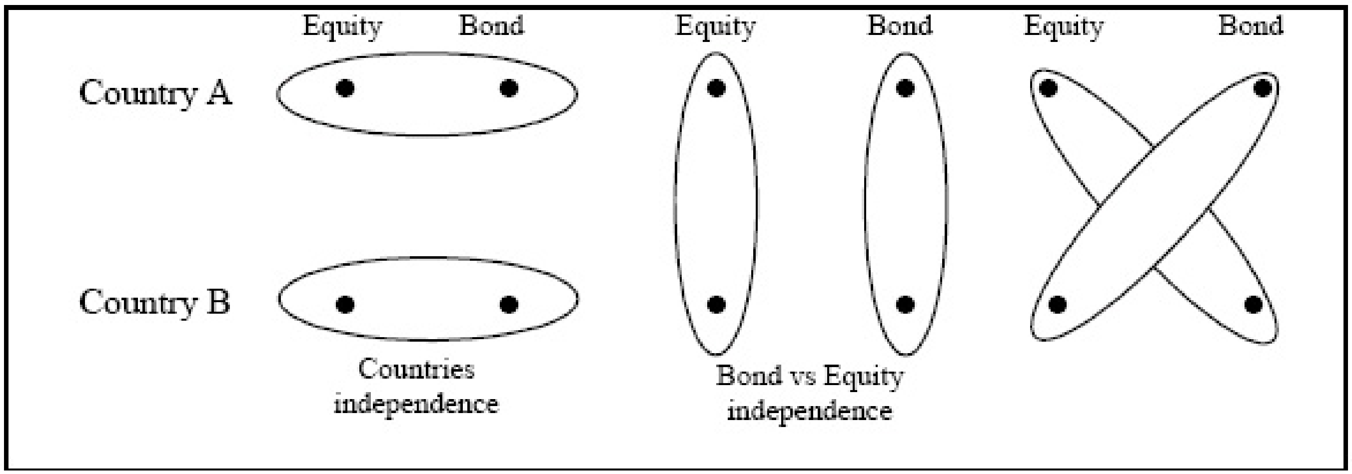


Figure 4: Illustration of the three components of asymmetric copula. Each component is the product of the two bivariate copulas representing the corresponding encircled couple of returns.

Figure 3: Asymmetric Copula

Restricted expression

$$
\begin{aligned}
C_A\left(u_1, \ldots, u_4; \rho^A\right) \equiv\ & \pi C_{GS}\left(u_1, u_2; \tau_1^L\right) \times C_{GS}\left(u_3, u_4; \tau_2^L\right) \\
& + (1-\pi) C_{GS}\left(u_1, u_3; \tau_3^L\right) \times C_{GS}\left(u_2, u_4; \tau_4^L\right)
\end{aligned}
$$

$$
C_{GS}\left(u, v; \tau^L\right) = u + v - 1 + \exp\left[-\left(\left(-\log\left(1-u\right)\right)^{\theta\left(\tau^L\right)} + \left(-\log\left(1-v\right)\right)^{\theta\left(\tau^L\right)}\right)^{1/\theta\left(\tau^L\right)}\right],
$$

where $\theta\left(\tau^L\right) = \dfrac{\log\left(2\right)}{\log\left(2-\tau^L\right)}$, $\tau^L \in [0,1)$ is the lower TDC and the upper TDC is zero.

Therefore, the asymmetry copula is characterized by five parameters $\rho^A = \left(\pi, \tau_1^L, \tau_2^L, \tau_3^L, \tau_4^L\right)$.

4. A model of international bond and equity markets

Estimation

The Log likelihood can be decomposed into two parts

$$L\left(\delta,\theta;\underline{X}_T\right)=\sum_{i=1}^{4}L_i\left(\delta_i,;\underline{X}_{iT}\right)+L_C\left(\delta,\theta;\underline{X}_T\right)$$

Two-step estimation

First step: $\widehat{\delta}=\underset{\delta=(\delta_1,\cdots,\delta_4)\in\Delta}{\arg\max}\sum_{i=1}^{4}L_i\left(\delta_i,;\underline{X}_{iT}\right)$

Second step: $\widehat{\theta}=\underset{\theta\in\Theta}{\arg\max}L_C\left(\widehat{\delta},\theta;\underline{X}_T\right)$

5. Empirical Evidence

Data

Type: bond and equity indices, and exchange rates.

Frequency: weekly.

Two pairs of Countries
North America: Canada and USA
Europe: France and Germany

Table 2: Unconditional correlations between different assets (bond and equity) of four considered countries.

	US Equity	US Bond	CA Equity	CA Bond	FR Equity	FR Bond	DE Equity
US Bond	0.0576						
CA Equity	0.7182	0.0116					
CA Bond	0.1783	0.4706	0.4392				
FR Equity	0.1957	-0.0182	0.1974	0.1065			
FR Bond	-0.0499	0.3386	-0.0080	0.2433	0.3066		
DE Equity	0.2089	-0.0536	0.1995	0.1009	0.8099	0.2625	
DE Bond	-0.0832	0.3081	-0.0234	0.2143	0.3084	0.9403	0.2847

5. Empirical Evidence

US-Canada Dependence Structure (Jan. 1985 – Dec 2004)

Large dependence for asset of same type (Equity-Equity or Bond-Bond) compared to Equity-Bond

Very strong asymmetry for cross-country dependence.

Persistence in each regime.

Cross-Country (US-CA) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)
US Equity - CA Equity		0.8739		0.9100	0.7917
		(0.1560)		(0.0185)	
US Bond - CA Bond		0.3870		0.6234	0.5424
		(0.0831)		(0.0124)	
			1-π	0.6897	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	US Bond	CA Bond		τ	TDC($\pi\tau$)
US Equity	-0.1101	0.1234	US Equity - US Bond	0.1300	0.0169
	(0.0416)	(0.0312)		(0.041)	
CA Equity	-0.0812	0.4085	CA Equity - CA Bond	0.1385	0.0180
	(0.0207)	(0.0103)		(0.0145)	
			π	0.3102	
				(0.0207)	
Parameters of transitional probability matrix					
	P	0.9020	Q	0.9586	
		(0.0207)		(0.0206)	

5. Empirical Evidence

France-Germany Dependence Structure **(Jan. 1985 – Dec 2004)**

Large dependence for asset of same type (Equity-Equity or Bond-Bond) compared to Equity-Bond

Very strong asymmetry for cross-country dependence.

Persistence in each regime.

Cross-Country (FR-DE) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)
FR Equity - DE Equity		0.9083		0.9554	0.7787
		(0.0267)		(0.0603)	
FR Bond - DE Bond		0.9901		0.8261	0.6733
		(0.058)		(0.027)	
			1-π	0.8151	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	FR Bond	DE Bond		τ	TDC($\pi\tau$
FR Equity	0.1893	0.2023	FR Equity - FR Bond	0.0923	0.0171
	(0.0170)	(0.0129)		(0.028)	
DE Equity	0.1175	0.1294	DE Equity - DE Bond	0.0969	0.0179
	(0.0214)	(0.030)		(0.029)	
			π	0.1849	
				(0.0294)	
Parameters of transitional probability matrix					
	P	0.8381	Q	0.9373	
		(0.0270)		(0.0373)	

5. Empirical Evidence

France-Germany Dependence Structure **(After fixed Exchange rate: Jan. 1999 – Dec 2004)**

More persistence in Normal regime.

Cross-Country (FR-DE) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
				τ	TDC((1-π)
FR Equity - DE Equity		0.9426		0.2598	0.2582
		(0.0950)		(0.0106)	
FR Bond - DE Bond		0.9937		0.8946	0.8892
		(0.0382)		(0.071)	
			1-π	0.9940	
Cross-Asset (Equity-Bond) Dependence					
Normal Regime Correlation Coefficient			Asymmetric Regime Tail Dependence Coefficient		
	FR Bond	DE Bond		τ	TDC($\pi\tau$)
FR Equity	0.2272	0.2350	FR Equity - FR Bond	0.2249	0.0013
	(0.0241)	(0.0177)		(0.024)	
DE Equity	0.1516	0.1573	DE Equity - DE Bond	0.9760	0.0059
	(0.0118)	(0.059)		(0.082)	
			π	0.0060	
				(0.012)	
Parameters of transitional probability matrix					
	P	0.9212	Q	0.2274	
		(0.0118)		(0.0117)	

5. Empirical Evidence

- **US-Canada**

$$\hat{P}_t = \underset{(6.81E-02)}{-1.26} + \underset{(2.28E+01)}{5.06E+02} \times Vol_t + e_t$$

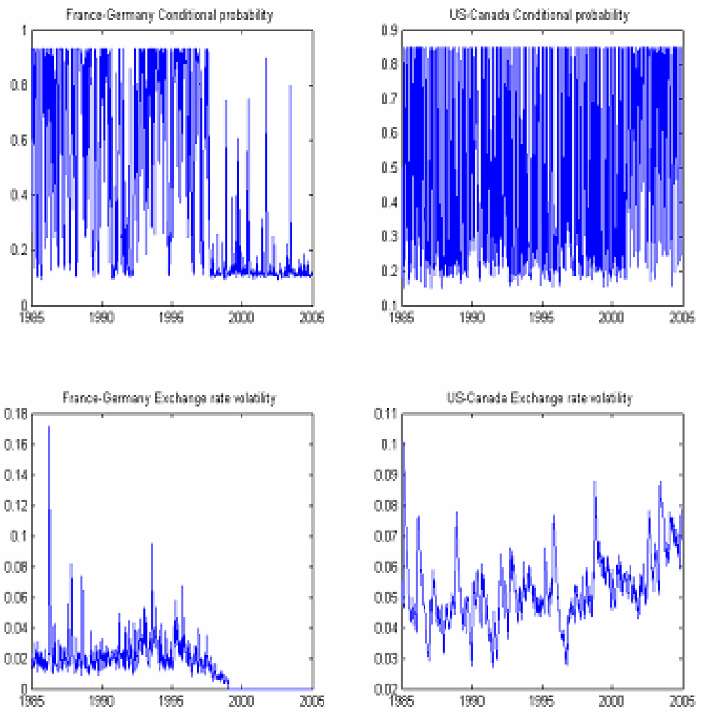
$$(R-square = 0.75)$$

- **France-Germany**

$$\hat{P}_t = \underset{(1.76E-01)}{-7.71E-01} + \underset{(2.36E+01)}{9.30E+01} \times Vol_t + e_t$$

$$(R-Square = 0.86)$$

$\hat{P}_t = \log\left[\frac{P_t}{1-P_t}\right]$ is the reverse of the logistic transformation of Conditional probability P_t, and Vol_t is the exchange rate volatility



5. Empirical Evidence

Test of asymmetry in dependence structure

	US-Canada	France-Germany
LR	0.0731	0.7889
$p-value$	0.0090	0.0000

The Monte Carlo results confirm the presence of asymmetric behaviour in dependence for both pairs of countries.

6. Portfolio Implications

Asymmetric dependence and Cross-Country Portfolio Diversification : "Home bias investment".

A strong dependence in lower returns creates a lower (or large negative) co-skewness.

A strong downside market dependence, which create co-skewness, combined with a large foreign risk, implies that the share invested in the domestic portfolio will increase compared with the share invested in a MV framework.

Asymmetric dependence effect on Domestic Diversification: "Flight to Safety".

The same intuition explains the fact that in the presence of asymmetric dependence, investors will increase the share of bonds in their portfolio relative to equity.

7. Conclusion

We show that Classical models such as GARCH and RS cannot clearly reproduce extreme asymmetry in dependence.

We propose an alternative model to investigate dependence structure which allows multivariate extreme tail dependence.

Empirically, we find large extreme dependence in cross-country dependence into each markets (bond or equity) and low dependence between bond and equity even in same country.

The exchange rate volatility amplifies the asymmetry in dependence.

7. Conclusion

Asymmetric dependence and portfolio diversification: Home bias investment and flight to safety are amplified by asymmetric dependence.

Implications of asymmetric dependence for risk management (Tsafack, 2007).