Coherent mortality forecasting for less developed countries

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Motivation

- Less developed countries (LD) have the majority of global population:
  - 6.13 billion in 2015, about **5 times** the population in the more developed (MD) world (1.25 billion).
- Drastic increase in life expectancy in LD over the past decades. According to World Population Perspective 2017 (United Nations)
  - **27.4 years** (41.7 to 69.1) for LD; **13.6 years** (64.8 to 78.4) for MD.
- WPP 2017 predicts that faster increase in life expectancy of LD will continue in the future.
- Credible and detailed mortality projections are needed.

Existing studies for mortality projections of LDs

- Extensive literature on the forecasting of life expectancy at birth for the LD countries. [Lin et al., 2012, Lutz et al., 2008]
- Little attention has been paid to the forecasting of age-specific mortality rates.
- The latter is rather important:
  - more detailed mortality information;
  - necessary inputs to estimate population structure, dependency ratio, life expectancy at 65, etc.

Modeling of age-specific mortality rates

- The Lee-Carter model [Lee and Carter, 1992]

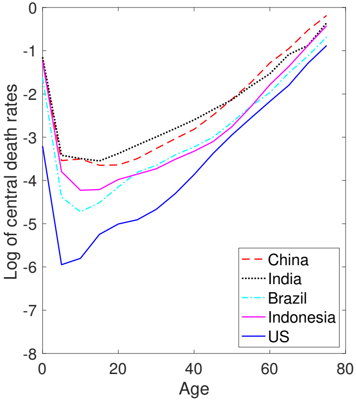
$$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$$

  - $\log m_{x,t}$ — log of death rate at age x and year t;
  - a_x — mortality level;
  - k_t — aggregate mortality trend;
  - b_x — loadings to the aggregate trend, i.e., the age effects.

Lee-Carter model

- Two crucial assumptions
  1. linear aggregate mortality trend
     $k_t = d + k_{t-1} + \epsilon_t$
  2. time-invariant age effects (b_x)
- Compatible with post-war data for MDs. [Tuljapurkar et al., 2000, Lee and Miller, 2001]
- But questionable for the LD.

Age-specific mortality rates, 1960 and 2015



Age-specific mortality rates, 1960 and 2015

- Compared to the US, LD countries have
  - Faster aggregate mortality declines;
  - Imbalanced age-specific improvements, faster for the young, slower for the old
- Lee-Carter model will extrapolate these patterns into the future.

The historical pattern of LDs will not persist

- Largely driven by modernization and better prevention of infectious diseases in the past decades [Austin and McKinney, 2012, Jeuland et al., 2013]
  - very fast improvements;
  - benefit mostly the younger ages;
- As the chronic diseases might be more prominent in the future, they are likely to become closer to the more developed countries
  - slower aggregate improvements;
  - more focused on old ages.

Existing solutions in the literature

- The Li-Lee model [Li and Lee, 2005]

$$\log m_{i,x,t} = a_{i,x} + B_x K_t + b_{i,x} k_{i,t} + \varepsilon_{i,x,t}, \qquad \forall i, x, t \qquad (1)$$
$$K_t = d_0 + K_{t-1} + v_t, \qquad (2)$$
$$k_{i,t} = \alpha_{0,i} + \alpha_{1,i} k_{i,t-1} + \epsilon_{i,t}. \qquad (3)$$

- The Li-Lee model
  - estimate B_x and K_t using a group of MD countries;
  - impose $\hat{B}_x$ and $\hat{K}_t$ on the LD country to produce coherent forecasts.
- Potential limitations:
  - historical mortality patterns of the LD country play no role in projecting its own mortality;
  - artificial structural break in the beginning of projection;

This paper

- We propose rotation method to produce coherent mortality forecasts for the LD countries. The main idea is :
  1. Start from their historical mortality pattern in the short term, **gradually** rotate to the mortality patterns of MD in the long term, inspired by [Li et al., 2013];
  2. In the rotation for LD, mortality pattern of MDs gains larger weights in projections, as the LD become more similar as MDs, i.e., life expectancy gap narrows down to a long-term value.
  3. **Coherent** projections in the long-run
- Our approach avoids the limitations of Li-Lee model.

Our approach: extended Lee-Carter model for LD

$$\log m_{x,s} = a_x + b_{x,s} k_s + \varepsilon_{x,s}$$
$$k_{s+1} = d_s + k_s + v_s, s \in \{T+1, ...\}$$

- Time-varying age effects $b_{x,s}$
- Time-varying aggregate mortality trend d_s.

Our approach: rotations for $b_{x,s}$ and d_s

For $\forall x$

$$d_{s+1} = \begin{cases} (1-w_s(g_s))\hat{d}^{LC} + w_s(g_s)\hat{d}_0^{LL}, & \text{if } g_s > g_0, \\ \hat{d}_0^{LL}, & \text{if } g_s \leq g_0, \end{cases} \quad (4)$$

$$b_{x,s+1} = \begin{cases} (1-w_s(g_s))\hat{b}_x^{LC} + w_s(g_s)\hat{B}_x^{LL}, & \text{if } g_s > g_0, \\ \hat{B}_x^{LL}, & \text{if } g_s \leq g_0, \end{cases} \quad (5)$$

- d_{s+1} is the time-varying weighted average of the aggregate mortality trend of LD ($\hat{d}^{LC}$) and the one of MDs ($\hat{d}_0^{LL}$). LD's d_{s+1} starts from historical $\hat{d}^{LC}$ and gradually rotates to MDs' benchmark $\hat{d}_0^{LL}$. Similar logic applies to the time-varying age effect $b_{x,s}$.
- The weights depend on life expectancy gap between LD and MDs g_s

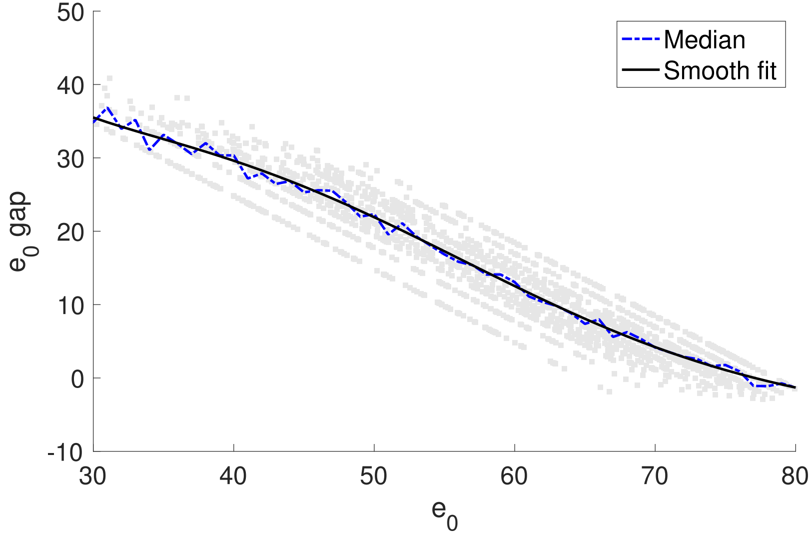
Our approach: weighting parameters

$$w_s = \frac{1}{2}\Big(1 + \sin\big[\frac{\pi}{2}(2w_{0,s} - 1)\big]\Big), \quad (6)$$

$$\text{where} \quad w_{0,s} = \frac{g_T - g_s}{g_T - g_0}. \quad (7)$$

- The weight of MDs' mortality pattern ($\hat{B}_x^{LL}$ and $\hat{d}_0^{LL}$) goes smoothly from 0 to 1, as the life expectancy gap decreases.
- The weight become 1 and LD's mortality projections are fully based on the MDs' mortality pattern ($\hat{B}_x^{LL}$ and $\hat{d}_0^{LL}$), when the life expectancy gap narrows down to a long-term value g_0, i.e., LD become similar enough as MDs.

Our approach: determining g_0



Our approach: determining g_0

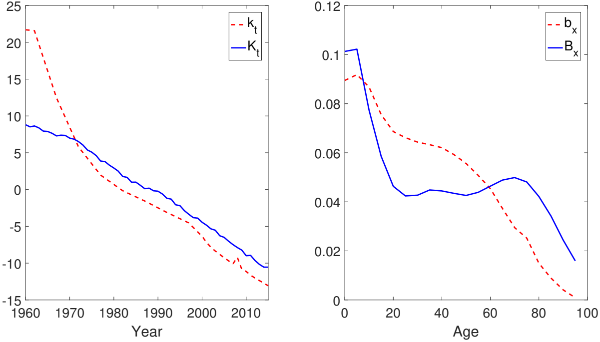
Optimal logistic function for general transition

$$g_t = k_1 + \frac{k_2 - k_1}{1 + \exp(-\frac{A_1}{b_2}(e_{0,t} - b_1 - A_2 b_2))}$$
$$+ \frac{g_0 - k_2}{1 + \exp(-\frac{A_1}{b_4}(e_{0,t} - \sum_{i=1}^{3} b_i - A_2 b_4))} + \epsilon_t, double\ or$$
$$g_t = k + \frac{g_0 - k}{1 + \exp(-\frac{A_1}{b_2}(e_{0,t} - b_1 - A_2 b_2))} + \epsilon_t, single$$

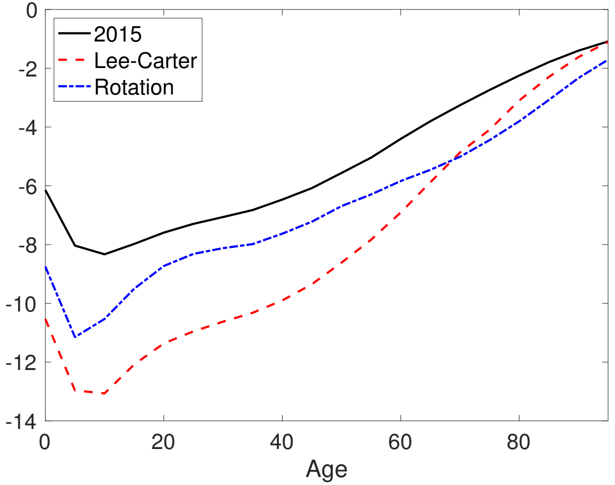
Empirical Study

- Case study for China, Brazil, and Nigeria — the most populous country in Asia, South America, and Africa.
- Mortality data
  - Age: 0-4, 5-9, ..., 95-99; Year: 1960 - 2015.
  - 3 LD countries — merge of two data sources: The United Nations and the World Health Organization .
  - 10 MD countries as benchmark: Germany, Denmark, Finland, France, The Netherlands, Switzerland, Sweden, UK, US, and Japan. Data from the Human Mortality Database.

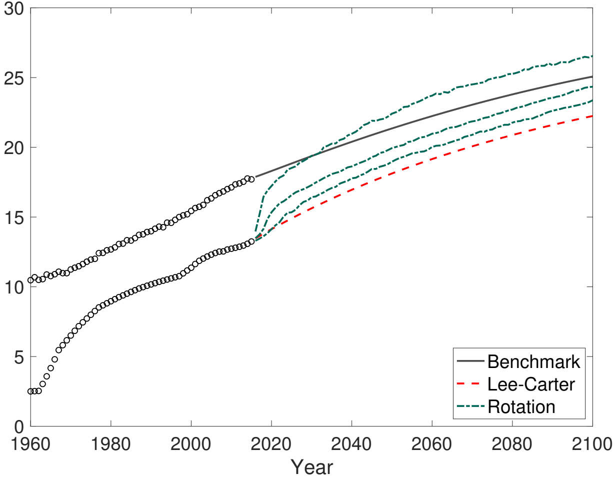
China: Mortality pattern



China: Projected age-specific log m in 2065



China: Remaining life expectancy at 65



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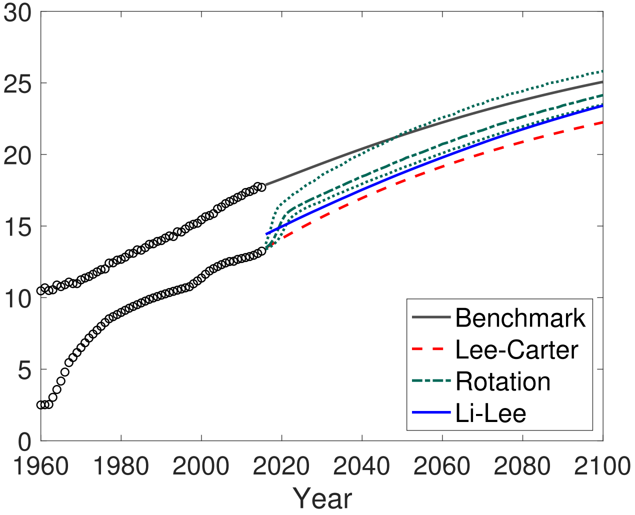
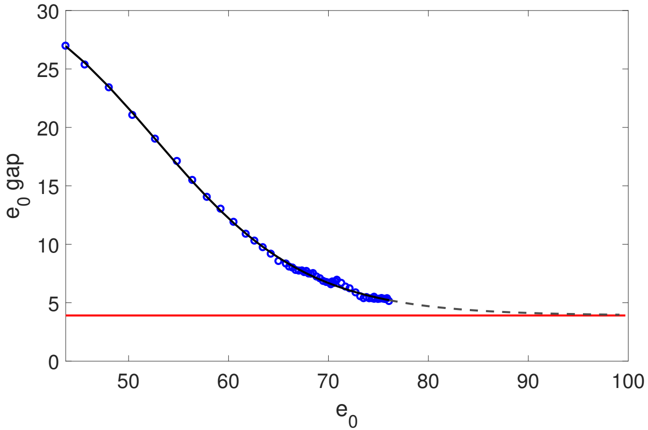
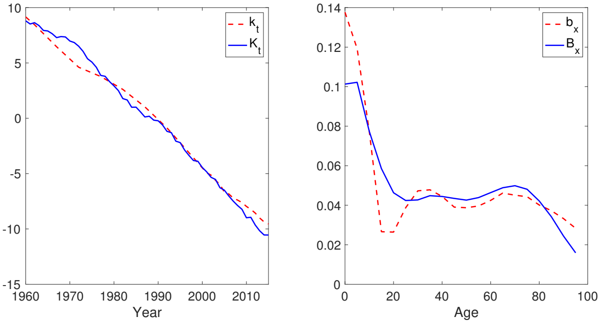


China: g_0

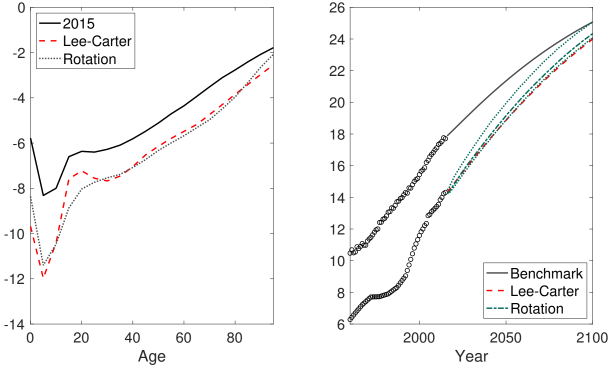


- AIC and BIC both suggest that single logistic function is optimal for China.
- g_0 for China is 3.9.
- Coherence achieved in 2022.

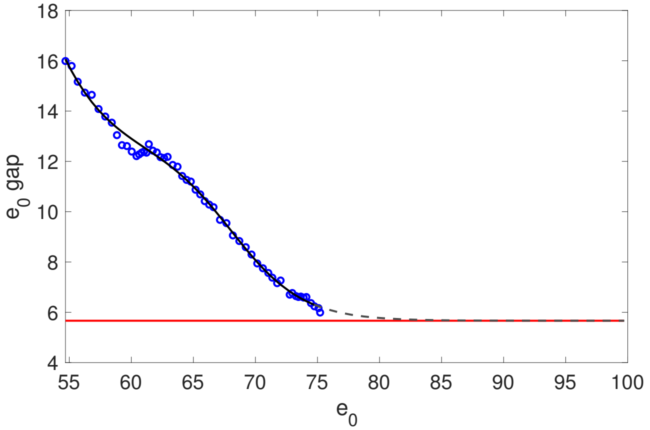
Brazil: Mortality pattern



Brazil: Projected age-specific log m and e_{65} in 2065

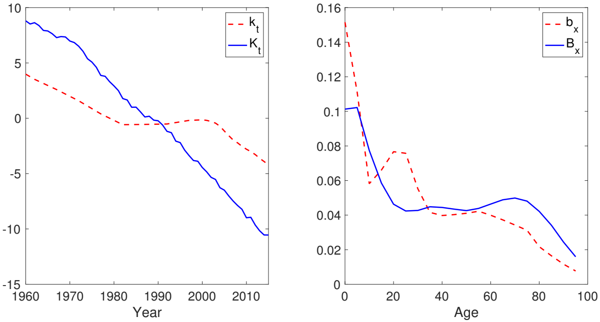


Brazil: g_0

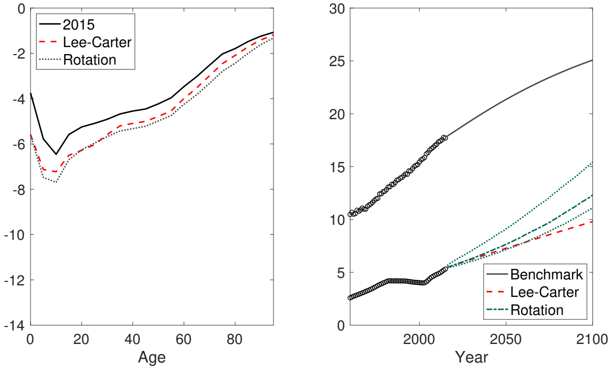


- AIC and BIC both suggest that double logistic function is optimal for Brazil.
- g_0 for Brazil is 5.6
- Coherence achieved in 2029.

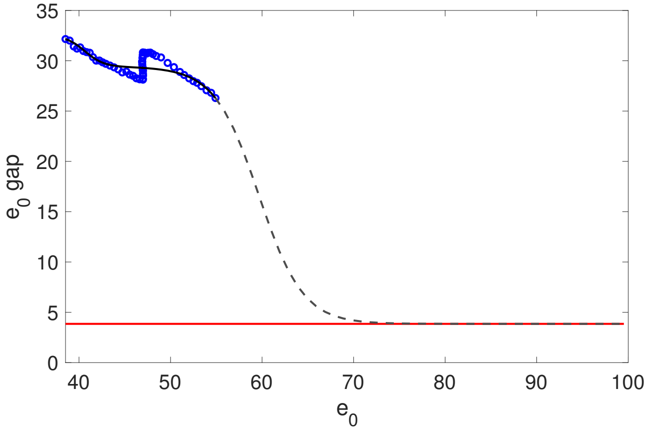
Nigeria: Mortality pattern



Nigeria: Projected age-specific log m and e_{65} in 2065



Nigeria: g_0



- AIC and BIC both suggest that double logistic function is optimal for Nigeria.
- g_0 for Nigeria is 3.9.
- Coherence achieved in 2100.

Conclusion

- This paper proposes an innovative method to generate coherent mortality forecasts for less developed countries.
- Mortality patterns of a less developed country
  - follow its own history in the beginning of projection;
  - gradually rotate to the patterns of a set of more developed countries;
  - achieved coherence in the long-run.
- More reasonable projections are obtained compared with the independent Lee-Carter model.

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