Detecting longevity common trends by multiple population approach

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Agenda

- The Motivation
- The Dependent Mortality Data
- The Lee Carter Model
- The Methodology
- Numerical Applications

Motivation

The presence of ***dependence*** in the data over time and across ages leads to systematic over-estimation or under-estimation of the uncertainty in the mortality estimates, caused by whether negative or positive dependence dominates.

Motivation

In particular, the objective of our work is to produce longevity projections by taking into account the presence of various forms of cross-sectional and temporal dependencies

Motivation

The dependence structure in the data has to be tackled. Otherwise prediction intervals for longevity projections underestimate the actual longevity risk.

In other words, it is necessary to assess a significant and further source of risk: a sort of ***dependency risk***.

The Dependent Mortality Data

On one hand either the dependence for adjacent age groups has to be taken into account, either the dependence structure across time in a single population setting: a sort of intra-dependence structure

The Dependent Mortality data

On the other hand, the dependence across different populations, here we mention it as inter-dependence, can be explored for capturing common long run relationships between countries

The Lee Carter Model

Lee and Carter (1992) suggested a log-bilinear form for the force of mortality:

$$m_{xt} = \exp(\alpha_x + \beta_x k_t + u_{xt})$$

$$\ln(m_{xt}) = \alpha_x + \beta_x k_t + u_{xt}$$

$$\sum_t k_t = 0 \qquad \sum_x \beta_x = 1$$

The Lee Carter Model

Because of the nonlinear nature of the quantities of interest, such as life expectancy, annuity premiums and so on, an analytic approach to the calculation of prediction intervals is intractable, so that a *simulation* approach is required.

The Lee Carter Model

In the literature, there is more than one **bootstrap** method for **dependent data** as for example Block, Local, Wild, Markov Bootstrap, Sub-sampling and **Sieve**.

The Lee Carter Model

Choi and Hall (2000) show that **the Sieve Bootstrap** has **substantial advantages over blocking methods**, to such an extent that block–based methods are not really competitive. In particular, other authors show that the Sieve Bootstrap outperforms the block bootstrap (Hardle et al. 2003).

The Methodology

We consider an extension of the basic Lee and Carter (1992) to consider for the presence of possibly multiple common stochastic trends in the log-mortality rates $y_{ij,t}$, and also for the possible presence of cross sectional correlation arising from stationary common factors in the DGP of the$y_{ij,t}$s.

The Methodology

We model the log-mortality rate $y_{ij,t}$, for country $i = 1, ..., N_c$, age group $j = 1, ..., N_{ag}$ and time $t = 1, ..., T$ as

$$y_{ij,t} = \lambda_{ij}^{F\prime} F_t + \lambda_{ij}^{G\prime} G_t + u_{ij,t}; \tag{1}$$

model (1) can be written more compactly by introducing the notation $\lambda_i^K \equiv [\lambda_i^{F\prime}, \lambda_i^{G\prime}]'$ and $K_t \equiv [F_t', G_t']'$, whence we write

$$y_{ij,t} = \lambda_{ij}^{K\prime} K_t + u_{ij,t}. \tag{2}$$

The Methodology

the model could be rewritten in terms of $y_{l,t}$ without any loss of generality, viz.

$$y_{lt} = \lambda_l^{K\prime} K_t + u_{lt}. \quad (3)$$

The Methodology

We assume that F_t is a k-dimensional nonstationary process; using standard notation, $F_t \sim I(1)$. Similarly, we assume that G_t is an h-dimensional stationary process, and we equivalently write $G_t \sim I(0)$.

As far as the DGP of the non-stationary common factors F_t is concerned, we partition F_t as $F_t = \left[F'_{1t}, F'_{k-1,t}\right]'$, where we let F_{1t} denote the first factor and $F'_{k-1,t}$ is a $k-1$-dimensional vector containing all the other factors. We thus model F_t as

$$\begin{bmatrix} F_{1t} \\ F_{k-1,t} \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} F_{1t-1} \\ F_{k-1,t-1} \end{bmatrix} + e_t^F, \tag{4}$$

The Methodology

The presence of serial dependence in ΔF_t and u_{it} requires a bootstrap algorithm that preserves the autocorrelation structure over time. This can be accomplished by approximating the infinite AR polynomials $\alpha(L)$ and $\Gamma(L)$ by truncating them at lags q_F and $q_{u,i}$ respectively:

$$\Delta F_t = \sum_{j=1}^{q_F} \alpha_{q,j} \Delta F_{t-j} + e_{t,q}^F, \tag{5}$$

$$u_{it} = \sum_{j=1}^{q_{u,i}} \gamma_{q,j}^{(i)} u_{it-j} + e_{it,q}^u. \tag{6}$$

The values of q_F and $q_{u,i}$ depend on n and T, as discussed in the following assumption.

The Methodology

We start with the estimation of F_t and G_t, for given values of k and h. Consider (3):

$$y_{lt} = \lambda_l^{K\prime} K_t + u_{lt},$$

and let $Y = [y_1, ..., y_n]$, where we define $y_l = [y_{l,t}, ..., y_{l,T}]'$ for $l = 1, ..., n$; thus, Y is an $T \times n$ matrix, and (3) can be written as

$$Y = K\Lambda + u, \tag{7}$$

where $K = [K_1', ..., K_T']'$, $\Lambda = [\lambda_1^K, ..., \lambda_n^K]$, and u is defined analogously to Y.

The Methodology

The estimation of K is based on the applying the Principal Components estimator (PC henceforth) to the $T \times T$ matrix YY'. In particular, after extracting the eigenvalues and eigenvectors of YY', sort the eigenvalue/eigenvector couple based on the magnitude of the eigenvalue in descending order. Then

- the non-stationary factor with drift, F_{1t}, is estimated by the first eigenvector of YY' multiplied by $T^{3/2}$;
- the next $k-1$ non-stationary factors $F_{k-1,t}$ are estimated by the next $k-1$ eigenvectors of YY' multiplied by T;
- the h stationary factors G_t are estimated by the next h eigenvectors of YY' multiplied by $\sqrt{T}$.

The Methodology

$$\hat{\lambda}_l^K = \left[\sum_{t=1}^{T} \hat{K}_t \hat{K}_t'\right]^{-1} \left[\sum_{t=1}^{T} \hat{K}_t y_{l,t}\right].$$

The Methodology

$$\widehat{k+h} = \arg \min_{0 \leq k+h \leq F_{\max}} PC\left(k+h\right),$$

with

$$PC\left(k+h\right) = V\left(k+h\right) + \left(k+h\right) \times g\left(n, T\right),$$

and

$$V\left(k+h\right) = \min_{\lambda^{k+h}, K^{k+h}} \frac{1}{nT} \sum_{l=1}^{n} \sum_{t=1}^{T} \left(\Delta y_{lt} - \hat{\lambda}_l^{K\prime} \Delta \hat{K}_t\right)^2,$$

and $g\left(n, T\right)$ a penalty function such that $g\left(n, T\right) \to 0$ and $\min\left\{n, T\right\} g\left(n, T\right) \to \infty$.

The Methodology

At the same time, k is determined as

$$\hat{k} = \arg \min_{0 \le k \le F_{\max}} IPC(k),$$

where

$$IPC(k) = V'(k) + k \times g(n, T),$$

and

$$V'(k) = \min_{\lambda^k, F^k} \frac{1}{nT} \sum_{l=1}^{n} \sum_{t=1}^{T} \left(y_{lt} - \hat{\lambda}_l^{K\prime} \hat{K}_t \right)^2.$$

Then a consistent estimate of h is given by $\hat{h} = \widehat{k+h} - \hat{k}$.

The Methodology

The bootstrap algorithm is a classical sieve bootstrap algorithm, with the only difference that it is applied to generated regressors such as $\hat{K}_t = \left[\hat{F}_t', \hat{G}_t\right]'$. In particular, the algorithm is based on fitting two autoregressions; letting ${}_{\Delta}K_t = [\Delta F_t', G_t']'$, we assume that the DGPs of the common factors K_t and of the error term u_{lt} in (3) can be approximated by

$$
\begin{aligned}
{}_{\Delta}K_t &= \sum_{j=1}^{q_K} A_{q,j} {}_{\Delta}K_{t-j} + e_{t,q}^{K}, \\
u_{lt} &= \sum_{j=1}^{q_{u,i}} \gamma_{q,j}^{(l)} u_{lt-j} + e_{lt,q}^{u}.
\end{aligned}
$$

$\hat{a} = T^{-1} \sum_{t=1}^{T} \Delta \hat{F}_{1t}$ to the first component of $F_{t,b}$.

The Methodology

The bootstrapping algorithm is as follows:

Step 1. (PC estimation)

(1.1) Determine k and h;

(1.2) Estimate λ_l^K and K_t in (3) using PC.

(1.3) Generate the residuals $\hat{u}_{lt} = y_{lt} - \hat{\lambda}_l^{K\prime} \hat{K}_t$ and define $\hat{\xi}_{lt} = \left[{}_\Delta \hat{K}_t^{\prime}, \hat{u}_{lt}\right]^{\prime}$, where ${}_\Delta \hat{K}_t = \left[\Delta \hat{F}_t^{\prime}, \hat{G}_t^{\prime}\right]^{\prime}$.

The Methodology

Step 2. (estimation)

(2.1) Estimate $A_{q,j}$ and $\gamma_{q,j}^{(l)}$ (obtaining $\hat{A}_{q,j}$ and $\hat{\gamma}_{q,j}^{(l)}$ respectively) by applying OLS (or some other estimator, e.g. the Yule-Walker estimator) to $\Delta\hat{K}_t = \sum_{j=1}^{q_K} A_{q,j}\Delta\hat{K}_{t-j} + e_{t,q}^K$ and $\hat{u}_{lt} = \sum_{j=1}^{q_{u,i}} \gamma_{q,j}^{(l)}\hat{u}_{lt-j} + e_{lt,q}^u$.

(2.2) Compute the residuals $\hat{e}_{t,q}^K = \Delta\hat{K}_t - \sum_{j=1}^{q_K} \hat{A}_{q,j}\Delta\hat{K}_{t-j}$ and $\hat{e}_{lt,q}^u = \hat{u}_{lt} - \sum_{j=1}^{q_{u,i}} \hat{\gamma}_{q,j}^{(l)}\hat{u}_{lt-j}$. Define $\hat{e}_{it,q} = \left[\hat{e}_{t,q}^{K\prime}, \hat{e}_{lt,q}^u\right]'$.

The Methodology

Step 3. (bootstrap) for $b = 1, ..., \beth$ iterations

(3.1) (resampling)

(3.1.a) Center the residuals $\hat{e}_{lt,q}$ around their mean, as $\bar{e}_{lt,q} = \hat{e}_{lt,q} - T^{-1}\sum_{t=1}^{T}\hat{e}_{lt,q}$.

(3.1.b) Draw (with replacement) T values from $\{\bar{e}_{lt,q}\}_{t=1}^{T}$ to obtain the bootstrap sample $\{e_{lt,b}\}_{t=1}^{T}$, with $e_{lt,b} = \left[e_{t,b}^{K\prime}, e_{lt,b}^{u}\right]^{\prime}$.

The Methodology

(3.2) (generation of the bootstrap sample)

(3.2.a) Generate recursively the pseudo sample $\xi_{it,b} = \left[{}_{\Delta}K'_{t,b}, u_{it,b}\right]'$ as ${}_{\Delta}K_{t,b} = \sum_{j=1}^{q_F} \hat{A}_{q,j}$ ${}_{\Delta}K_{t-j,b} + e^K_{t,b}$ and $u_{lt,b} = \sum_{j=1}^{q_{u,i}} \hat{\gamma}^{(l)}_{q,j} u_{lt-j,b} + e^u_{lt,b}$, using as initialization $\left\{\xi_{lq,b}, ..., \xi_{l1,b}\right\} = \left\{\xi_{lq}, ..., \xi_{l1}\right\}$.

(3.2.b) Generate $F_{t,b}$ as $F_{t,b} = F_{0,b} + \sum_{j=1}^{t} \Delta F_{j,b}$, with initialization is $F_{0,b} = \hat{F}_0$, or alternatively $T^{-1} \sum_{t=1}^{T} \hat{F}_t$, and add $\hat{a} = T^{-1} \sum_{t=1}^{T} \Delta \hat{F}_{1t}$ to the first component of $F_{t,b}$.

(3.2.c) Generate the pseudo sample $\{y_{lt,b}\}_{t=1}^{T}$.

Numerical Applications

Application scheme:

-1) Fitting the LC model,

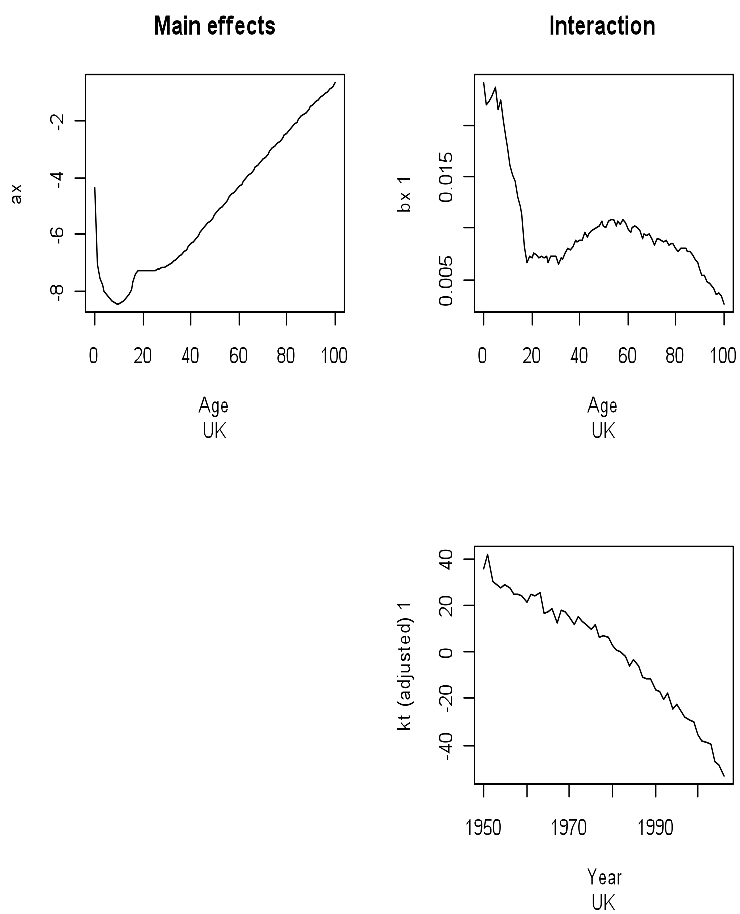
-2) Measuring Dependence Structure,

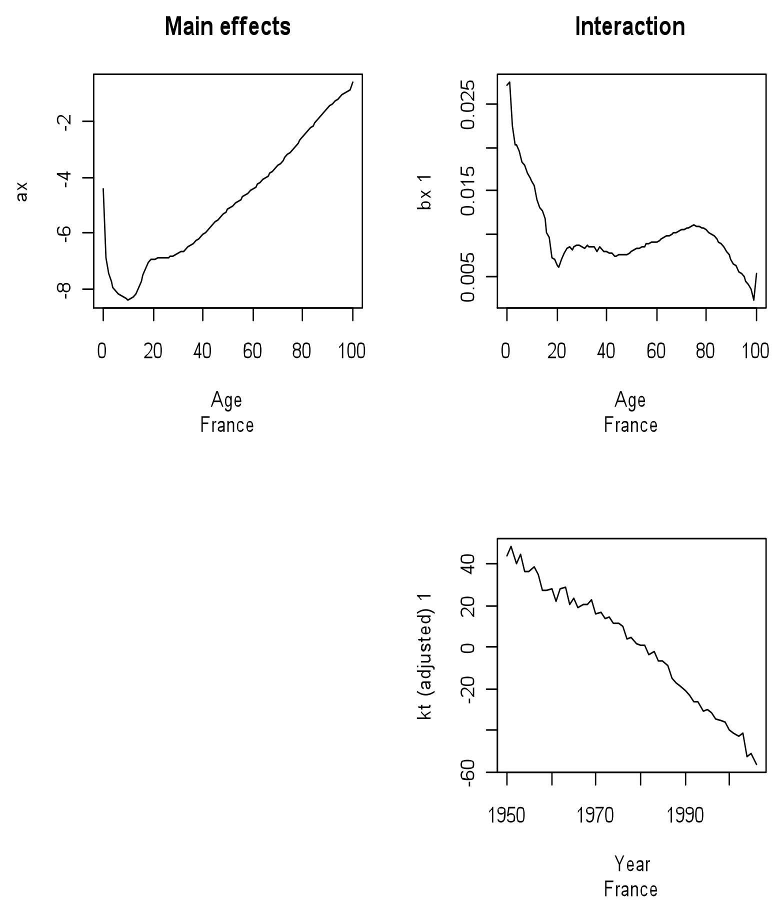
-3) Projecting mortality.

Dataset

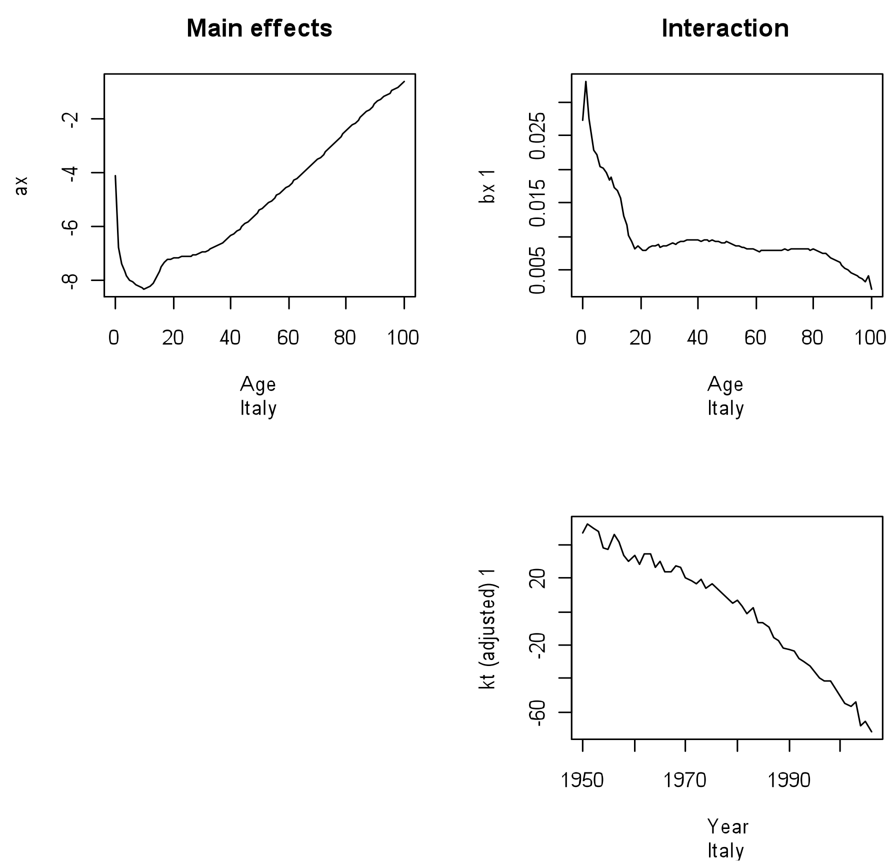
The analysis considers the following countries: United Kingdom (henceforth UK), France, Italy, Spain, Belgium. The study is performed for each country on total population (male and female) ranging from 1950 to 2006, for ages from 0 up to 100 years, considered by single calendar year and by single year of age, where the class of age above 100 years is collected in an open age group 100+.

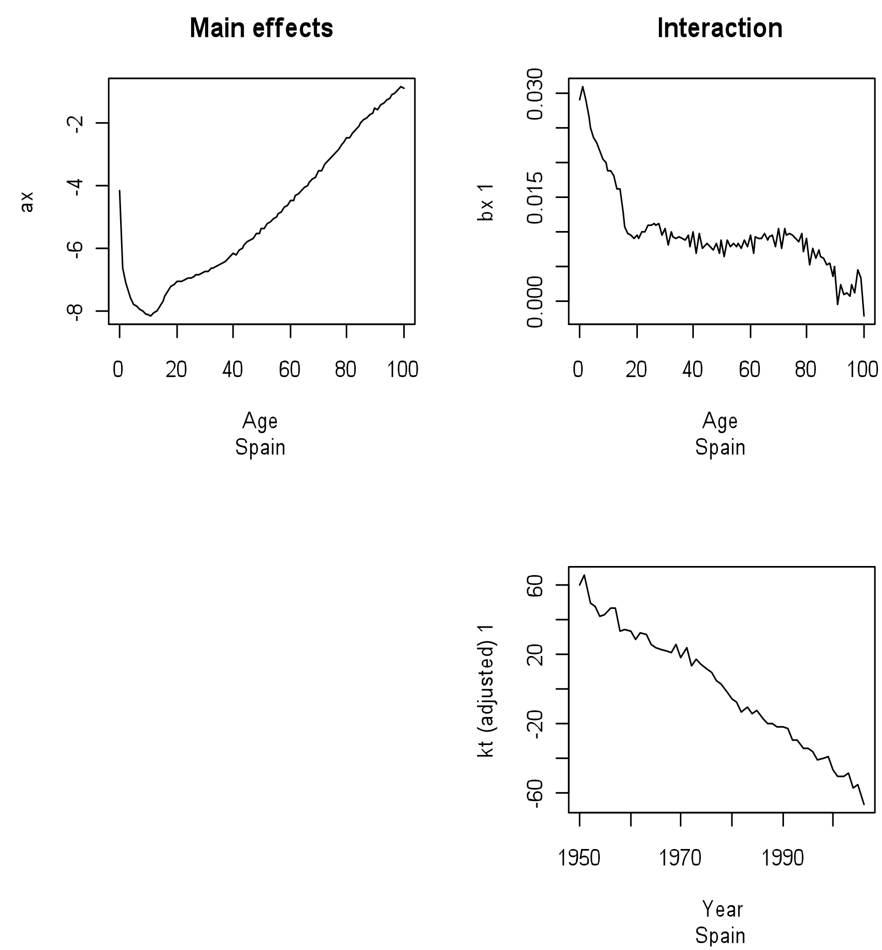
Fitting the LC model



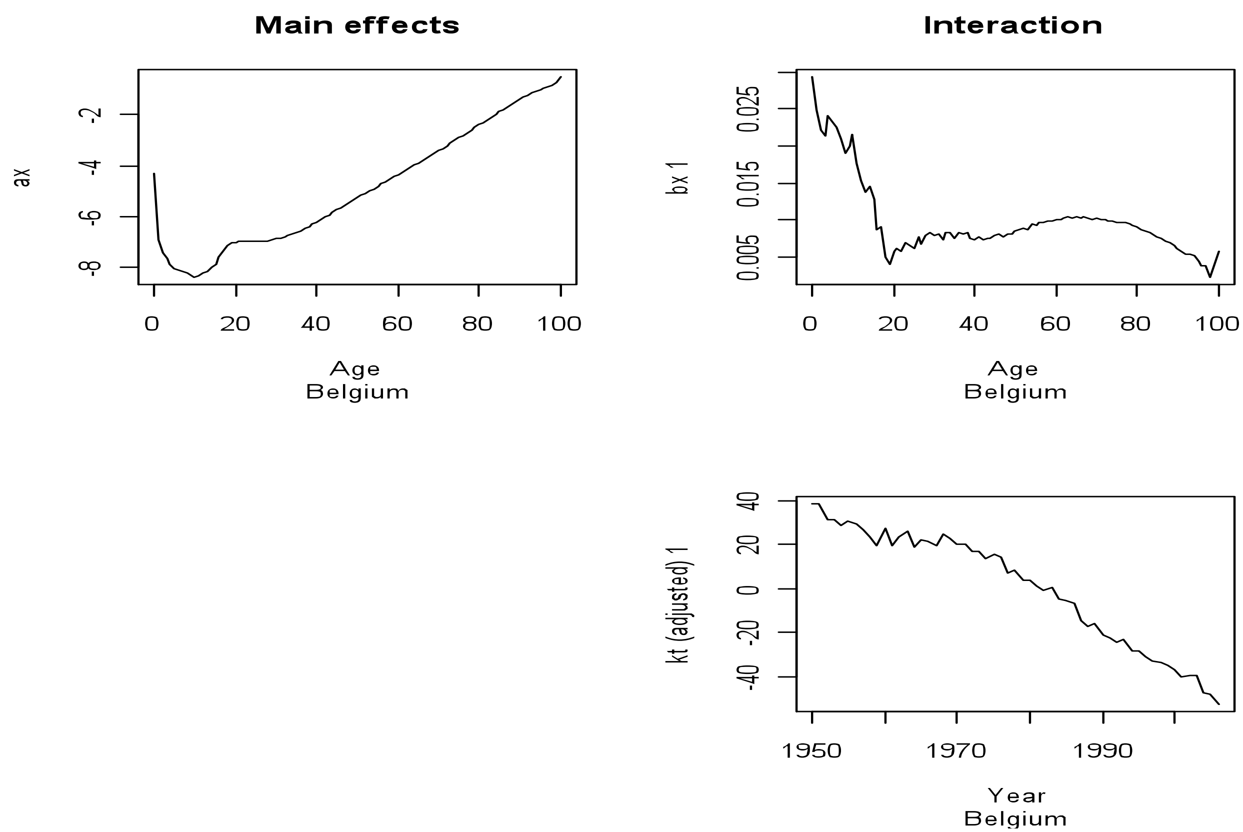


Fitting the LC model





Fitting the LC model



Fitting the LC model

	Averages across ages:			
	ME	**MSE**	**MPE**	**MAPE**
UK	**-0.00001**	**0.00005**	**0.00527**	**0.05513**
France	**0.00005**	**0.00009**	**0.00741**	**0.05645**
Italy	**0.00008**	**0.00008**	**0.0111**	**0 0.07269**
Spain	**-0.00010**	**0.00008**	**0.00994**	**0.09113**
Belgium	**-0.00021**	**0.00038**	**0.00926**	**0.07280**
	Averages across years:			
	IE	**ISE**	**IPE**	**IAPE**
UK	**-0.00034**	**0.00395**	**0.52529**	**5.41629**
France	**0.00601**	**0.00612**	**0.73746**	**5.52106**
Italy	**0.00833**	**0.00573**	**1.11463**	**7.16246**
Spain	**-0.00944**	**0.00693**	**0.99988**	**8.97029**
Belgium	**-0.01393**	**0.02343**	**0.90969**	**6.92477**

Measuring Dependence Structures

We measure either the dependence within each single population, either the dependence between different populations: respectively the intra-dependence and the inter-dependence.

Measuring Dependence Structures

In that respect by focusing on the intra-dependence, we include graphical analysis on autocorrelation functions by age and time, formal statistical tests as the Ljung-Box test based on the autocorrelation plot and Pearson test of independence.

Measuring Dependence Structures

We calculate the correlogram for each country, constructed considering the correlation between ages for each year of the dataset. The evidence shows the persistence of correlation for UK, Italy and Spain almost always during the years. In a different way, for France and Belgium the dependence outcome seems to be not so quite marked.

Measuring Dependence Structures

We construct the correlogram by considering the correlation between years for each age. In this case, for each age, we are dealing with a time series generated from a stochastic process and verify the autocorrelation during the time. In other words, by considering the dependence for a fixed time, we verify a temporal dependence for each age during the years.

Measuring Dependence Structures

Italy and Spain prove a strong dependence structure for almost all ages, in particular for younger ones, which tends to decrease for adult ages.

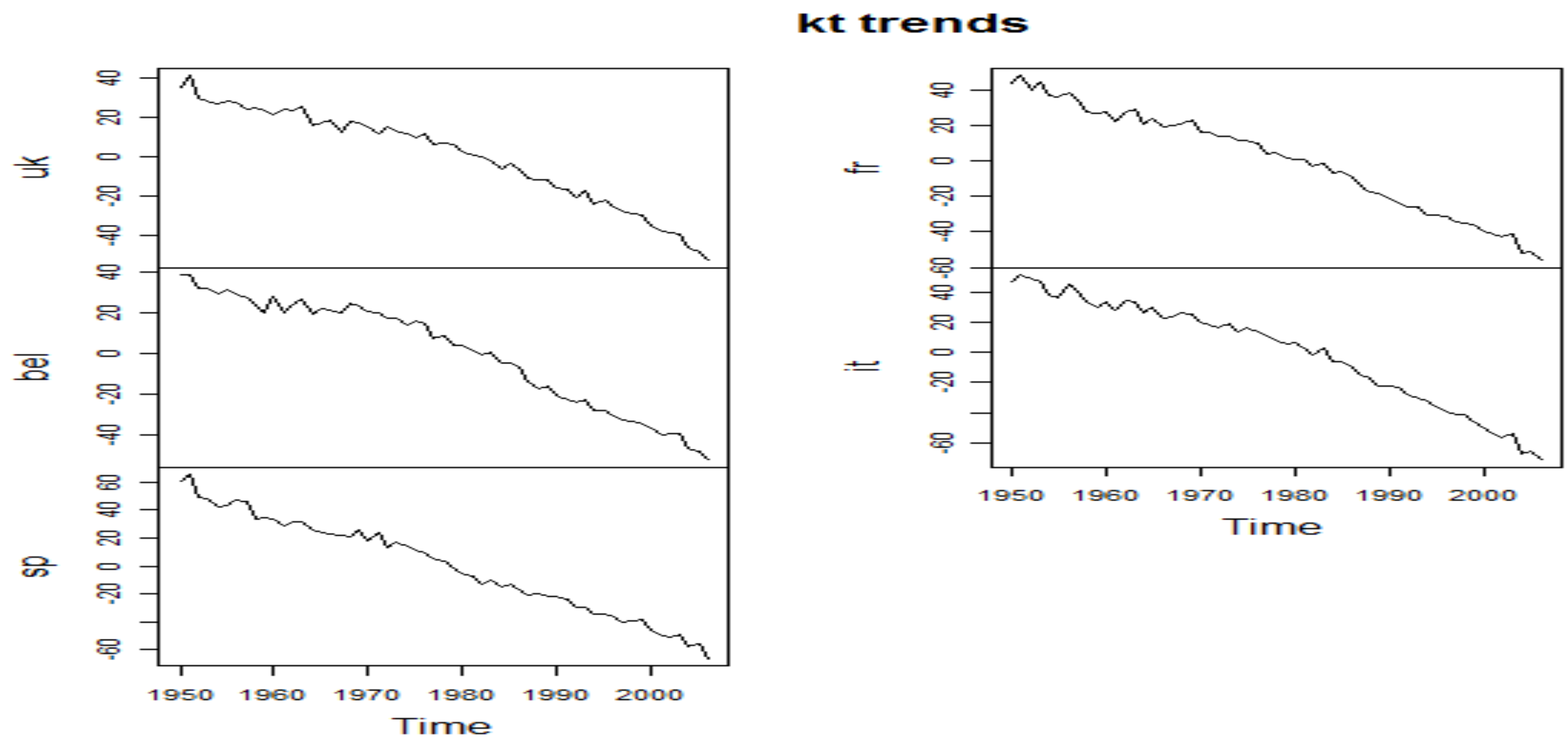
Measuring Dependence Structures

The case of UK shows a low correlation for younger ages, different from Italy, while it increases from 24-25 years up to 70 years.

Measuring Dependence Structures

Also for France the correlation is stronger for the *central* ages instead of the extremes (lower or higher ages). Belgium seems to have less marked dependence structure.

Projecting Mortality



plots estimates of simultaneously, for the total population of the five Countries considered. As shown, declines roughly linearly from 1950 to 2006, specially for France and Italy.

Projecting Mortality

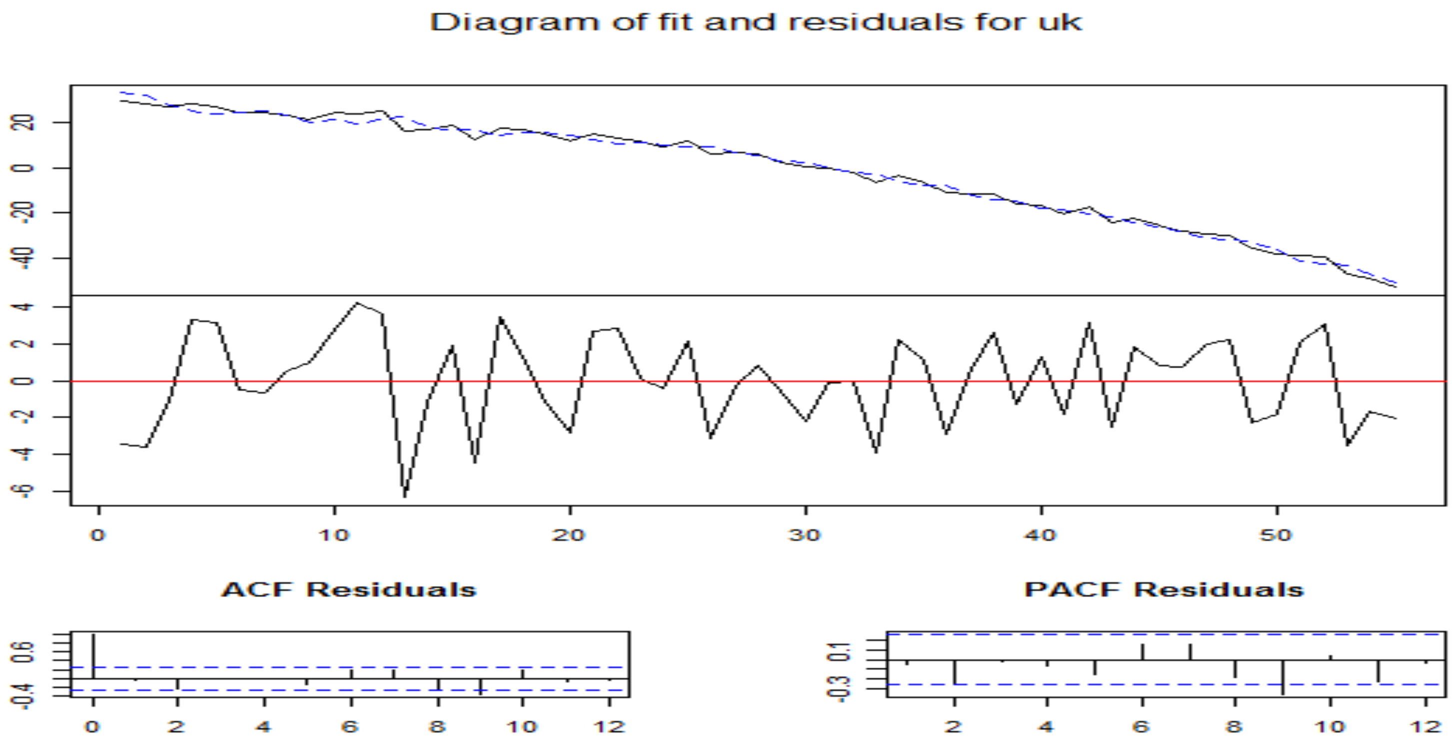


Diagram of fit, residuals, ACF and PACF of residuals for UK

Projecting Mortality

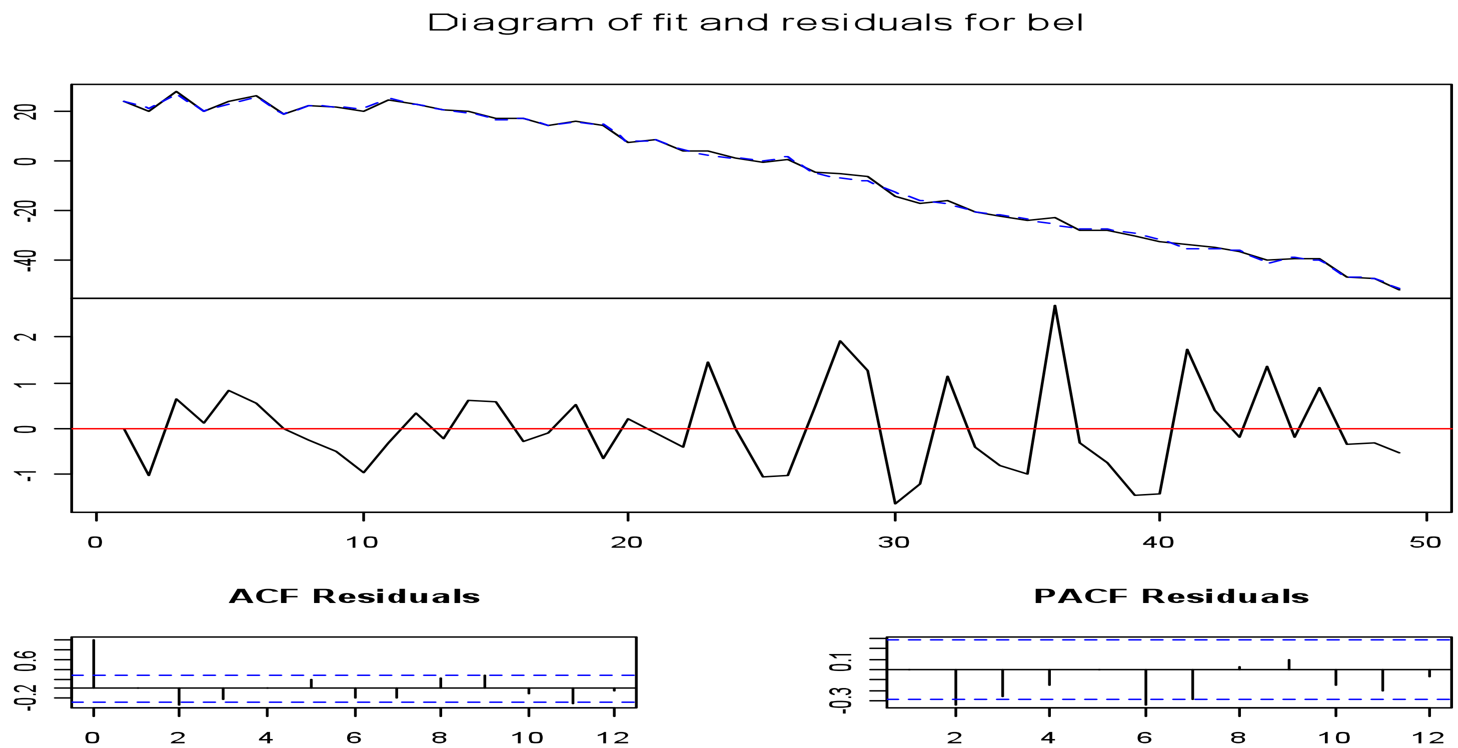


Diagram of fit, residuals, ACF and PACF of residuals for Belgium

Projecting Mortality

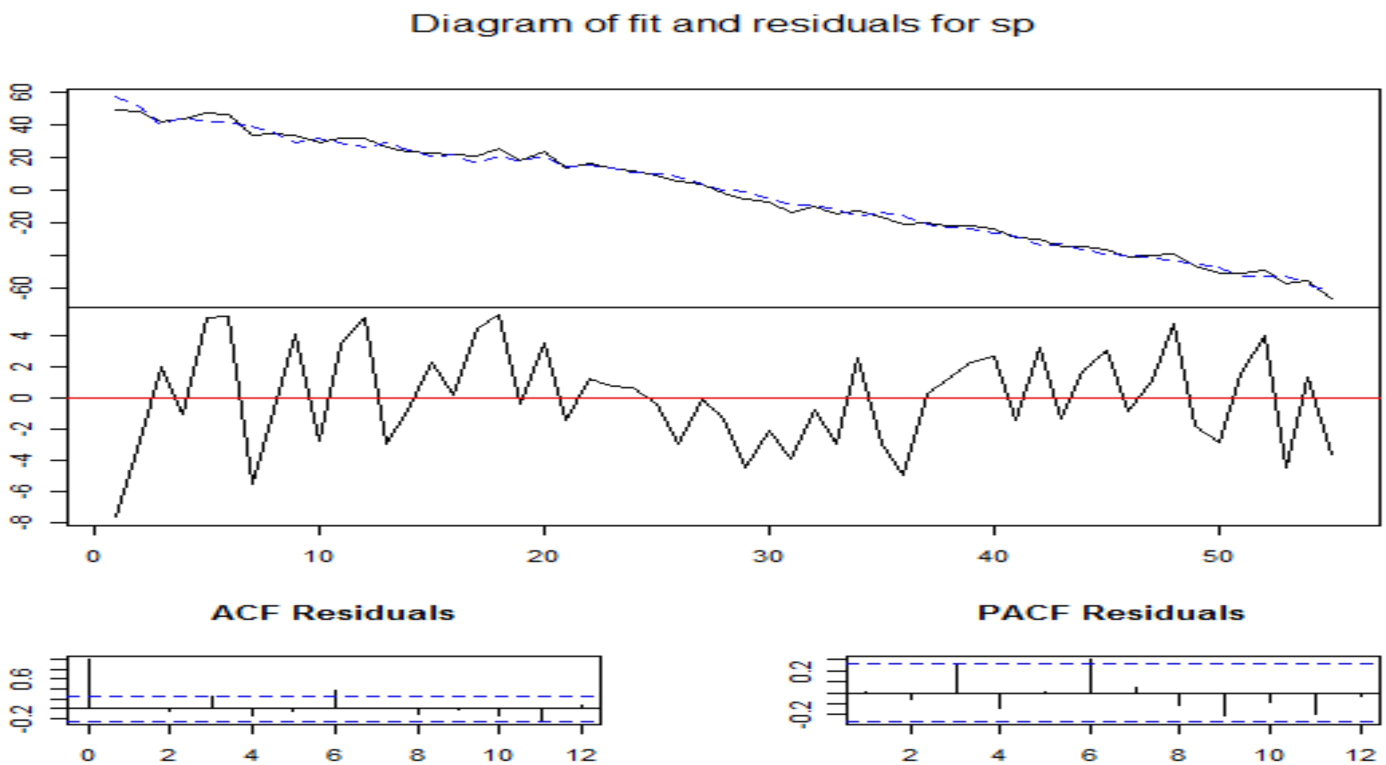


Diagram of fit, residuals, ACF and PACF of residuals for Spain

Projecting Mortality

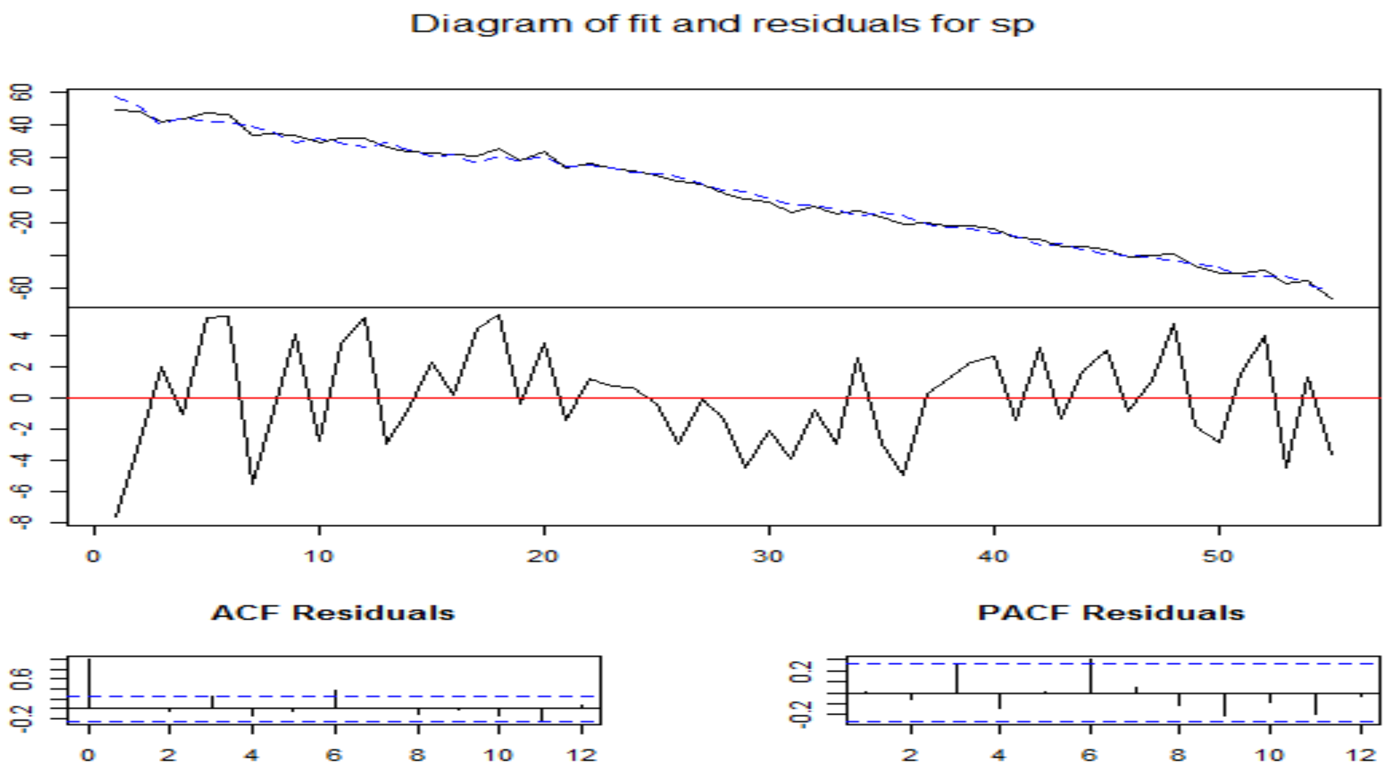


Diagram of fit, residuals, ACF and PACF of residuals for France

Projecting Mortality

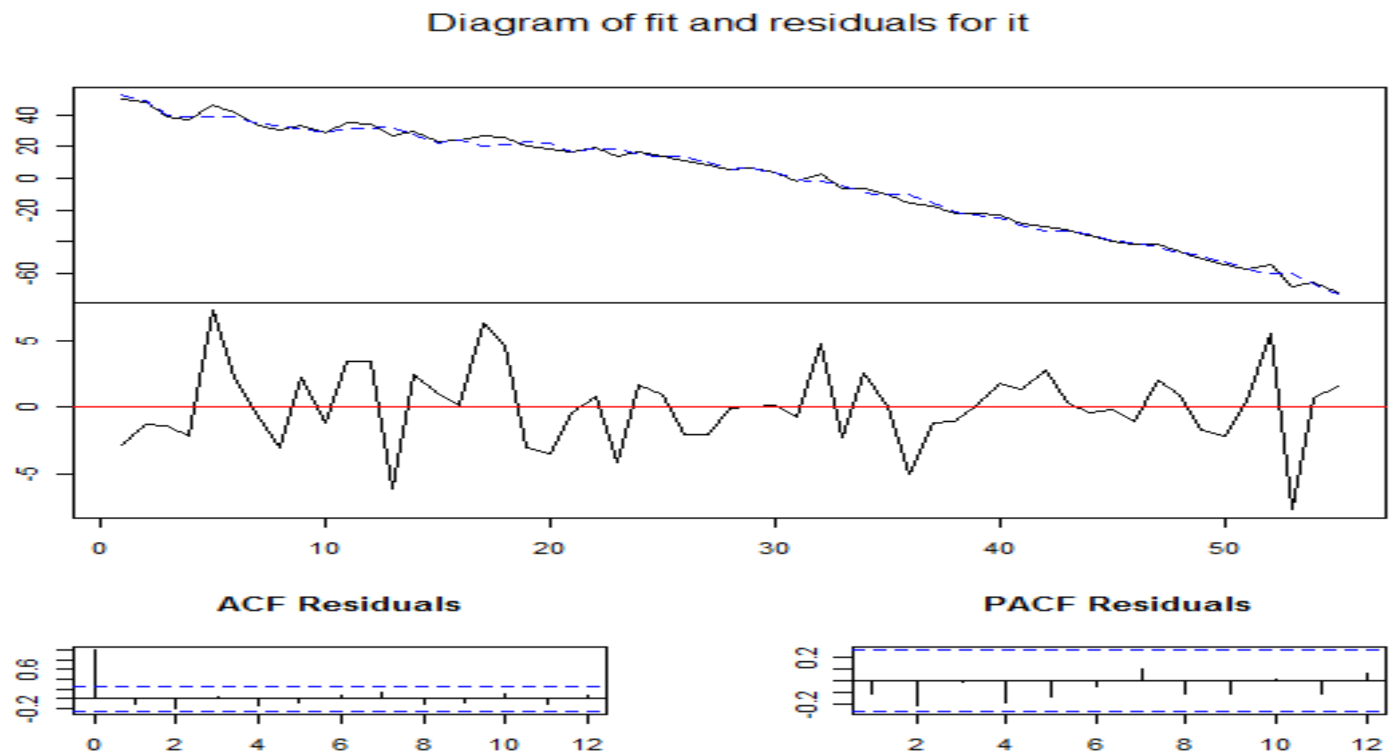
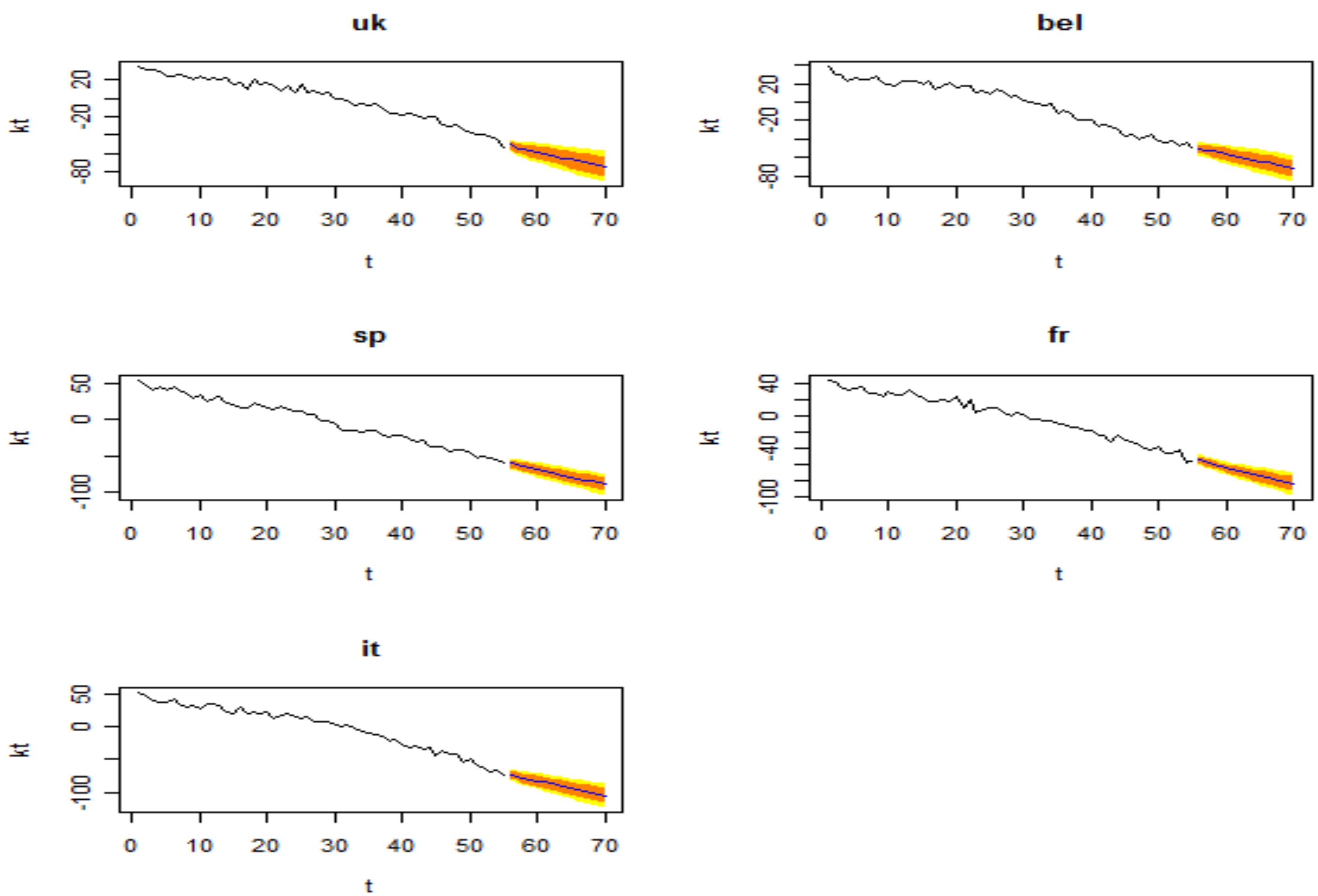


Diagram of fit, residuals, ACF and PACF of residuals for Italy

Projecting Mortality



Projecting Mortality

UK					
h	**Mean**	**80%**		**95%**	
1	-50.87365	-55.18137	-46.56592	-57.46174	-44.28555
2	-55.19162	-59.84698	-50.53625	-62.31138	-48.07185
3	-55.13368	-60.82036	-49.44701	-63.83070	-46.43666
4	-57.65851	-63.82319	-51.49382	-67.08658	-48.23044
5	-58.65893	-65.47290	-51.84496	-69.08000	-48.23786
6	-60.55909	-67.85024	-53.26794	-71.70995	-49.40823
7	-61.92820	-69.73011	-54.12630	-73.86019	-49.99622
8	-63.61075	-71.85655	-55.36495	-76.22161	-50.99989
9	-65.10830	-73.79504	-56.42157	-78.39352	-51.82309
10	-66.71505	-75.81023	-57.61987	-80.62492	-52.80517
11	-68.25734	-77.74970	-58.76498	-82.77465	-53.74003
12	-69.83767	-79.70766	-59.96769	-84.93252	-54.74283
13	-71.39556	-81.63129	-61.15983	-87.04976	-55.74136
14	-72.96669	-83.55438	-62.37901	-89.15916	-56.77422
15	-74.53000	-85.45898	-63.60103	-91.24443	-57.81558

Projecting Mortality

BEL					
h	Mean	80%		95%	
1	-50.16977	-54.76127	-45.57827	-57.19186	-43.14768
2	-51.71650	-56.83237	-46.60064	-59.54054	-43.89247
3	-53.26324	-58.85450	-47.67197	-61.81434	-44.71213
4	-54.80997	-60.83927	-48.78067	-64.03098	-45.58895
5	-56.35670	-62.79430	-49.91911	-66.20215	-46.51125
6	-57.90343	-64.72493	-51.08194	-68.33601	-47.47086
7	-59.45017	-66.63508	-52.26525	-70.43854	-48.46179
8	-60.99690	-68.52772	-53.46608	-72.51429	-49.47951
9	-62.54363	-70.40514	-54.68212	-74.56678	-50.52049
10	-64.09037	-72.26921	-55.91152	-76.59883	-51.58190
11	-65.63710	-74.12142	-57.15277	-78.61275	-52.66145
12	-67.18383	-75.96301	-58.40466	-80.61042	-53.75724
13	-68.73056	-77.79501	-59.66612	-82.59343	-54.86770
14	-70.27730	-79.61830	-60.93630	-84.56313	-55.99147
15	-71.82403	-81.43363	-62.21443	-86.52065	-57.12741

Projecting Mortality

SP					
h	Mean	80%		95%	
1	-61.06797	-65.80709	-56.32885	-68.31583	-53.82011
2	-63.12137	-68.33701	-57.90573	-71.09800	-55.14474
3	-65.17478	-70.82690	-59.52265	-73.81896	-56.53059
4	-67.22818	-73.28542	-61.17094	-76.49193	-57.96443
5	-69.28158	-75.71850	-62.84467	-79.12599	-59.43718
6	-71.33499	-78.13039	-64.53959	-81.72766	-60.94232
7	-73.38839	-80.52430	-66.25249	-84.30182	-62.47497
8	-75.44180	-82.90268	-67.98092	-86.85223	-64.03136
9	-77.49520	-85.26749	-69.72292	-89.38189	-65.60852
10	-79.54861	-87.62029	-71.47693	-91.89318	-67.20404
11	-81.60201	-89.96238	-73.24165	-94.38808	-68.81594
12	-83.65542	-92.29482	-75.01601	-96.86825	-70.44259
13	-85.70882	-94.61853	-76.79911	-99.33505	-72.08259
14	-87.76222	-96.93428	-78.59017	-101.78967	-73.73478
15	-89.81563	-99.24273	-80.38853	-104.23314	-75.39812

Projecting Mortality

FR					
h	**Mean**	**80%**		**95%**	
1	-54.40954	-59.09458	-49.72451	-61.57469	-47.24440
2	-57.53012	-62.21537	-52.84486	-64.69560	-50.36463
3	-60.27504	-65.02405	-55.52603	-67.53802	-53.01205
4	-62.75654	-67.67673	-57.83634	-70.28133	-55.23175
5	-65.05331	-70.24118	-59.86544	-72.98747	-57.11915
6	-67.22054	-72.74407	-61.69700	-75.66805	-58.77302
7	-69.29692	-75.19643	-63.39741	-78.31944	-60.27440
8	-71.30960	-77.60455	-65.01465	-80.93690	-61.68231
9	-73.27761	-79.97363	-66.58159	-83.51829	-63.03693
10	-75.21429	-82.30852	-68.12006	-86.06397	-64.36461
11	-77.12900	-84.61372	-69.64429	-88.57589	-65.68212
12	-79.02831	-86.89325	-71.16336	-91.05670	-66.99991
13	-80.91681	-89.15061	-72.68301	-93.50932	-68.32430
14	-82.79774	-91.38874	-74.20673	-95.93655	-69.65893
15	-84.67335	-93.61013	-75.73658	-98.34097	-71.00573

Projecting Mortality

IT					
h	**Mean**	**80%**		**95%**	
1	-73.52702	-79.08849	-67.96555	-82.03255	-65.02148
2	-75.76981	-81.99289	-69.54672	-85.28720	-66.25242
3	-78.01260	-84.83342	-71.19178	-88.44415	-67.58105
4	-80.25539	-87.62563	-72.88515	-91.52720	-68.98358
5	-82.49818	-90.37963	-74.61673	-94.55182	-70.44454
6	-84.74097	-93.10243	-76.37950	-97.52872	-71.95321
7	-86.98376	-95.79914	-78.16838	-100.46572	-73.50180
8	-89.22655	-98.47359	-79.97951	-103.36867	-75.08442
9	-91.46934	-101.12877	-81.80991	-106.24216	-76.69651
10	-93.71213	-103.76705	-83.65721	-109.08980	-78.33445
11	-95.95492	-106.39035	-85.51948	-111.91453	-79.99530
12	-98.19771	-109.00025	-87.39516	-114.71878	-81.67663
13	-100.44050	-111.59809	-89.28290	-117.50456	-83.37643
14	-102.68329	-114.18497	-91.18160	-120.27359	-85.09298
15	-104.92608	-116.76185	-93.09030	-123.02733	-86.82482

Projecting Mortality

IT					
h	**Mean**	**80%**		**95%**	
1	-73.52702	-79.08849	-67.96555	-82.03255	-65.02148
2	-75.76981	-81.99289	-69.54672	-85.28720	-66.25242
3	-78.01260	-84.83342	-71.19178	-88.44415	-67.58105
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15	-104.92608	-116.76185	-93.09030	-123.02733	-86.82482

Measuring Dependence Structures

Multi-population Modeling is useful for

- management of the longevity basis risk
- longevity swap reinsurance

It involves the study of the dependence for having good estimators.

We develop a specific methodology for country – based projections in order take into account the dependence structure

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