Longevity Risk in Last Survivor Annuities

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Outline

Introduction

Research Motivations

- Remarkable mortality improvement in the last century
- Inadequate allowance for longevity risk in the annuity markets
- Dependence between joint-live mortality has been well recognized.
- Little research in joint-life longevity risk.

Aim of This Research

Examine how the annuity market accounts for future improvements in mortality rates and life expectancy, especially in the case of last survivor annuities.

- Incorporate stochastic mortality processes into the semi-Markov joint-life mortality model studied by Ji et al. (2011)
- Demonstrate joint-life longevity risk.

A Semi-Markov Joint-life Longevity Risk Model

Model Specification

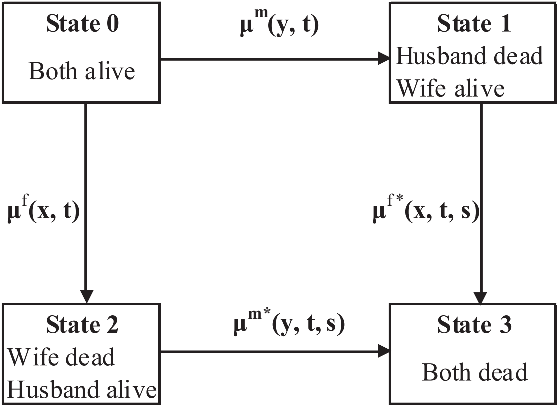

Figure: **A Semi-Markov Multi-state Model for Joint Life Longevity Risk**

Semi-Markov Property in the model

We assume that force of mortality in the widowed status is exponentially proportional to the corresponding mortality in the married status, due to decaying bereavement effect.

$$\mu^{f*}(x,t,s) = (1 + a^f e^{-k^f s})\mu^f(x,t); \tag{1}$$

and,

$$\mu^{m*}(y,t,s) = (1 + a^m e^{-k^m s})\mu^m(y,t), \tag{2}$$

where s is the elapsed time after bereavement; $\mu^f(x,t)$ and $\mu^m(y,t)$ represent the force of mortality for married women and men, respectively, at age $x(y)$ and time-t; a^m, a^f, k^m, and k^f are the semi-Markov parameters.

Stochastic Transition Intensities

The forces of mortality $\mu^f(x,t)$ and $\mu^m(y,t)$ are expressed as

$$\mu^f(x,t) = \xi_t^f \exp\{\xi_t^f(x - \gamma_t^f)\}, \tag{3}$$

and

$$\mu^m(y,t) = \xi_t^m \exp\{\xi_t^m(y - \gamma_t^m)\}, \tag{4}$$

where ξ_t^f and ξ_t^m are Gompertz aging parameters at time t for females and males respectively; γ_t^f and $\gamma_t^m$are the Gompertz mode parameters at time t.

Historic Gompertz Parameters in the US

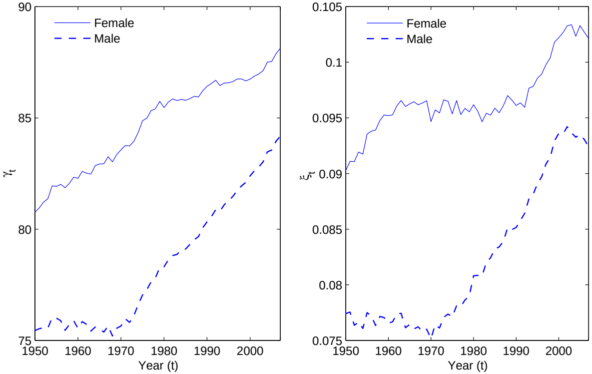

Figure: Estimated values of ξ_t and γ_t for the US historic mortality data for ages 60 to 109 from 1950 to 2007

Historic Gompertz Parameters in the UK

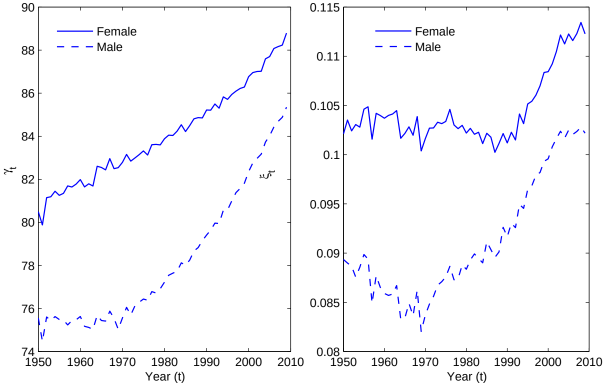

Figure: Estimated values of ξ_t and γ_t for the England and Wales population mortality data for ages 60 to 109 from 1950 to 2009

Project Female and Male Mortality Separately

The vector processes $z^f(t) = [\xi_t^f, \gamma_t^f]'$ and $z^m(t) = [\xi_t^m, \gamma_t^m]'$ are expressed as

$$dz^f(t) = \nu^f \ dt + \sigma^f \ dW(t), \tag{5}$$

and

$$dz^m(t) = \nu^m \ dt + \sigma^m \ dW(t), \tag{6}$$

where $W(t)$ is the 2-dimensional standard Brownian motion, ν^f and ν^m are the drift vector for females and males respectively, and

$$V^f = \sigma^f \sigma^{f*} = \begin{pmatrix} V_{11}^f & V_{12}^f \\ V_{21}^f & V_{22}^f \end{pmatrix} \quad \text{and} \quad V^f = \sigma^m \sigma^{m*} = \begin{pmatrix} V_{11}^m & V_{12}^m \\ V_{21}^m & V_{22}^m \end{pmatrix},$$

are the 2×2 - dimensional diffusion matrix for females and males respectively.

Project Female and Male Mortality Consistently

The vector processes $z(t) = [\xi_t^f, \gamma_t^f, \xi_t^m, \gamma_t^m]'$ are expressed as

$$dz(t) = \nu \; dt + \sigma \; dW(t), \tag{7}$$

where $W(t)$ is the 4-dimensional standard Brownian motion, ν is the drift vector, and

$$V = \sigma\sigma^* = \begin{pmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{pmatrix}$$

are the 4×4 - dimensional diffusion matrix.

Investigating Joint-life Longevity Risk

Impact of Joint-life Longevity risk, the US Example

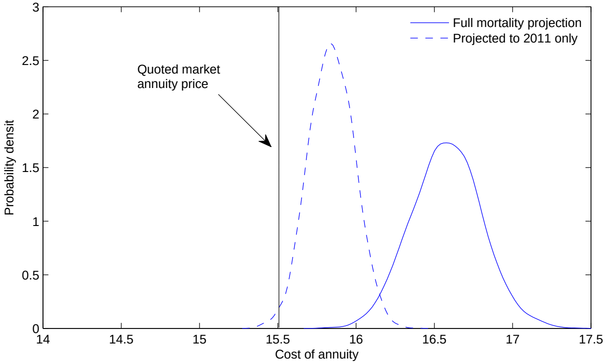

Figure: **Distribution of the cost of last survivor annuity in 2011 with and without allowance for mortality improvement for the US, $x = 65$ and $y = 65$, fisk-free interest rate 4.25%**

Impact of Joint-life Longevity risk, the US Example

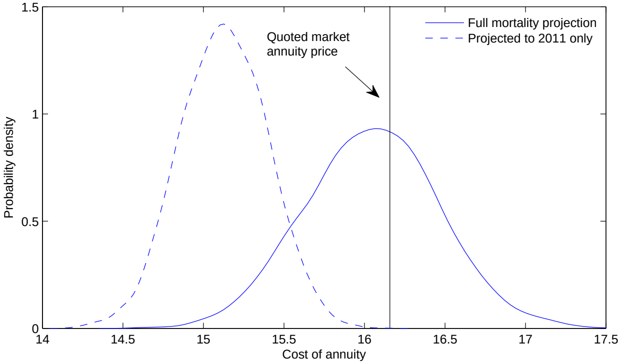

Figure: **Distribution of the cost of last survivor annuity in 2011 with and without allowance for mortality improvement for the UK, $x = 65$ and $y = 65$, risk-free interest rate 4.25%**

Pricing Longevity Risk

Assume that market players act in an equilibrium setting and this equilibrium selects a market-consistent risk-neutral measure, in which

$$
\begin{aligned}
z(t+1) &= z(t) + \nu + \sigma\Big(\tilde{Z}(t+1) + \eta\Big) \\
&= z(t) + \tilde{\nu} + \sigma \tilde{Z}(t+1),
\end{aligned}
$$

where $\tilde{\nu} = \nu + \sigma\eta$, and $\eta = [\eta^f, \eta^f, \eta^m, \eta^m]'$ assuming the market prices of longevity risk associated with the stochastic processes for two Gompertz parameters are equal. $\tilde{Z}(t+1)$ is a standard two dimensional normal random variable under Q-measure.

Pricing Longevity Risk

Define $\delta(t)$ to be the risk-free interest rate at time t. In the risk neutral measure Q,

$$P(s,\tau) = \mathsf{E}_Q\Big[\exp\Big(-\int_s^\tau \delta(t)dt\Big)|\mathcal{F}_s\Big],$$

where $\{\mathcal{F}_s, s = 0, 1, \dots\}$ is the natural filtration for the process.

In the risk neutral measure Q, assuming the longevity and interest rate risks are independent, the price of last survivor immediate annuity with unit annual payment issued to a y-year old husband and x-year old wife in year 2011 is

$$\ddot{a}_{\overline{xy}}^{market}(2011) = 1 + \sum_{\tau \geq 1} P(0,\tau)\mathsf{E}_{Q(\eta^f,\eta^m)}\big[{}_\tau p_{\overline{xy}}|\mathcal{G}_0\big], \qquad (8)$$

The Market Prices of Longevity Risk for Joint Lives

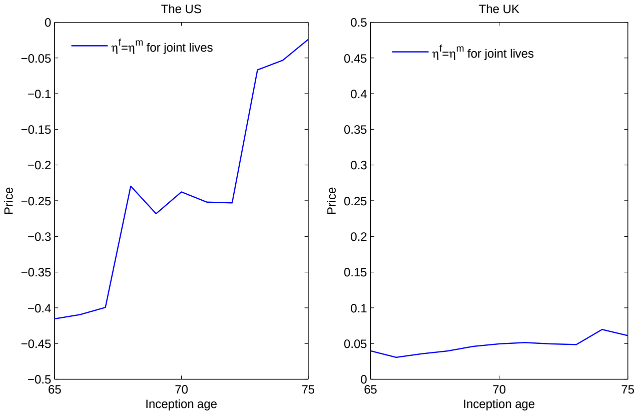

Figure: **The estimated market prices of longevity risk in last survivor annuities, in the US market (Left) and the UK market (Right)**

Joint-life VS Single-life

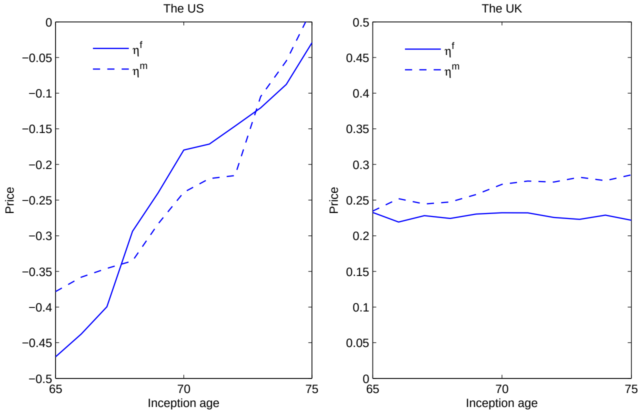

Figure: **The estimated market prices of longevity risk in single-life annuities, in the US market (Left) and the UK (Right) market**

5. Conclusions

Concluding Remarks

- A joint-life mortality model is needed for pricing last survivor annuities;
- According to the proposed stochastic mortality model, the US annuity market appears to underprice last survivor annuities; while the underpricing problem is lesser in the UK market.
- Pricing and risk management of joint-life longevity risks need more work.

Future work

- Joint-life longevity risk based on semi-Markov setting VS joint-life longevity risk with a copula method.
- Explore two-population mortality forecasting model for females and males.
- Explore high age mortality improvement model.

Thank You !

Q & A