Multi-population mortality models

Fitting, Forecasting, Comparison and Applications

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Outline

- Contribution:
  Comparing a new multipopulation model
  Kleinow (2015) $\leftrightarrow$ Li and Lee (2005)
  and applications.
- Data
- Model fit and robustness
- Correlations between populations
- Financial risk management applications

Data specifics

Six populations considered:

		Exposure to risk at the age of 60 in 2010	
i	Country	Male	Female
1	Austria	47023	49526
2	Belgium	65344	66434
3	Czech Republic	71575	71575
4	Denmark	34420	35132
5	Sweden	59759	59742
6	Switzerland	46527	47078

- Thirty ages 60 to 89
- Fifty calendar years 1961 to 2010
- Closely related countries
- Similar size and features

Multi-population mortality models

- **Model 0:** (Li and Lee, 2005)
  Country i, Age x, Year t.

  $$\log m(x,t,i) = \alpha(x,i) + B(x)K(t) + \beta(x,i)\kappa(t,i)$$

  Mixture of common parameters, and country-specific parameters.

- Models 1 and 2: special cases of Model 0.
- **Model 3:** (Kleinow, 2015)

  $$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\kappa^1(t,i) + \beta^1(x)\kappa^1(t,i)$$

Multi-population mortality models

Model 0: $\log m(x,t,i) = \alpha(x,i) + B(x)K(t) + \beta(x,i)\kappa(t,i)$

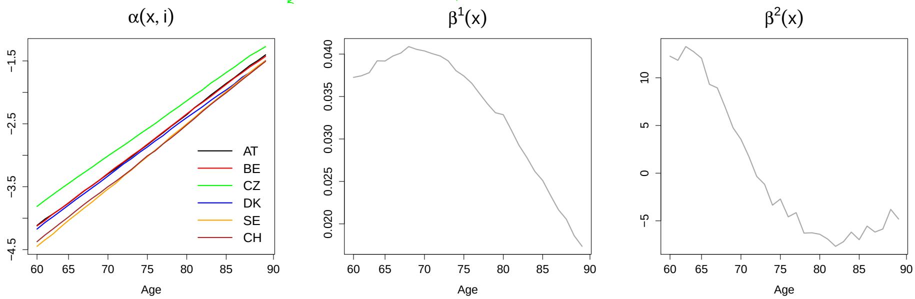
Model 3: $\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\kappa^1(t,i) + \beta^1(x)\kappa^1(t,i)$

- Parameter estimation: maximum likelihood (Conditional Poisson)
- Model selection using Bayes Information Criterion (BIC)

	BIC	Rank
Model 0	100043.08	(2)
Model 1	101,628.38	(4)
Model 2	101,525.12	(3)
Model 3	99,805.38	(1)

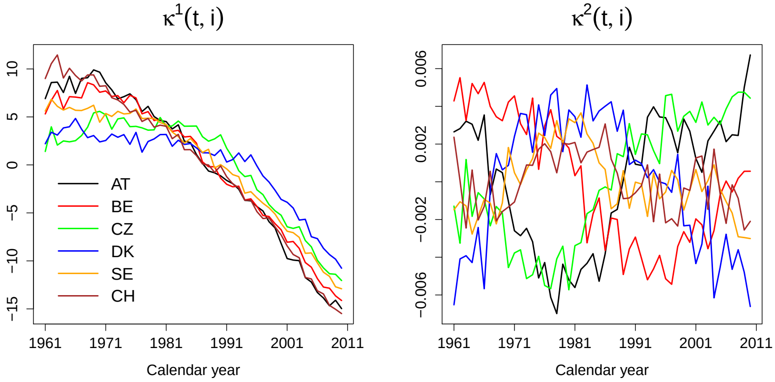
Estimated age effects in Model 3

$$\log m(x,t,i) = \boxed{\alpha(x,i)} + \boxed{\beta^1(x)}\kappa^1(t,i) + \boxed{\beta^2(x)}\kappa^2(t,i)$$



Estimated period effects in Model 3

$$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\boxed{\kappa^1(t,i)} + \beta^2(x)\boxed{\kappa^2(t,i)}$$

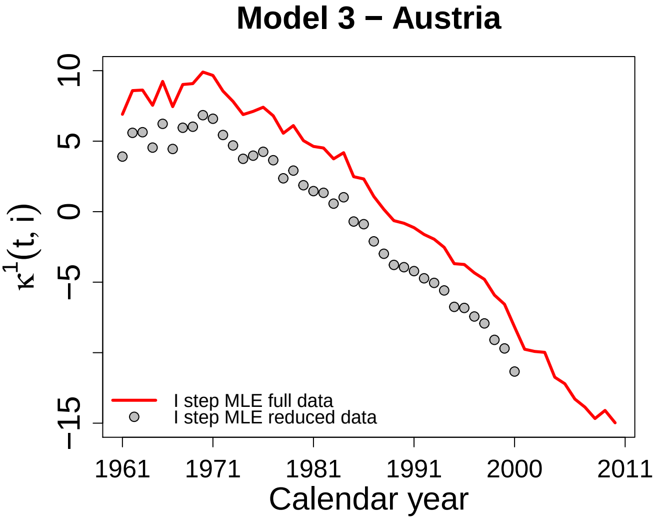


Robustness of the estimated parameters

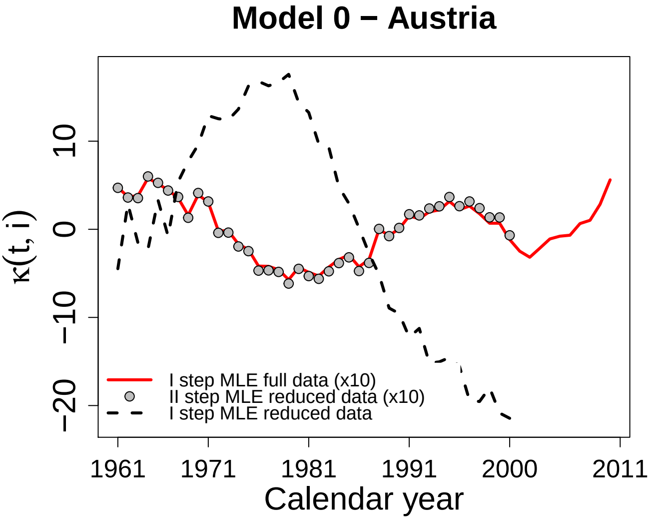
- Compare parameter estimates for two time periods
  1961-2010 versus 1961-2000
- Should get similar estimates for age and period effects
- For Model 0: Two approaches to estimation:
  - One step MLE
  - Two step MLE:
    1. Fit the Lee-Carter model to the combined data
    2. Optimise over all country-specific parameters

Robustness of the estimated parameters: Model 3

e.g. Austria $\kappa^1(t, i)$



e.g. Austria $\kappa(t, i)$



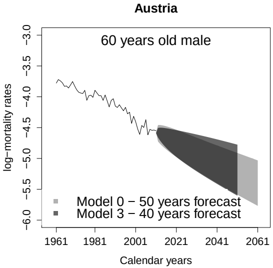
Robustness observations

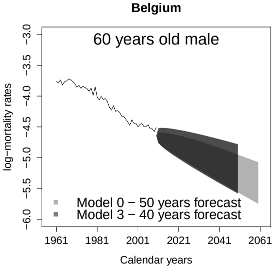
- 1961-2000 $\Rightarrow$ Li and Lee model has robustness problems
- One and two-step procedures lead to qualitatively different estimates for period effects.
- Log likelihood function is not concave
- Leading to problems with multiple maxima
- Different initial values $\Rightarrow$ potentially different parameter estimates
- Model 3: robust (so far!)
- Robustness strengthens conclusion that Model 3 is the better model.

Forecasting joint mortality rates

Simulations:

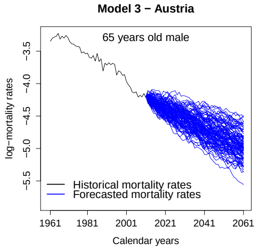
- Model 0:
  - $K(t)$: univariate random walk
  - $\kappa(t,i)$: Vector AR(1)
- Model 3:
  - $\kappa^1(t,i)$: multivariate random walk
  - $\kappa^2(t,i)$: vector AR(1)

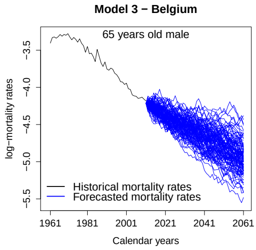




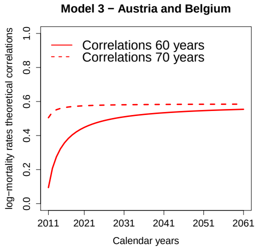
$$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\kappa^1(t,i) + \beta^2(x)\kappa^2(t,i)$$

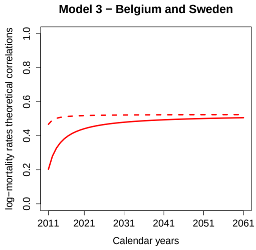
Joint forecasts





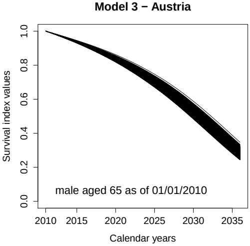
Theoretical correlations

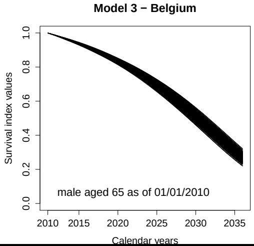




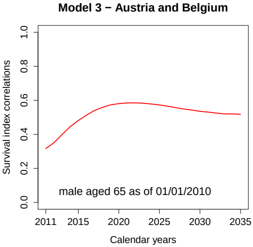
$$S(x, T, i) = \exp\left[-m(x, 1, i) - m(x + 1, 2, i) - \ldots - m(x + T - 1, T, i)\right]$$

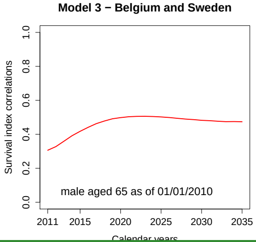
Survival index values





Empirical correlations

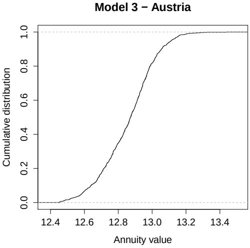


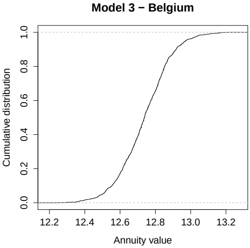


Multi-population mortality models applications

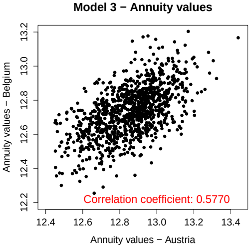
$$A(x,i) = S(x,1,i)v + S(x,2,i)v^2 + \ldots$$

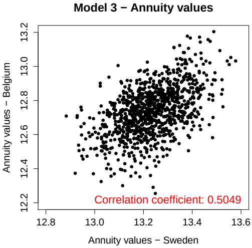
Annuity ECDF





Annuity scatter plots





Multi-population mortality models applications

Other applications of multi-population mortality models include

- Improved risk reserving (e.g. NL, BEL)
- Ability to assess benefits of diversification across countries
- SCR values: single and multi country
- Model 3: potential application to sub-population data
- Hedging portfolio risk
  - Q-forward contracts
  - S-forward contracts

http://www.macs.hw.ac.uk/∼andrewc/papers/Enchev2015.pdf (Scandinavian Actuarial Journal)

Paper link

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Thank You!

Questions?