Optimal Investment and Net Contribution Rules for a Public Investment Fund

(comments welcome)

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Motivation

- Governments everywhere have amassed large pools of resources held and managed for the public sector, called Public Investment Funds (e.g., SWF, Pension Reserve Funds, Currency Stabilization Funds, etc.)
- The literature so far has place its attention on the considerations that should surround the definition of a transparent investment policy
- Our research project focus instead on the quantitative considerations that should guide the strategic asset allocation of a PIF, in view of the nature of the embedded liabilities that they must finance in the future

Paper overview

- This paper
  1. proposes and solves a dynamic, continuous-time ALM problem, subject to convex portfolio constraints
  2. uses the model to assess the optimality of the current contribution and investment rules of the Chilean Pension Reserve Fund (PRF).
- The paper´s main contributions
  1. on the methodology front, it shows how to compute near-optimal policies based on the analytical characterization of the optimal (yet, numerically unfeasible) ones.
  2. on the application front, it shows how to use reduced-from econometrics models to describe the exogenous inflows/outflows processes, based on a full blown simulated structural (economic/statistical) model for the Chilean economy.
  3. on the empirical front, it computes the welfare loss due to the unhedged longevity risk implicit in the liabilities faced by the Chilean PRF.

Related literature

- dynamic investment with liquidity constraints
  - El Karoui & Jeanblanc (1998, FS), and Detemple & Serrat (2003, RFS) solve a liquidity constrained consumption-investment problem with analytical methods
- optimal dynamic investment in an ALM problem
  - Detemple & Rindisbacher (2008, IME) solve a dynamic ALM problem in complete mkts
- near-optimal policies
  - Haugh et al. (2006, OR) propose an approximation method of the optimal investment rule that relies on an approximation of the value function
  - Bick et al. (2013, MS) propose an approximation method for the process (identified by Cvitanic & Karatzas, 1992, AAP) that keeps the optimal investment within the confinements of the constraint set

Model overview

- The model has two components:
  1. a financial mkt comprised of one riskless asset, and $d \geq 1$ risky assets that follow general dynamics (i.e., a stochastic investment opportunity set), based on Brownian uncertainty
     - financial frictions (e.g., mkt incompleteness, portfolio restrictions) are introduced in the form of convex portfolio constraints
     - the assumption of Brownian uncertainty is made for simplicity, but it could be relaxed to include, for example, Poison jumps
  2. a benevolent social planner whose utility depends upon the investment rule and the flow of net contributions made to, or taken from, a Public Investment Fund (PIF)
     - The dynamics of the PIF is subject to al exogenous (stochastic) stream of inflows/outflows, making the asset allocation problem of the ALM type
     - longevity risk is embedded in the liabilities of the PIF

The uncertainty

- The uncertainty of the environment is summarized by the vector $Y_t \in \mathbb{R}^d$ $(d \geq 1)$, whose dynamics is dictated by a diffusion vector process (where $W_t \in \mathbb{R}^d$ is a d-dimensional BM process)

$$\mathrm{d}Y_t = \mu_Y(t, Y_t)\mathrm{d}t + \sigma_Y(t, Y)\mathrm{d}W_t, \quad Y_0 \in \mathbb{R}^d \text{ given,}$$

where $\mu_Y \in \mathbb{R}^d$ and $\sigma_Y \in \mathbb{R}^{d \times d}$ satisfy regular conditions (i.e., $Y \equiv (Y_t)_{t \geq 0}$ is an integrable vector process).

The financial market

- The PIF's assets can be held in cash for a riskless return of r_t, between t and $t+\mathrm{d}t$, or be invested in a set of d (non-redundant) risky asset classes whose return (between t and $t+\mathrm{d}t$) evolves according to:

$$\mathrm{d}S_{i,t} + \delta_t \mathrm{d}t = I_{S,t}(\mu_S(t, Y_t)\mathrm{d}t + \sigma_S(t, Y_t)\mathrm{d}W_t),\ S_0 \in \mathbb{R}^d \text{ given},$$

  where $I_S \in \mathbb{R}^{d\times d}$ is a diagonal matrix, with $S_t \in \mathbb{R}^d$ on its main diagonal, δ and $\mu_S \in \mathbb{R}^d$ are the dividend yield and the expected return vectors of the risky assets, and $\sigma_S \in \mathbb{R}^{d\times d}$ is its (a.e. invertible) volatility matrix.

Exogenous contribution/withdrawal processes

- The exogenous stream of inflow/outflows to/from the PIF are represented by two (reduced-form) processes, denoted by e and l, whose dynamics is dictated by:

$$\mathrm{d}a_t/a_t = \mu_a(t, Y_t)\mathrm{d}t + \sigma_a(t, Y_t)\mathrm{d}W_t, \ \ a_0 \in \mathbb{R}_{++} \text{ given},$$

where $a \in \{e, l\}$, $\mu_a \in \mathbb{R}$, and $\sigma_a \in \mathbb{R}^d$ satisfy regular conditions.

The dynamics of the PIF

- The value process of the PIF, $X \equiv (X_t)_{t \geq 0}$ evolves according to:

$$\begin{cases} \mathrm{d}X_t = X_t(1 - \pi_t'\mathbf{1})r_t\mathrm{d}t + X_t\pi_t'(\mu_{S,t}\mathrm{d}t + \sigma_{S,t}\mathrm{d}W_t) \\ \qquad\qquad + (e_t - l_t + c_t)\mathrm{d}t; \quad X_0 \geq 0; \\ c_t \in [-C_t, +\infty), \pi_t \in \mathcal{K} \subseteq \mathbb{R}^d, X_t \geq 0, \forall t \geq 0; \end{cases}$$

where

  - $\pi_t \in \mathcal{K} \subseteq \mathbb{R}^d$ is vector of proportions of the PIF invested in the set risky assets,
  - $\mathcal{K} \subseteq \mathbb{R}^d$ is a convex set of constraints (long only + incomplete mkts),
  - $(')$ denotes the transposition of vector and matrices, $\mathbf{1} \equiv (1, \ldots, 1) \in \mathbb{R}^d$,
  - $c_t \geq -C_t$ is the injection/withdrawal process to/from the PIF, and
  - $C_t \geq 0$ is an exogenous (wide economy consumption process) defined below

The preferences of the social planner

- The preferences of the benevolent social planner are represented by a standard expected utility function,

$$U(c) = \mathbf{E}\left[\int_0^{+\infty} u(c_t + C_t, t)\mathrm{d}t\right]$$

where $u(\cdot, t)$ is a vN-M utility function, and the process $C \equiv (C_t)_{t\geq 0}$ accounts for the exogenous (or un-modeled) sources of consumption available to the representative consumer in the economy. The dynamics of C also follows a diffusion process.

The optimal policy

- Using the convex duality approach of Cvitanic & Karatzas (1992, AAP), and the martingale representation of Detemple & Serrat (1998 WP, 2003 RFS) we obtain:

$$c_t^* = I(y\gamma_t \xi_t \eta_t, t) - C_t, \quad \text{and} \quad \pi_t^* = (\sigma_t')^{-1} \mathscr{D}_t \left(X_t^* \right),$$

where $I(\cdot, t)$ is the inverse marginal utility, $\gamma \equiv (\gamma_t)_{t \geq 0}$ is the *liquidity multiplier* process, $\xi \equiv (\xi_t)_{t \geq 0}$ is the stochastic discount factor (SDF) that prevails in an economy without portfolio restrictions (i.e., $\mathcal{K} = \mathbb{R}^d$), $\eta \equiv (\eta_t)_{t \geq 0}$ is an (investor specific) adjustment to the SDF, which serves the purpose of enforcing the portfolio constraints contained in the set $\mathcal{K}$, $y > 0$ is a constant,

$$X_t^* = \mathbf{E}_t \left[\int_t^{+\infty} \xi_{t,s} \eta_{t,s} \left(I(y\gamma_s \xi_s \eta_s, s) - C_s - e_s + l_s \right) \mathrm{d}s \right],$$

and $\mathscr{D}_t(\cdot)$ is the Malliavin derivative operator.

The "near-optimal" policy

- Unfortunately, the optimal policy cannot be obtained in general (even numerically), as the processes (γ, η) involved a hard numerical problem.
- As an alternative, we use "near-optimal" rules that use approximations of (γ, η). In particular, we approximate:
  - γ_t by $\gamma_t^0 = \inf_{v \in 0,t} m_v$, where m_v is a known function of $(C_t, e_t, l_t, \xi_t, \eta_t)$.
  - η_t by η_t^0, where η_t^0 is a known function of λ_t^0, which solves

$$\lambda_t^0 \in \arg\min \left\| \pi_t^0 - \pi_t^{u,*} \right\|^2 \in \mathbb{R}^d,$$

where $\pi^{u,*}$ is the optimal *unconstrained* investment rule, and π^0 is a "local" approximation of the optimal *constrained* investment rule.

The Chilean Pension Reserve Fund (PRF)

- We applied the model to assess the optimality of the investment and contribution rules of the Chilean PRF.
- The Chilean PRF was created in 2006, it currently has nearly USD 7bn in AUM, and so far has only receive injections of funds.
- It current injection policy follows an (S,s)-policy between 0.2% and 0.5% of domestic GDP, and its current strategic asset allocation policy is restricted to a subset of international assets.

Calibration of the model

- Financial market (μ, r, and σ)
  - we considered five asset classes (including an unspanned longevity risk asset), and relied on an international CAPM model for μ (with a conservative assumption for the world's equity premium of 3%), a Vasicek specification for r, and a (continuous-time) CCC GARCH (1,1) for σ.
- Injection/Withdrawal processes (e and l)
  - we used the "structural" economic/statistical model of Castañeda et al. (2013, Report) to first simulate the pair (e, l) for the next 50 years, determine the factors Y, and fit reduced-form models for (e, l)
- Exogenous consumption process (C)
  - we model $C_t = (y_C \gamma^0_{C,t} \xi_t \eta^0_{C,t} e^{\beta_C t})^{-1/R}$ as optimal domestic consumption, which comes as a result of an intertemporal consumption-investment problem, provided an exogenous GDP inflow process.