Longevity Risk in the Pension Context

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Agenda

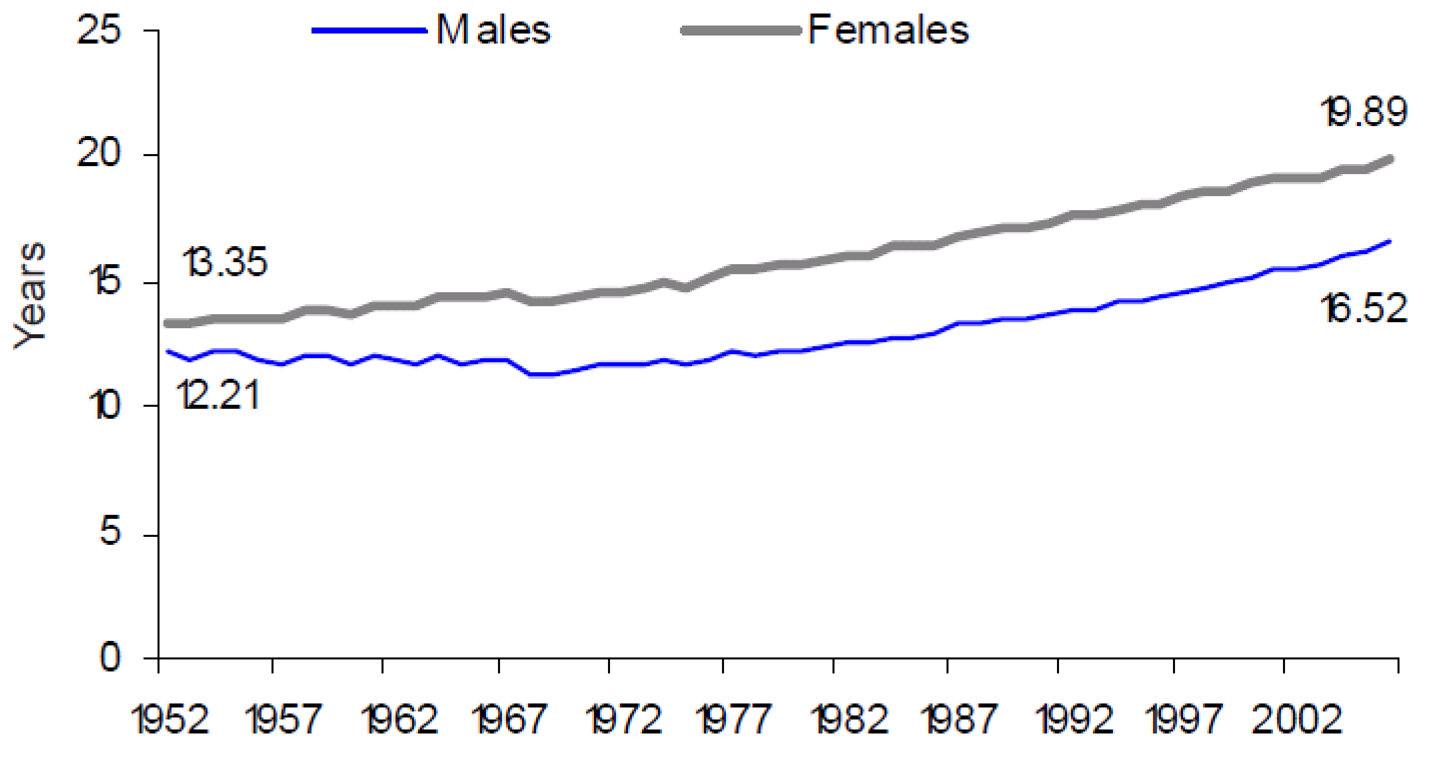
1. Introduction and motivation
2. Stochastic model for mortality rates
3. Quantification of longevity risk
4. Results

Introduction and motivation

1

Life expectancy

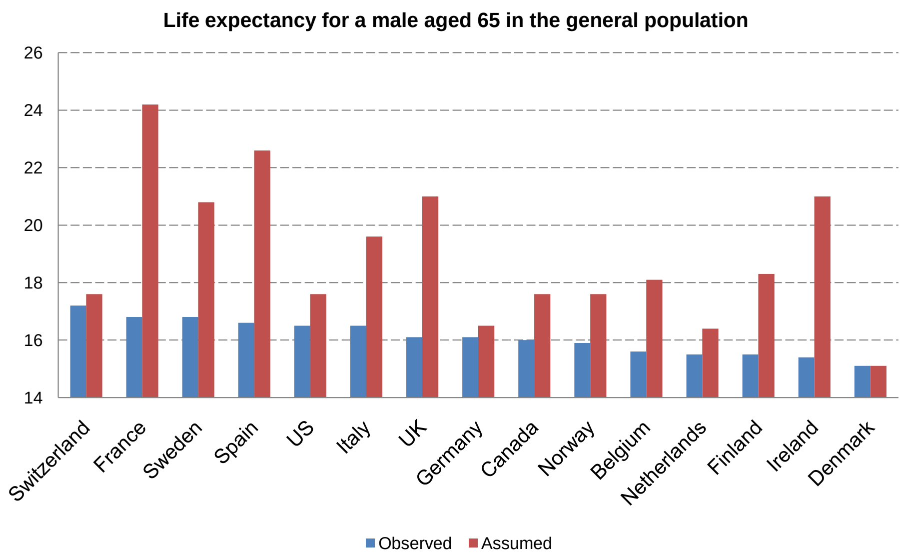
Increasing life expectancy for a 65-year old person in Germany



Source: LifeMetrics - German Longevity Index

Life expectancy
Observations vs. assumptions

- Differences in observed life expectancies across countries
- Large deviations between mortality assumptions in different countries
- Deviations between mortality assumptions and observations within selected countries



Source: Cass Business School (2005)

Motivation

- The positive trend of increasing life expectancy causes several problems for pension plans
- Most of the pension plan sponsors have been focused on interest rate and inflation risk so far
- Increasing attention to longevity risk -
  according to [Loeys et al. 2007] the longevity exposure for DB plans was about 300 bn USD in the USA in 2006
- No consistent methodology for quantification of longevity risk and calculation of the best estimate for the pension plan liabilities
- No consistent framework for the analysis of longevity risk for a pension plan along with interest and inflation risks

Stochastic model for mortality rates

2

Mortality model
Notation and definitions

- Random lifetime of a person aged x at $t = 0$ is modeled as a stopping time $\tau(x)$ of a counting process $N_t(x+t)$ with corresponding mortality intensity $\mu_t(x+t)$

- Introduce two filtrations $\mathbb{G}$ and $\mathbb{F}$, generated by

$$\mathcal{G}_t = \sigma(\mu_s(x+s) : s \leq t), \quad \mathcal{F}_t = \sigma(\mathbb{1}_{\tau(x) \leq s} : s \leq t)$$

- **Definition 1.** ***Survival probability*** *is defined as a probability that a person at the age of $x+t$ at time t survives at least up to time T:*

$$p_t(x+t, T|\mathcal{G}_t) := \mathbb{P}(\tau(x) > T|\mathcal{G}_t \vee \mathcal{F}_t),$$

$p_t(x+t) := p_t(x+t, t+1|\mathcal{G}_t)$ - *is called* ***one-year survival probability***

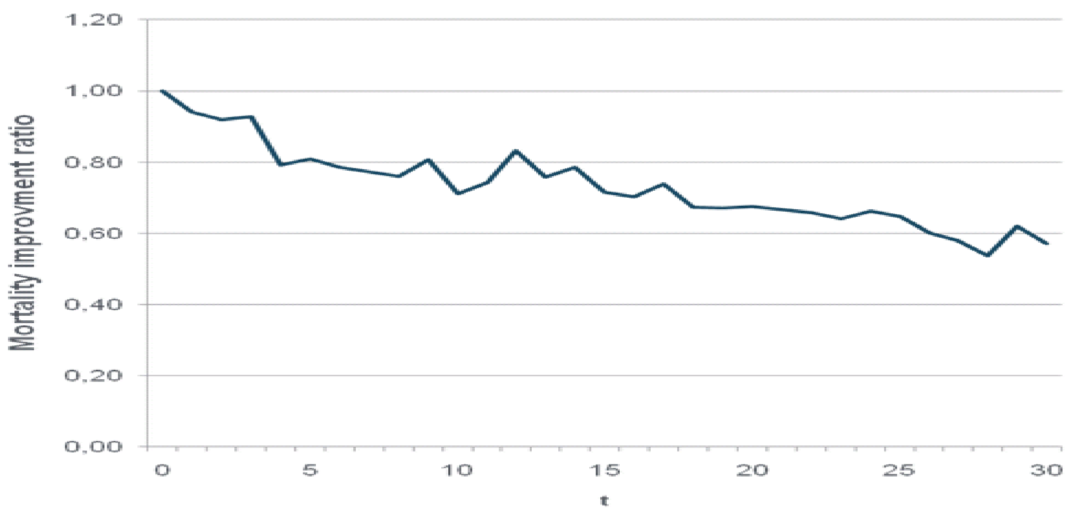
- For the survival probability measured at time t of a person at the age of $x+t$ at time t it holds that

$$p_t(x+t, T|\mathcal{G}_t) = \mathbb{E}\left[e^{-\int_t^T \mu_s(x+s)ds} |\mathcal{G}_t \vee \mathcal{F}_t\right]$$

Mortality model
Mortality improvement ratio

- Compare the mortality intensity at time 0 with mortality intensity at time t
- Introduce **mortality improvement ratio** as

$$\xi_t(x+t) = \frac{\mu_t(x+t)}{\mu_0(x+t)}$$



Mortality improvement ratio of a cohort aged 30 in 1978, $\xi_t(30+t)$

Mortality model

- Following [Dahl et al. 2006] we model ξ_t as an extended CIR process

$$d\xi_t = \delta(e^{-\gamma t} - \xi_t)dt + \sigma\sqrt{\xi_t}dW_t$$

- Initial mortality intensity is described via Gompertz model

$$\mu_0(x+t) = bc^{x+t}$$

  and is calibrated to the current life table

- Future mortality intensity can be calculated as

$$\mu_t(x+t) = \mu_0(x+t) \cdot \xi_t$$

- Survival probabilites can be expressed as

$$p_t(x+t, T|\mathcal{G}_t) = e^{A(t,T) - B(t,T)\mu_t(x+t)},$$

  where $A(t,T)$ and $B(t,T)$ satisfy two ordinary differential equations

Quantification of longevity risk

3

Goals

- Quantification of longevity risk in different DB pension plans for different time horizons
- Comparison of longevity risk with inflation and interest-rate risks

General approach

- Calculate the best estimate of the liabilities, $L^{BE}(t_i)$, at time t_i
- Simulate the underlying risk factors and calculate the actual value of the liabilities, $L^{actual}(t_i)$, at time t_i
- Determine the **overall longevity risk** as (absolute or relative)

$$Q_{99\%}(L^{actual}(t_i)) - L^{BE}(t_i), \quad \frac{Q_{99\%}(L^{actual}(t_i)) - L^{BE}(t_i)}{L^{BE}(t_i)}$$

- Determine the **deviation longevity risk** as (absolute or relative)

$$Q_{99\%}(L^{actual}(t_i)) - Q_{50\%}(L^{actual}(t_i)), \quad \frac{Q_{99\%}(L^{actual}(t_i)) - Q_{50\%}(L^{actual}(t_i))}{Q_{50\%}(L^{actual}(t_i))}$$

Calculation of the best estimate of the liabilities

- We calculate the best estimate of the liabilities at time t_i for a person aged x at time 0 (i.e. aged $x + t_i$ at time t_i) and who retires at time T as:

$$L^{BE}(t_i) = \sum_{t=t_i+1}^{120-x-t_i} B(t, T|\mathcal{I}_0) \cdot d(0, t_i, t) \cdot p_0(x, t_i|\mathcal{G}_0) \cdot p_{t_i}(x + t_i, t|\mathcal{G}_0)$$

- $B(t, T|\mathcal{I}_0)$ is the value of the benefit at time t based on the information available at 0

- $d(0, t_i, t)$ is the forward discount factor, determined at time 0 for the period between t_i and t

Calculation of the actual liabilities

- In each scenario we calculate the actual liabilities at time t_i for a person aged x at time 0 (i.e. aged $x + t_i$ at time t_i) and who retires at time T as:

$$L^{actual}(t_i) = \sum_{t=t_i+1}^{120-x-t_i} B(t, T|\mathcal{I}_{t_i}) \cdot d(t_i, t) \cdot p_0(x, t_i|\mathcal{G}_{t_i}) \cdot p_{t_i}(x + t_i, t|\mathcal{G}_{t_i})$$

- $B(t, T|\mathcal{I}_{t_i})$ is the value of benefit at time t based on the information available at t_i

- $d(t_i, t)$ is the discount factor, determined at time t_i for the period between t_i and t

- To analyze e.g. the interest risk only, we use

$$L^{interest}(t_i) = \sum_{t=t_i+1}^{120-x-t_i} B(t, T|\mathcal{I}_0) \cdot d(t_i, t) \cdot p_0(x, t_i|\mathcal{G}_0) \cdot p_{t_i}(x + t_i, t|\mathcal{G}_0)$$

Calculation of the actual liabilities

1. Simulation of risk factors
   - Inflation and interest rates via Economic Scenario Generator [Zagst et al. 2007]
   - Mortality rates via Dahl and Lee-Carter models
2. Estimation of liabilities with different time horizons ($t = 5$, $t = 15$) for the following cases:
   - Stochastic interest rates, constant inflation and mortality rates
   - Stochastic inflation, constant interest and mortality rates
   - Stochastic mortality, constant inflation and interest rates
   - Simultaneous simulation of all risk factors

Results

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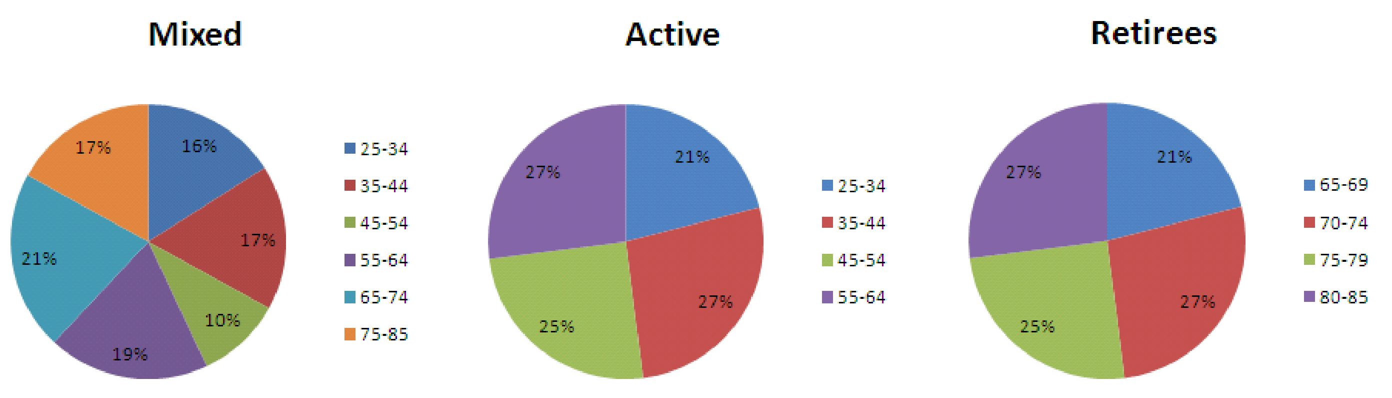
Pension plans

- General assumptions
  - Age of retirement - 65
  - Number of persons - 10000
  - Plan type - closed final pay (i.e. depends on the last salary and the years of service)
  - No consideration of widows and disabled persons
- Age structure



Best estimate of the liabilities

Which life table should be used to calculate the best estimate of the liabilities?

- Current life table (no improvements)
- Generation life table (low improvements based on the historical development of the mortality rates)
- Generation life table (high improvements based on the historical development of the mortality rates)
- Model based life table (Dahl vs. Lee-Carter)

Best estimates of the liabilities for the mixed plan at $t = 5$ in bn euro:

	Current life table	Life table(low impr.)	Life table(high impr.)	Lee-Carter model	Dahl model
Best estimate	1.03	1.11	1.18	1.13	1.27

Current and generation life tables are taken from Federal Statistical Office of Germany.

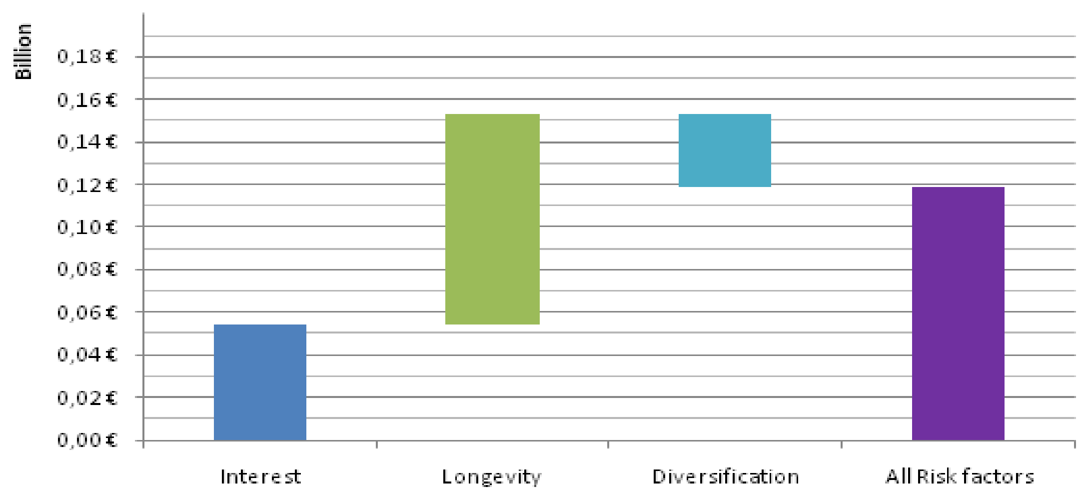
Results for $t = 5$ in the plan with retirees only

Best estimate of the liabilities[a]: 0.31 bn euro

Actual liabilities in bn euro:

	Interest	Longevity	All risk factors
Median	0.33	0.40	0.38
99% quantile	0.36	0.41	0.43
Absolute overall risk	0.05	0.10	0.12
Relative overall risk	18%	32%	38%
Absolute deviation risk	0.04	0.01	0.05
Relative deviation risk	11%	3%	13%

Absolute risk for different risk factors at time $t = 5$:



[a] Here and in the following the generation life table with low improvement is used.

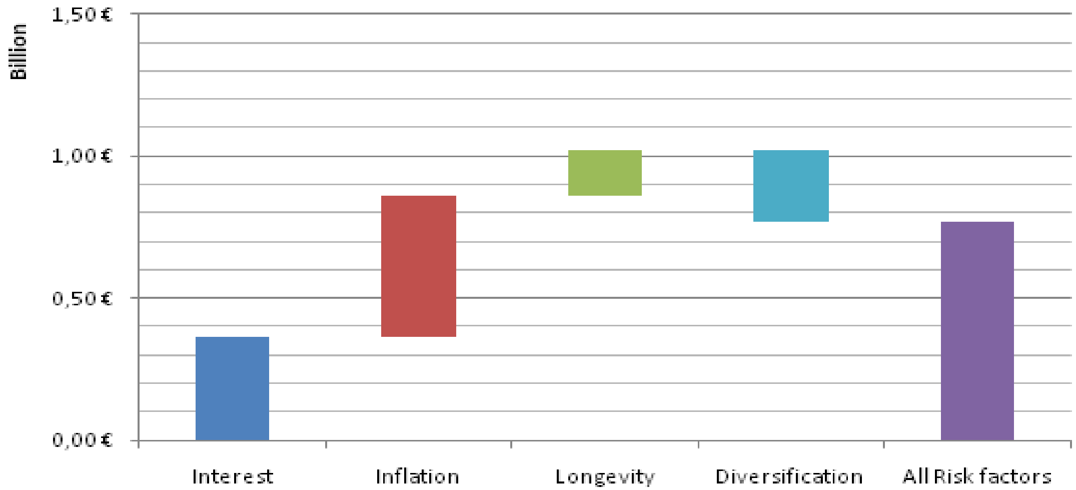
Results for $t = 5$ in the mixed plan

Best estimate of the liabilities: 1.11 bn euro

Actual liabilities in bn euro:

	Interest	Inflation	Longevity	All risk factors
Median	1.11	1.19	1.27	1.33
99% quantile	1.47	1.61	1.28	1.88
Absolute overall risk	0.36	0.50	0.16	0.77
Relative overall risk	32%	45%	15%	69%
Absolute deviation risk	0.36	0.42	0.01	0.54
Relative deviation risk	33%	36%	1%	41%

Absolute risk for different risk factors at time $t = 5$:



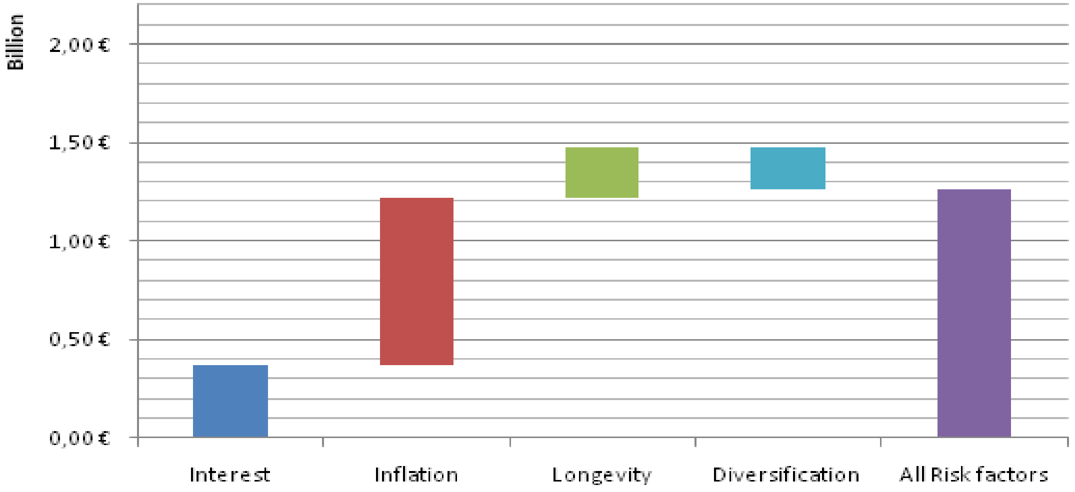
Results for $t = 15$ in the mixed plan

Best estimate of the liabilities: 1.52 bn euro

Actual liabilities in bn euro:

	Interest	Inflation	Longevity	All risk factors
Median	1.40	1.66	1.76	1.73
99% quantile	1.89	2.37	1.78	2.78
Absolute overall risk	0.37	0.85	0.25	1.26
Relative overall risk	24%	56%	17%	83%
Absolute deviation risk	0.50	0.71	0.02	1.05
Relative deviation risk	36%	43%	1%	61%

Absolute risk for different risk factors at time $t = 15$:



Summary and conclusion

- Analysed two mortality models and their impact on pension liabilities
- Calibrated both mortality models to historical data
- Presented a methodology to analyse main risks for pension liabilities
- Exemplarily applied the presented methodology for different pension plans

- Longevity risk is a long term risk
- Longevity risk is higher in the pension plans with a higher average age
- Strong dependence of the results on best estimate calculation approach
- Hedging strategies?

Thank you for your attention.

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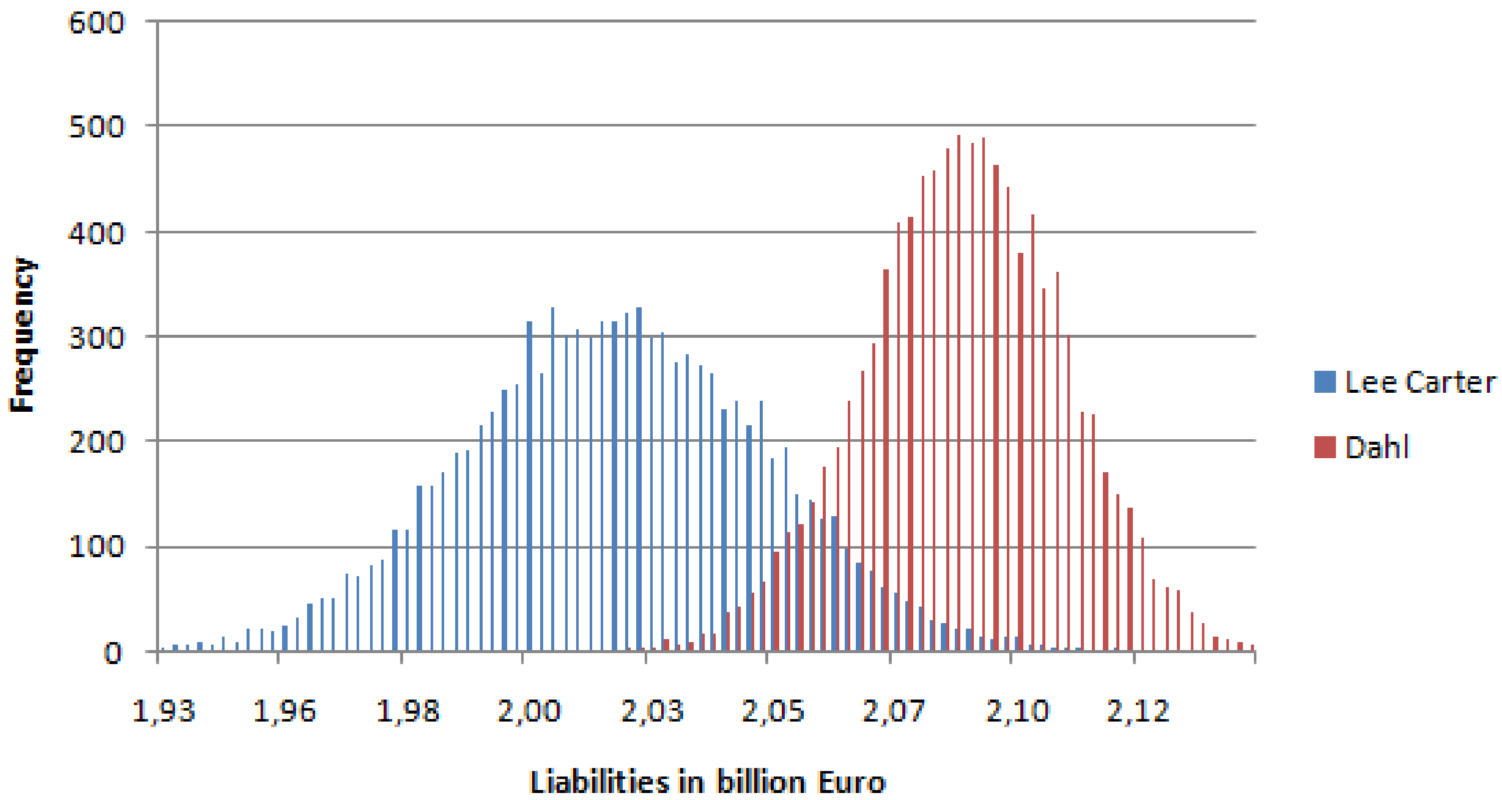
Appendix

Liabilities distribution in Lee-Carter and Dahl model



Distribution of the liabilities for the mixed plan at time $t = 30$.

Survival probabilities in the Dahl model

Survival probabilites can be expressed as

$$p_t(x+t,T|\mathcal{G}_t) = e^{A(t,T)-B(t,T)\mu_t(x+t)},$$

where $A(t,T)$ and $B(t,T)$ satisfy the following ordinary differential equations

$$\frac{\partial B(t,T)}{\partial t} = (\delta - ln(c))B(t,T) + \frac{1}{2}\sigma^2 bc^{x+t}B(t,T)^2 - 1$$

$$\frac{\partial A(t,T)}{\partial t} = \delta e^{-\gamma t} bc^{x+t}B(t,T)$$

with boundary conditions $A(T,T) = 0$ and $B(T,T) = 0$.