Impacts of Longevity Risk on the Formula of Mortality Dividend for Participating Policies

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- Formula
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Motivation

Motivation

- The paper aims to investigate the correctness of the mortality dividend formula for participating policies under the environment of mortality improvement.
- How to build the rules to pay the mortality dividend to policyholders for insurance companies?

The formula of mortality gain or loss

- Pricing: price the product
- Simulation: simulate future true event
- $R = \text{mortality gain or loss} - \text{mortality dividend}$
- For term insurance,

  mortality gain or loss

$$= B\sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_x^{pricing} - B\sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_x^{simulation}$$

$$= B\sum_{k=0}^{n-1} v^{k+1}\left({}_{k|}q_x^{pricing} - {}_{k|}q_x^{simulation}\right)$$

Build the mortality dividend formula for term insurance

$$VMD_t = max(AV_t - (1+i)AV_{t-1}, 0) \text{，} t = 1,2,\dots,n$$

$$AV_0 = 0$$

$$AV_t - (1+i)AV_{t-1}$$

$$= B\begin{bmatrix} max\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}, 0\right) \\ +min\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}, 0\right) \end{bmatrix}$$

$$= B\left[{}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}\right]$$

Build the mortality dividend formula for term insurance

Mortality dividend

$$= \sum_{t=1}^{n} v^t VMD_t$$

$$= \sum_{t=1}^{n} v^t max(AV_t - (1+i)AV_{t-1}, 0)$$

$$= \sum_{t=1}^{n} v^t max\left(B\left[{}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}\right], 0\right)$$

$$= \sum_{t=1}^{n} max\left(v^t B\left[{}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}\right], 0\right)$$

$$= \sum_{k=0}^{n-1} max\left(v^{k+1} B\left[{}_{k+1|}q_x^{pricing} - {}_{k+1|}q_x^{simulation}\right], 0\right)$$

$$\geq \sum_{k=0}^{n-1} v^{k+1} B\left({}_{k|}q_x^{pricing} - {}_{k|}q_x^{simulation}\right)$$

$$= \text{Mortality gain or loss}$$

Mortality gain or loss for endowment insurance

Mortality gain or loss

$$= B\sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_x^{pricing} + v^n\,{}_{n}p_x^{pricing}$$

$$-B\left[\sum_{k=0}^{n-1} v^{k+1}\,{}_{k|}q_x^{simulation} + v^n\,{}_{n}p_x^{simulation}\right]$$

$$= B\sum_{k=0}^{n-1} v^{k+1}\left({}_{k|}q_x^{pricing} - {}_{k|}q_x^{simulation}\right)$$

$$+Bv^n\left(\boxed{{}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}}\right)$$

Build the mortality dividend formula for endowment insurance

$VMD_t = max(AV_t - (1+i)AV_{t-1}, 0)$，$t = 1,2,\dots,n$

$AV_0 = 0$

For $t = 1, 2, \dots, n-1$，

$$AV_t - (1+i)AV_{t-1}$$

$$= B\begin{bmatrix} max\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}, 0\right) \\ +min\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}, 0\right) \end{bmatrix}$$

$$= B\left[{}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}\right]$$

Build the mortality dividend formula for endowment insurance

For $t = n$，

$$AV_n - (1+i)AV_{n-1}$$

$$= B\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation} + {}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right)$$

Mortality dividend

$$= \sum_{t=1}^{n} v^t VMD_t$$

$$= \sum_{t=1}^{n} v^t max(AV_t - (1+i)AV_{t-1}, 0)$$

$$= \sum_{t=1}^{n-1} v^t max(AV_t - (1+i)AV_{t-1}, 0)$$

$$+ v^n max(AV_n - (1+i)AV_{n-1}, 0)$$

Build the mortality dividend formula for endowment insurance

$$= \sum_{t=1}^{n-1} v^t max\left(B\left[{}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation}\right], 0\right)$$
$$+v^n max\left(B\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation} + {}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right), 0\right)$$
$$= \sum_{k=0}^{n-2} max\left(v^{k+1} B\left[{}_{k+1|}q_x^{pricing} - {}_{k+1|}q_x^{simulation}\right], 0\right)$$
$$+max\left(v^n B\left({}_{t|}q_x^{pricing} - {}_{t|}q_x^{simulation} + {}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right), 0\right)$$
$$= \sum_{k=0}^{n-1} max\left(v^{k+1} B\left[{}_{k+1|}q_x^{pricing} - {}_{k+1|}q_x^{simulation}\right], 0\right)$$
$$+max\left(v^n B\left({}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right), 0\right)$$
$$\geq \sum_{k=0}^{n-1} v^{k+1} B\left({}_{k|}q_x^{pricing} - {}_{k|}q_x^{simulation}\right) + v^n B\left({}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right)$$
$$= \text{Mortality gain or loss}$$

Numerical Analysis

Mortality model

- Data: Unique experience mortality data, ml, ma, fl, fa, obtained from more than 50,000,000 policies issued by Taiwanese life insurance companies from 1980 to 2007.

- Mitchell et al. (2013; IME)

$$\ln[m(x,t+1)] - \ln[m(x,t)] = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \varepsilon_{x,t}$$

Which mortality model is better?

		ml	ma	fl	fa
RSSE	Lee and Carter (1992)	3.9568	3.2048	6.7547	2.9870
	Mitchell et al. (2013)	3.5167	1.7872	4.6079	2.2529
MAPE	Lee and Carter (1992)	5.0338	5.0280	4.5968	2.9088
	Mitchell et al. (2013)	4.1402	2.9196	3.6204	2.5961

Test for normality of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$

	ml		ma		fl		fa	
	kt1	kt2	kt1	kt2	kt1	kt2	kt1	kt2
skewness	-0.774	1.006	-0.202	0.820	-0.295	1.030	0.165	-0.121
skew std.	0.594	0.594	0.594	0.594	0.594	0.594	0.594	0.594
kurtosis	0.507	0.790	2.300	0.996	-0.914	0.735	0.543	-1.055
kur std.	1.188	1.188	1.188	1.188	1.188	1.188	1.188	1.188
JB	1.882	3.310	3.864	2.607	0.839	3.391	0.286	0.829
p-value	0.126	0.055	0.044	0.078	0.482	0.053	>0.500	0.491

ARMA(p,q) for $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$

$$\kappa_t = \mu_t + \sum_{i=1}^{p} \varphi_i \kappa_{t-i} + \sum_{j=0}^{q} \theta_i \varepsilon_{t-j}$$

	ml		ma		fl		fa	
	kt1	kt2	kt1	kt2	kt1	kt2	kt1	kt2
p	0	0	0	0	0	1	0	0
q	1	1	1	1	1	1	1	1

Numerical Analysis

- Data: ml
- Pricing: 90%*period moratity
- Simulation: 100,000 paths
- Term insurance and endowment insurance

For term insurance,
Mortality dividend

$$= \sum_{k=0}^{n-1} max\left(v^{k+1}B\left[{}_{k+1|}q_x^{pricing} - {}_{k+1|}q_x^{simulation}\right], 0\right)$$

$$\geq \sum_{k=0}^{n-1} v^{k+1}B\left({}_{k|}q_x^{pricing} - {}_{k|}q_x^{simulation}\right)$$

$$= \text{Mortality gain or loss}$$

For endowment insurance,
Mortality dividend

$$= \sum_{k=0}^{n-1} max\left(v^{k+1}B\left[{}_{k+1|}q_x^{pricing} - {}_{k+1|}q_x^{simulation}\right], 0\right)$$

$$+max\left(v^{n}B\left({}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right), 0\right)$$

$$\geq \sum_{k=0}^{n-1} v^{k+1}B\left({}_{k|}q_x^{pricing} - {}_{k|}q_x^{simulation}\right) + v^{n}B\left({}_{n}p_x^{pricing} - {}_{n}p_x^{simulation}\right)$$

$$= \text{Mortality gain or loss}$$

Term insurance							
age=20, period=30				age=40, period=40			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0060	0.0037	0.0023	mortaltiy gain	0.2154	0.1119	0.0608
mortaltiy dividend	0.0068	0.0044	0.0029	mortaltiy dividend	0.2412	0.1250	0.0677
R	-0.0008	-0.0007	-0.0006	R	-0.0258	-0.0131	-0.0068
age=20, period=50				age=40, period=60			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0333	0.0159	0.0079	mortaltiy gain	0.0456	0.0510	0.0385
mortaltiy dividend	0.0371	0.0178	0.0090	mortaltiy dividend	0.2892	0.1459	0.0769
R	-0.0038	-0.0019	-0.0011	R	-0.2436	-0.0949	-0.0384
age=20, period=80				age=60, period=20			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0374	0.0285	0.0149	mortaltiy gain	0.2597	0.1893	0.1397
mortaltiy dividend	0.2366	0.0814	0.0298	mortaltiy dividend	0.2894	0.2114	0.1565
R	-0.1992	-0.0529	-0.0149	R	-0.0296	-0.0222	-0.0168
age=40, period=20				age=60, period=40			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0209	0.0163	0.0129	mortaltiy gain	0.0516	0.0793	0.0808
mortaltiy dividend	0.0213	0.0166	0.0131	mortaltiy dividend	0.3531	0.2524	0.1831
R	-0.0004	-0.0003	-0.0002	R	-0.3014	-0.1731	-0.1023

Endowment insurance							
age=20, period=30				age=40, period=40			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0003	0.0005	0.0005	mortaltiy gain	0.0117	0.0189	0.0177
mortaltiy dividend	0.0068	0.0044	0.0029	mortaltiy dividend	0.2412	0.1250	0.0677
R	-0.0066	-0.0039	-0.0024	R	-0.2295	-0.1061	-0.0499
age=20, period=50				age=40, period=60			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0035	0.0047	0.0037	mortaltiy gain	0.0361	0.0480	0.0376
mortaltiy dividend	0.0371	0.0178	0.0090	mortaltiy dividend	0.2893	0.1459	0.0769
R	-0.0336	-0.0131	-0.0053	R	-0.2533	-0.0979	-0.0393
age=20, period=80				age=60, period=20			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0296	0.0269	0.0145	mortaltiy gain	0.0092	0.0200	0.0245
mortaltiy dividend	0.2367	0.0814	0.0298	mortaltiy dividend	0.2895	0.2115	0.1565
R	-0.2070	-0.0545	-0.0153	R	-0.2803	-0.1915	-0.1320
age=40, period=20				age=60, period=40			
interest rate	1%	3%	5%	interest rate	1%	3%	5%
mortaltiy gain	0.0014	0.0031	0.0039	mortaltiy gain	0.0396	0.0738	0.0783
mortaltiy dividend	0.0213	0.0166	0.0131	mortaltiy dividend	0.3532	0.2525	0.1831
R	-0.0199	-0.0135	-0.0092	R	-0.3136	-0.1787	-0.1049

Conclusions

Conclusions

- If insurance companies ignore the mortality loss, they will pay too much the mortality dividend.
- Considering the survival probabilities for endowment insurance, they are mortality losses and should be put into the formula of mortality dividend.
- Forecast precisely mortality improvement, we do not pay more mortality dividend to policyholders.

Thanks for your attention