

A Study on Pricing of the Reverse Mortgage with an Embedded Redeemable Option

— — An analysis based on China's market

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Introduction

- Reverse mortgage:
 - Residents use property rights as collateral, to obtain loan on a regular basis, mainly as pension.
 - The loans are mainly used for daily consumption.
 - The principal and interest will due after the borrower dead.
 - The source of repayment is the rental income or sales revenue of the collateral when the mortgages expire.

Introduction

- The most significant function of this mortgage is to help old people who have considerable assets, such as realty, while have little cash to look for new cash flow
- The maximum loan amount the borrowers can get is determined by three factors:
 - the value of the house
 - the loan interest rate
 - age of the borrower

Introduction

- According to internationally recognized indicators of an aging society, China has been recognized as an aging society
- China is a developing country
 - income per capita is relatively low
 - public pension is insufficient
- China encounters more difficulty in solving aging problem, there exists a huge pension funding gap

Introduction

- The innovation, design and pricing of the reverse mortgage products can provide a supplement for the pension and strengthen the social security system
- It provides an effective approach to reduce the burden of aging problem for the government and the family

The Reverse Mortgage Pricing Model

- Assumption
 - perfectly competitive market
 - no excess income
 - actuarially fair
 - risk premium and cost of opportunities will be included in the loan interest rate

The Reverse Mortgage Pricing Model

- For the lender, who will pay the annuity to the borrower, the total discounted value of annuity paid to the borrower is :

$$\sum_{t=1}^T A(t) * \prod_{s=1}^t (1 + r(s))^{-1}$$

Where

$A(t)$: Reverse mortgage annuity paid in year t .

$r(s)$: Product interest rate in year s .

The Reverse Mortgage Pricing Model

- Total actuarial discounted value of the loans

$$\sum_{t=1}^{\infty} A(t) * {}_tP_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

The Reverse Mortgage Pricing Model

- For lenders, the final value of the underlying housing is the income to receive
- China's real estate market is affected by:
 - macroeconomic regulation and control policies
 - speculation behavior of irrational investors
- the value of the collateral is only realized after the death of the borrower

The Reverse Mortgage Pricing Model

- We use geometric Brownian motion model to describe the long term real estate price movement

$$\frac{dH}{H} = \mu dt + \sigma dW$$

The Reverse Mortgage Pricing Model

- After the product expires, the lender will sell the collateral and realize its income
- Discounted value of the lender's income is:

$$H(t) * \prod_{s=1}^t (1 + r(s))^{-1}$$

- Actuarial discounted value of the lender's income is:

$$\sum_{t=1}^{\infty} H(t) * {}_t|u_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

The Reverse Mortgage Pricing Model

- Based on the Actuarially fair:

lenders expect income = expect expenditure

$$\sum_{t=1}^{\infty} A(t) * {}_tP_x * \prod_{s=1}^t (1 + r(s))^{-1} = \sum_{t=1}^{\infty} H(t) * {}_t\mu_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

- Annuity amount paid to the borrow each year:

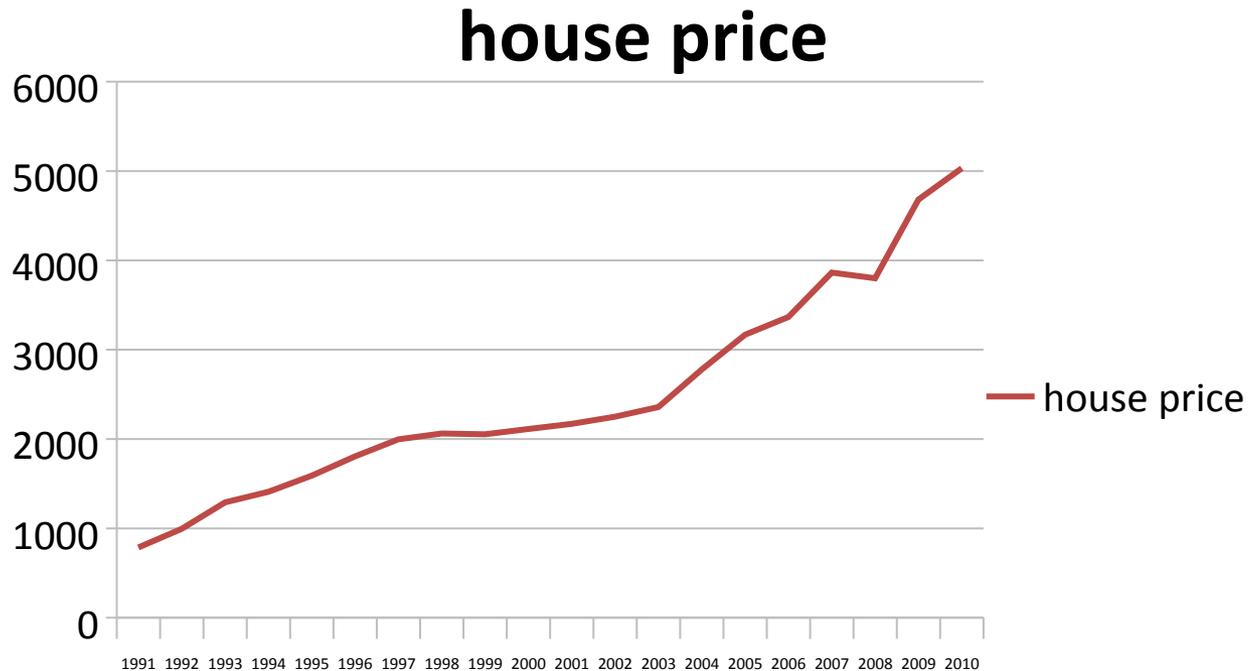
$$A = \sum_{t=1}^{\infty} H(t) * {}_t\mu_x * \prod_{s=1}^t (1 + r(s))^{-1} / \sum_{t=1}^{\infty} {}_tP_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

Numerical Explanation

- The pricing formula includes a number of random variables, expect annuity cannot be derived directly
- We use Monte Carlo methods for reverse mortgage pricing

Numerical Explanation

- $H(t)$
 - In the past 20 years, Chinese real estate market has experienced rapid development, price of house has risen fast.



Numerical Explanation

- In long term, housing price will not be sustained such a high growth rate.
- We use regression model to predicts the house price:

$$Y = 0.002307 + 0.794266X + \varepsilon$$

where

Y: GDP growth rate

X: house price growth rate

Numerical Explanation

- According to the forecast in the World Bank's report "China 2030", the long-term level of China's GDP growth will maintain at 5.5%. Therefore, long-term house price growth rate in China will be 4.6%.

Numerical Explanation

- Interest rate $r(t)$: Vasicek model

$$\Delta r_t = 0.018(2.01\% - r_t) + \varepsilon_t,$$
$$\varepsilon_t \sim N(0, 0.08\%)$$

Numerical Explanation

- ${}_t\mu_x$:
 - “Experience Life Table of Chinese Life Insurance Industry (2000-2003)”, issued by China Insurance Regulatory Commission.

Numerical Explanation

- Assumption
 - value of collateral is 2 million Yuan today
 - borrower signed the contract of reverse mortgage product on birthday
 - both lenders and borrowers since do not exist in default
 - annuity paid in the beginning of each year, from the next year of contract is entered into, to the borrower dead

Numerical Explanation

- Assumption
 - no redemption option
 - houses have permanent property right
 - no depreciation
 - on the death of the borrower, property rights transferred to the lender to repay loans

Numerical Explanation

RMB: yuan

Age	Male			Female		
	PV of house	Annuity	Payment coefficient	PV of house	Annuity	Payment coefficient
60	1165316	111359.1	0.0557	1087869	97817.6	0.0489
65	1265933	134842.3	0.0674	1201598	118709.8	0.0594
70	1380784	168988.8	0.0845	1317490	147365.3	0.0737
75	1492315	217543.8	0.1088	1433070	188154.6	0.0941
80	1598977	289818.9	0.1449	1544529	248663.4	0.1243

Numerical Explanation

- The older the borrowers buy a reverse mortgage contract, the greater the actuarial present value of the house, and the more annuity will be received each year.
- Due to life expectancy of female is longer than men's, the actuarial present value of female's house would be lower than the male's, the annuity and payment coefficient are less than men.

Embedded Redeemable Option

Reverse Mortgages

- Due to the long duration of reverse mortgages of generally more than ten years, the collateral has room for potential appreciation, especially in China.
- China's elderly generally have the habit of leaving property to the next generation.
- If house redemption can be selected by the heirs of the borrower, it can satisfy the demand of many older.

Embedded Redeemable Option Reverse Mortgages

- We consider a redemption option embedded in the original reverse mortgage pricing model.
 - At the death of the borrower, the heir of the borrower can choose to give up the house. The house is then recovered by the borrower and used to repay the loan.
 - The heir can also choose to repay the loan while redeeming house ownership.

Embedded Redeemable Option Reverse Mortgages

- Assuming the borrower death in year t , denote $H(t)$ as the housing price at the time of the death of the borrower, $BAL(t)$ as the loan account balance at the time of the death of the borrower. The heir's gain through redeeming the house is:

$$\begin{cases} 0, & H(t) \leq BAL(t) \\ H(t) - BAL(t), & H(t) \geq BAL(t) \end{cases}$$

$$\text{Or } [H(t) - BAL(t)]^+$$

Embedded Redeemable Option Reverse Mortgages

- The actuarial present value of the redemption option expected return is:

$$\sum_{t=1}^{\infty} [H(t) - \text{BAL}(t)]^+ * {}_t p_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

Embedded Redeemable Option Reverse Mortgages

- Consider the impact of the cost of the redemption option on the borrower's financial condition; we amortize the value of the redemption option to every mortgage period. Such amortization installments have the same number of periods as the annuity. The option fee is deducted from the borrower's annuity, and takes the loan balance into account.

Embedded Redeemable Option Reverse Mortgages

Denote the borrower's installment option fee as $\pi(t)$; the probability of survival of the borrower in year t as ${}_t p_x$; and the present value of the total option fee as:

$$\sum_{t=1}^{\infty} \pi(t) * {}_t p_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

The present value of the possible future revenue from the redemption of collateral should be equal to the present value of the option fee paid by the borrower. Thus, the following equation:

$$\sum_{t=1}^{\infty} \pi(t) * {}_t p_x * \prod_{s=1}^t (1 + r(s))^{-1} = \sum_{t=1}^{\infty} [H(t) - \text{BAL}(t)]^+ * {}_t \mu_x * \prod_{s=1}^t (1 + r(s))^{-1}$$

Assuming that the installment option fee $\pi(t)$ does not change with time t , $\pi(t)$ by the following formula:

$$\pi(t) = \frac{\sum_{t=1}^{\infty} [H(t) - \text{BAL}(t)]^+ * {}_t \mu_x * \prod_{s=1}^t (1 + r(s))^{-1}}{\sum_{t=1}^{\infty} {}_t p_x * \prod_{s=1}^t (1 + r(s))^{-1}}$$

Embedded Redeemable Option Reverse Mortgages

- We still use the foregoing assumption that the collateral housing present value is 2 million Yuan.
- By performing simulations of 60-, 65-, 70-, 75-, and 80-year-old male and female borrowers, we obtain the value of the annuity amount and the value of the redemption rights of borrowers of different age groups

Embedded Redeemable Option Reverse Mortgages

Sex	Age	With redemption option			Without redemption option		
		PV of house	Annuity	Payment coefficient	Value of option	Annuity	Payment coefficient
Male	60	1165316	111359.1	0.0557	291724.3	83481.6	0.0417
	65	1265933	134842.3	0.0674	321372.6	100611.0	0.0503
	70	1380784	168988.8	0.0845	345900.8	126655.4	0.0633
	75	1492315	217543.8	0.1088	372123.3	163297.1	0.0816
	80	1598977	289818.9	0.1449	386637.0	219739.9	0.1099
Female	60	1087869	97817.6	0.0489	275256.5	73067.5	0.0365
	65	1201598	118709.8	0.0594	296694.3	89398.4	0.0447
	70	1317490	147365.3	0.0737	320258.8	111543.4	0.0558
	75	1433070	188154.6	0.0941	349098.0	142319.9	0.0712
	80	1544529	248663.4	0.1243	371386.3	188871.6	0.0944

Sensitivity Analysis Of Main Factors

- Influence of Interest Rate
 - Vasicek model is used to characterize the random behavior of risk-free interest rate
 - spread between lending rate and risk-free interest rate is set to a fixed value
 - Consider the pricing changes under different spread of interest rate

Sensitivity Analysis Of Main Factors

Age:65

Sex	Spread (BP)	With redemption option			Without redemption option		
		PV of house	Annuity	Payment coefficient	Value of option	Annuity	Payment coefficient
Male	300	1500532	147527.4	0.0738	347296.6	113382.4	0.0567
	350	1371445	140429.2	0.0702	334870.1	106140.2	0.0531
	400	1265933	134842.3	0.0674	321372.6	100611.0	0.0503
	450	1169848	129512.5	0.0648	303737.3	95886.1	0.0479
	500	1079180	124018.7	0.0620	293615.4	90276.6	0.0451
Female	300	1444507	130954.1	0.0654	333874.6	100686.2	0.0503
	350	1313710	124388.5	0.0622	317047.8	94368.9	0.0472
	400	1201598	118709.8	0.0594	296694.3	89398.4	0.0447
	450	1095035	112715.6	0.0564	278923.8	84005.0	0.0420
	500	1002747	107425.8	0.0537	265014.5	79034.4	0.0395

Sensitivity Analysis Of Main Factors

- in the case of different spreads, the annuity paid by the lender are very different
- amount of annuity of the female borrowers is more sensitive to changes in spread
- reverse mortgage products have a negative convexity

Sensitivity Analysis Of Main Factors

- Influence of Changes in House Price
 - growth rate of real estate prices will significantly affect the pricing of the product
 - when the growth rate gets larger, the annuity will be increased; otherwise, the annuity will be reduced

Sensitivity Analysis Of Main Factors

Sex	Growth rate	With redemption option			Without redemption option		
		PV of house	Annuity	Payment coefficient	Value of option	Annuity	Payment coefficient
Male	0.00	664319	70761.4	0.0354	246925.2	44459.6	0.0222
	0.02	892477	95054.3	0.0475	278139.7	65430.7	0.0327
	0.042	1265933	134842.3	0.0674	321372.6	100611.0	0.0503
	0.06	1734210	184726.7	0.0924	361058.5	146267.0	0.0731
	0.08	2469104	263014.0	0.1315	442970.0	215827.9	0.1079
Female	0.00	570202	56332.3	0.0282	208374.5	35746.3	0.0179
	0.02	803033	79314.9	0.0397	247916.8	54828.4	0.0274
	0.042	1201598	118709.8	0.0594	296694.3	89398.4	0.0447
	0.06	1702787	168226.3	0.0841	347873.8	133858.2	0.0669
	0.08	2543983	251296.6	0.1256	466990.2	205166.9	0.1026

Sensitivity Analysis Of Main Factors

- When the house prices growth rate changes, the annuity amount will be different for many times
- when the prices growth rate is low, the discounted value of house of male is higher than female
- when the prices growth rate is high, the discounted value of house of male is lower than female

Conclusion

- What we have done
 - propose a pricing model for reverse mortgage product
 - use geometric Brownian motion model and Vasicek model to describe stochastic dynamic behavior
 - Include both of price risk and interest risk
 - comprehensive sensitivity analysis

Conclusion

- What we have found
 - The reverse mortgage products can strikingly increase the cash flow and income for the elder in China
 - reverse mortgage products can relieve the burden of government, society and family's financial pressures, and significantly improve the living standard of old people in China

Conclusion

- What we have found
 - sensitive analysis indicates that the reverse mortgage is sensitive to changes in interest rate and changes in house price
 - lenders should pay strict attention to control risks