The CBD Mortality Indexes: Modeling and Applications

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1. What are Mortality Indexes?

- Most extrapolative stochastic mortality models are constructed in a similar manner. Specifically, when they are fitted to historical data, one or more time-varying indexes (k_t) are identified.
- By extrapolating these indexes to the future, we can obtain a forecast of future death probabilities and consequently other demographic quantities such as life expectancies. They are important for quantifying longevity in pension risks and for constructing benchmarks for longevity-linked capital markets.

The Lee-Carter Model

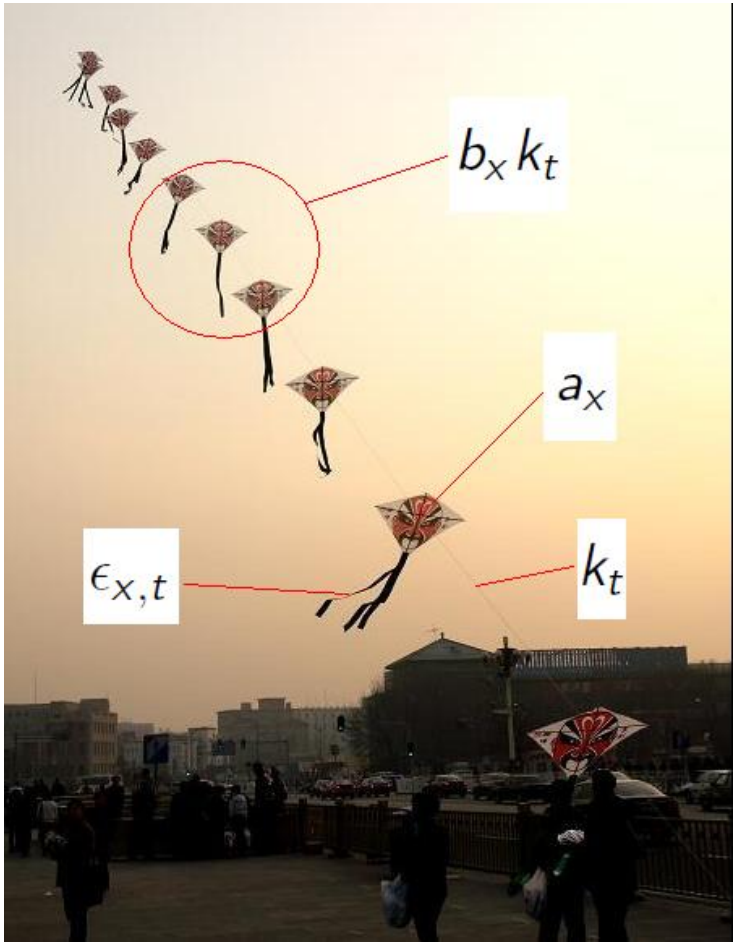
In the Lee-Carter model, k_t is a univariate time-series:

$$\ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}$$

where $m_{x,t}$ is the central death rate at age x and in year t.

- a_x – an age-specific parameter; the set of $\{a_x\}$ reflects the general shape of the mortality schedule.
- k_t – a time-varying parameter; the time-trend of k_t signifies the general speed of mortality improvement.
- b_x – an age-specific parameter which characterizes the sensitivity of to k_t at age x.
- $\epsilon_{x,t}$ – the error-term, which has no long-term trend.

The k_t in the Lee-Carter Model



The Cairns-Blake-Dowd (CBD) Model

- Cairns et al. (2006) propose a two-factor stochastic mortality model

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_t^{(1)} + k_t^{(2)}(x-\bar{x}), \tag{1}$$

where $q_{x,t}$ is the realized single-year death probability at age x and time t, $\bar{x}$ is the average age over the age range we consider, and $k_t^{(1)}$ and $k_t^{(2)}$ are period mortality indexes.

The Cairns-Blake-Dowd (CBD) Model

- After fitting equation (1) to historic death probabilities, the period indexes $k_t^{(1)}$ and $k_t^{(2)}$ are modeled by a bivariate random walk with drift, that is,

$$k_{t+1} = k_t + \mu + CZ(t+1) \tag{2}$$

where $k_t = (k_t^{(1)}, k_t^{(2)})'$, $\mu = (\mu_1, \mu_2)'$ is a constant 2×1 vector, C is a constant 2×2 upper triangular matrix, and $Z(t)$ is a 2-dimensional standard normal random vector.

The Cairns-Blake-Dowd (CBD) Model

- Trajectory of mortality predicted rates for a particular birth cohort (age x in year u) can be obtained by

$$\ln\left(\frac{\hat{q}_{x+s,u+s}}{1-\hat{q}_{x+s,u+s}}\right) = k_u^{(1)}(s) + k_u^{(2)}(s)(x+s-\bar{x}),$$

where $k_u^{(1)}(s)$ and $k_u^{(2)}(s)$ are the minimum sqaure error (MMSE) forecasts of $k_{u+s}^{(1)}$ and $k_{u+s}^{(2)}$, respectively.

The GCBD Model

- To model cohort effects, we may consider the following generalization of the CBD model:

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_t^{(1)} + k_t^{(2)}(x-\bar{x}) + k_t^{(3)}((x-\bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)}, \tag{3}$$

where $k_t^{(1)}$, $k_t^{(2)}$, and $k_t^{(3)}$ are time period risk factors, $\gamma_{t-x}^{(4)}$ is a cohort risk factor, and $\hat{\sigma}_x^2$ is the mean of $(x-\bar{x})^2$ over the age range we consider.

The GCBD Model

- Having fitted equation (3) to historic data, the period mortality indexes are modeled by a trivariate random walk with drift:

$$k_{t+1} = k_t + \mu + CZ(t+1), \tag{4}$$

where $k_t = (k_t^{(1)}, k_t^{(2)}, k_t^{(3)})'$, $\mu = (\mu_1, \mu_2, \mu_3)'$ is a constant 3×1 vector, C is a constant 3×3 upper triangular matrix, and $Z(t)$ is a 3-dimensional standard normal random vector.

The GCBD Model

- Trajectory of mortality predicted rates for a particular birth cohort (age x in year u) can be obtained by

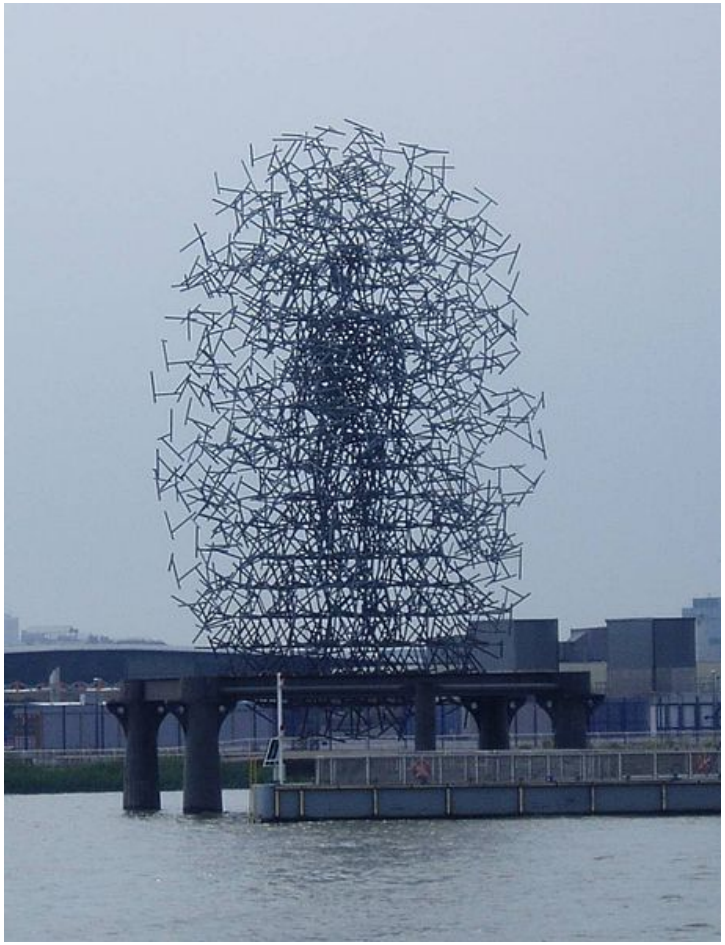
$$\begin{aligned}\ln\left(\frac{\hat{q}_{x+s,u+s}}{1-\hat{q}_{x+s,u+s}}\right) = \; & k_u^{(1)}(s) + k_u^{(2)}(s)(x+s-\bar{x}) \\ & + k_u^{(3)}(s)((x+s-\bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{u-x}^{(4)}, \end{aligned} \tag{5}$$

where $k_u^{(i)}(s) = k_u^{(i)} + s\mu_i$, $i = 1, 2, 3$, is the MMSE forecast of $k_{u+s}^{(i)}$.

Summary of the Introduction

- The general principle of extrapolative mortality modeling: extract one or more signals (mortality indexes) from historical data; project the index(es) forward to obtain a mortality forecast.
- In the Lee-Carter model (M1), k_t is a univariate time-series; and it is often modelled by a univariate random walk.
- In the CBD model (M5), k_t is a bivariate vector time-series; and it is often modelled by a bivariate vector random walk.
- In the GCBD model (M7), k_t is a trivariate vector time-series; and it is often modelled by a trivariate vector random walk.

Does Human k_t follow a Random Walk?



2. Properties of Mortality Indexes

- All mortality indexes represent some information about the mortality of a population at a certain time point, but not all mortality indexes can be used as an indicator of longevity risk. By an indicator, we mean a value that can be, for example, announced by national governments every year.

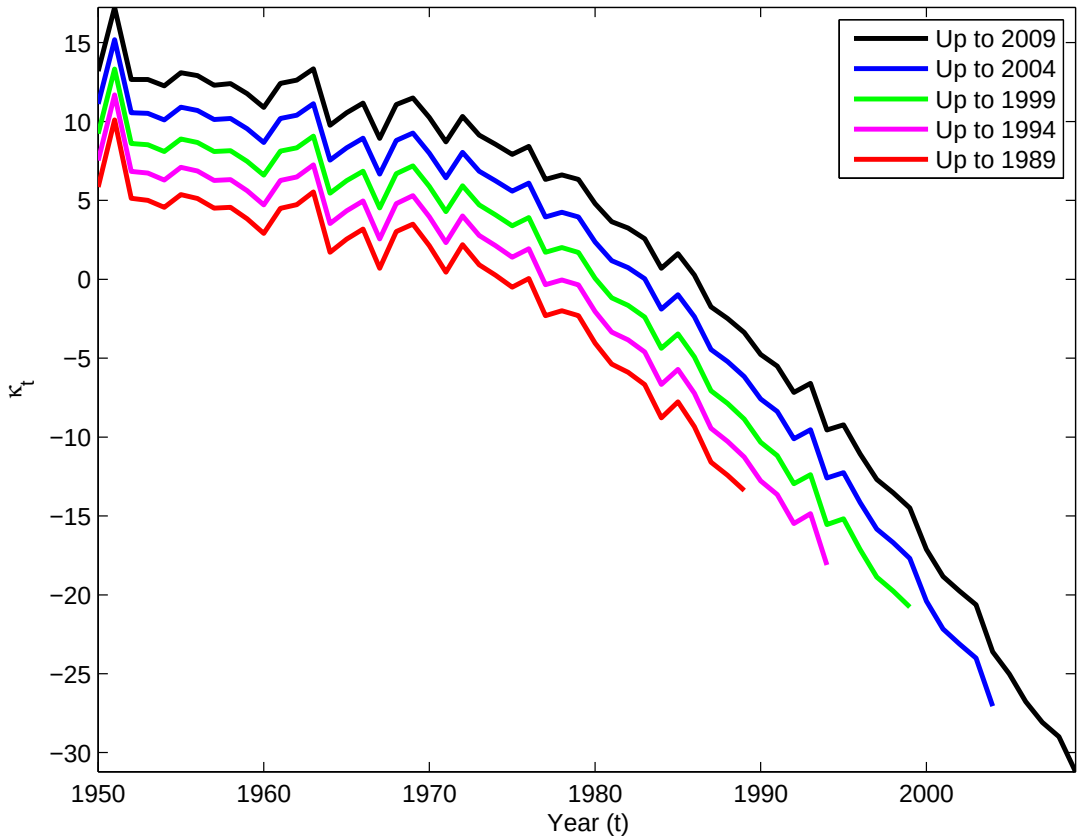
- For example, consider the Lee-Carter model:

$$\ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}$$

The time-varying parameter may be interpreted as the general level of mortality at a particular time t. However, this index is not appropriate as an indictor of risk.

Properties of the Lee-Carter Mortality Indexes

- See the following diagram:



Properties of the Lee-Carter Mortality Indexes

- The previous index (k_t) values from the Lee-Carter model would change if new data are considered.

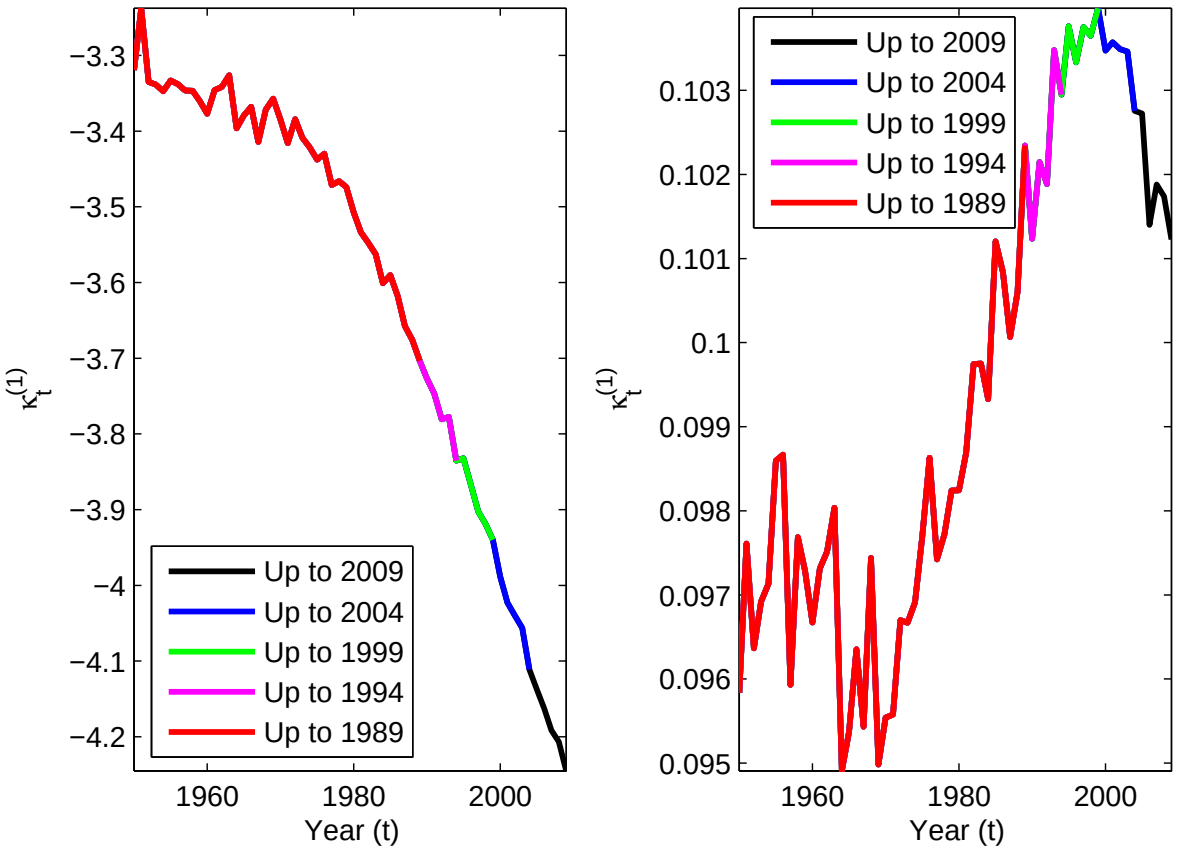
- Technically speaking, this is because:
  - When one more year of data is included, the maximum likelihood estimates of all model parameters, that is, a_x, b_x and k_t for all x and t will be updated.
  - Parameter constraints are involved in the estimation process. In particular, the constraint $\sum_t k_t$ re-scale the series of k_t as new data are included.

Properties of the CBD Mortality Indexes

- For the original CBD Model (M5), the inclusion of new data will **NOT** affect previous index values.
- We shall call this property as "new-data invariant".
- Reasons for this special property:
  - It can be shown that adding one year of data will have no effect on the parameters that are already estimated.
  - For Model M5, no constraint is needed to stipulate uniqueness.
- None of the stochastic mortality models (M1 to M8) discussed in Cairns *et. al.* (2009), except the original CBD model (M5), process this property.

Properties of the CBD Mortality Indexes

- An illustration of the data-invariant property of MLE estimates of mortality indexes from the CBD model using English and Welsh Data:



Interpretations of the CBD Mortality Indexes

- $k_t^{(1)}$ presents the level of the logit-transformed mortality curve. A reduction in $k_t^{(1)}$, that is, a parallel downward shift of the logit-transformed mortality curve, represents an overall mortality improvement.

- $k_t^{(2)}$ presents the steepness of the logit-transformed mortality curve. An increase in $k_t^{(2)}$, that is an increase in the steepness of the logit-transformed mortality curve, means that mortality (in logit scale) at younger ages improves more rapidly than at older ages.

Implication of the CBD Mortality Indexes

- For annuity providers and pension sponsors, their payouts are positively related to the improvement of mortality at older ages. Their financial obligations are larger when, of course, the overall mortality improvement is higher than expected (i.e., $k_t^{(1)}$ is lower than expected).

- For a fixed overall mortality improvement, the problem to annuity providers and pension sponsors would be worse when the improvement at older ages is higher than that at younger ages (i.e., when $k_t^{(2)}$ is lower than expected).

Implication of the CBD Mortality Indexes

- For life insurers selling term-life insurance products, their payouts are negatively related to the improvement of mortality at younger ages. Their payouts are larger when the overall mortality improvement is lower than expected (i.e., $k_t^{(1)}$ is higher than expected).

- For a fixed overall mortality improvement, the problem to life insurers is worse when mortality improvement at younger ages is less than that at higher ages (i.e., $k_t^{(2)}$ is lower than expected).

3. Multiple Time-Series Modelling

- In the last section, we argue that, among various mortality indexes, those encompassed in the Cairns-Blake-Dowd (CBD) model (also known as Model M5) are most suitably used as indictors of longevity risk.
- Random Walk models are often employed for extrapolating future mortality indexes.
- A vector random walk model implies, after the first differencing, the vector time-series does not exhibit any serial- and cross-correlations.
- However, serial- and cross-correlations are often observed in mortality indexes obtained from real data.

Multiple Time-Series Modelling

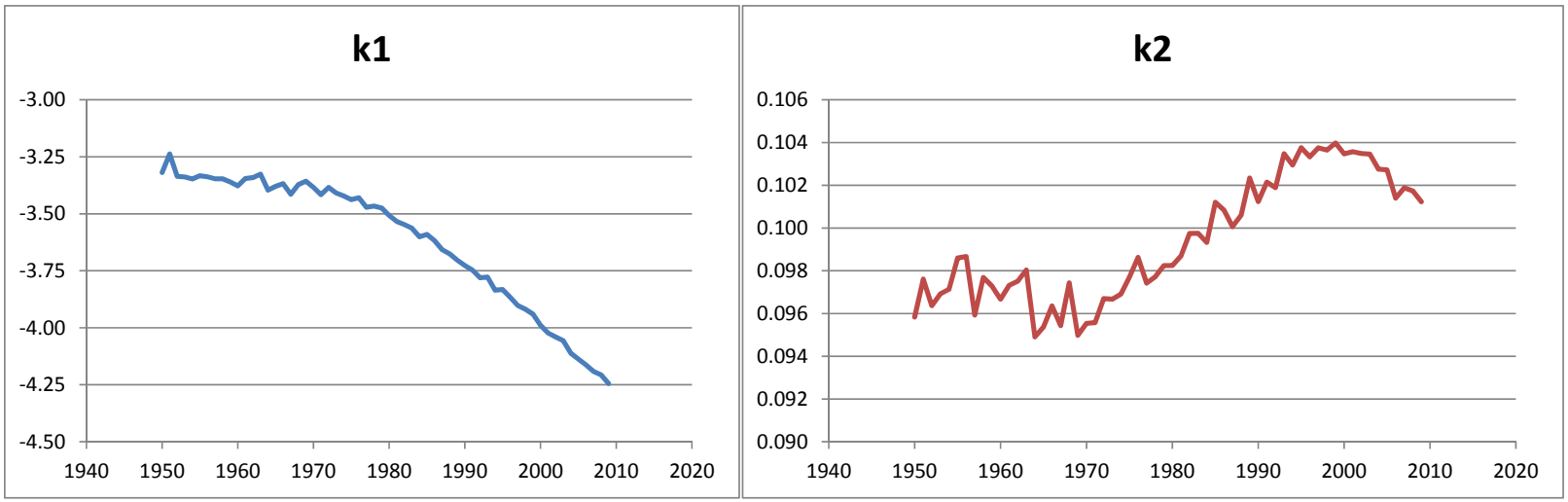
- Sims (1980) proposes characterizing possible cross-correlations among time-series by Vector Autoregressive (VAR) models. In this section we shall employ the multiple time-series modelling approach for VARMA processes due to Tiao & Box (1981).

- The orthodox modelling strategy (iterative stages of model identification, estimation and diagnostic checking) proposed by Box & Jenkins (1976) for univariate time-series can be extended and applied to this type of M5 bivariate mortailty index time-series. We shall restrict the discussion to points necessary for describing the applications in this paper. Further details can be found in Tiao & Box (1981).

Data

- Historic mortality data for English and Welsh (male) populations from year 1950 to 2009.
- The required data, death counts and exposures-to-risk, are obtained from the Human Mortality Database (2012).
- The maximum likelihood estimates (MLE) of $\mathbf{Z}(t) = (k_t^{(1)}, k_t^{(2)})'$, for $t = 1950, \ldots, 2009$, with age $40 - 90$, from the CBD model are obtained.
- Note that the methods we propose do not require a specifc choice of population gender and a sample period. They are chosen just purely for illustration purposes.

Time-Series Plots of the CBD Mortality Indexes



Model Identification of $\mathbf{Z}(t) = (k_t^{(1)}, k_t^{(2)})'$

lag (l)

	1	2	3	4	5
(a) Sample cross-correlation matrices (SCCM)	$\begin{pmatrix} + & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \end{pmatrix}$
(b) Sample partial autoregression matrices (SPAM)	$\begin{pmatrix} + & - \\ \cdot & + \end{pmatrix}$	$\begin{pmatrix} + & \cdot \\ + & \cdot \end{pmatrix}$	$\begin{pmatrix} + & \cdot \\ + & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ + & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ + & \cdot \end{pmatrix}$
$M(l)$	332.13	12.41	10.01	8.03	7.62

Model Identification of $\Delta\mathbf{Z}(t)$

	lag (l)				
	1	2	3	4	5
(a) Sample cross-correlation matrices (SCCM)	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
(b) Sample partial autoregression matrices (SPAM)	$\begin{pmatrix} \cdot & \cdot \\ \cdot & - \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} + & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & \cdot \end{pmatrix}$
$M(l)$	12.68	3.35	8.76	4.60	15.28

Model Estimation of VARIMA (5,1,0) for $\mathbf{Z}(t)$

(a) Full Model

$$\mathbf{C_0} = \begin{pmatrix} -.006 \\ (.005) \\ \\ .0002 \\ (.0016) \end{pmatrix}$$

$$\mathbf{A}_1 = \begin{pmatrix} -.282 & -1.426 \\ (.149) & (3.400) \\ \\ -.017 & -.363 \\ (.006) & (.144) \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} -.099 & 1.457 \\ (.141) & (3.582) \\ \\ -.011 & -.054 \\ (.006) & (.152) \end{pmatrix}$$

$$\mathbf{A}_3 = \begin{pmatrix} .406 & -2.188 \\ (.135) & (3.512) \\ \\ .000 & .101 \\ (.006) & (.149) \end{pmatrix}$$

$$\mathbf{A}_4 = \begin{pmatrix} .311 & -3.333 \\ (.138) & (3.458) \\ \\ .005 & .238 \\ (.006) & (.147) \end{pmatrix}$$

$$\mathbf{A}_5 = \begin{pmatrix} .272 & -7.827 \\ (.129) & (3.181) \\ \\ .016 & .009 \\ (.005) & (.135) \end{pmatrix}$$

$$\hat{\mathbf{\Sigma}} = \begin{pmatrix} .000350 & .000008 \\ \\ .000008 & .000001 \end{pmatrix}$$

Model Estimation of VARIMA (5,1,0) for $\mathbf{Z}(t)$

(b) Final Model

$$\mathbf{C_0} = \begin{pmatrix} -.008 \\ (.004) \\ \\ .0001 \\ (.0001) \end{pmatrix}$$

$$\mathbf{A}_1 = \begin{pmatrix} -.277 & 0 \\ (.122) & \\ & \\ -.014 & -.326 \\ (.006) & (.121) \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} 0 & 0 \\ & \\ & \\ -.011 & 0 \\ (.005) & \end{pmatrix}$$

$$\mathbf{A}_3 = \begin{pmatrix} .365 & 0 \\ (.100) & \\ & \\ 0 & 0 \\ & \end{pmatrix}$$

$$\mathbf{A}_4 = \begin{pmatrix} .201 & 0 \\ (.102) & \\ & \\ 0 & 0 \\ & \end{pmatrix}$$

$$\mathbf{A}_5 = \begin{pmatrix} .231 & -6.159 \\ (.114) & (2.667) \\ & \\ .011 & 0 \\ (.004) & \end{pmatrix}$$

$$\hat{\mathbf{\Sigma}} = \begin{pmatrix} .000368 & .000008 \\ & \\ .000008 & .000001 \end{pmatrix}$$

Diagnostic Checking of Fitted Residuals

	lag (l)				
	1	2	3	4	5
(a) Sample cross-correlation matrices (SCCM)	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
(b) Sample partial autoregression matrices (SPAM)	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & + \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
$M(l)$	1.98	1.23	1.95	7.15	1.81

Diagnostic Checking of Fitted Squared Residuals

	lag (l)				
	1	2	3	4	5
(a) Sample cross-correlation matrices (SCCM)	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & + \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
(b) Sample partial autoregression matrices (SPAM)	$\begin{pmatrix} \cdot & \cdot \\ + & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$
$M(l)$	5.56	0.98	4.40	5.40	2.84

4. An Indicator of Longevity Risk

- National governments and the WHO announce life expectancies of different populations every year. To financial institutions, life expectancy is **not** an adequate measure of risk, because all it does not give people any idea about how mortality rates at different ages vary over time.
- On the other hand, indicators of longevity risk cannot be too complicated. An indicator that is composed by a huge array of numbers is difficult to interpret and will lose the purpose as a "**summary**" of a mortality pattern.
- In this paper we **propose** using the M5 mortality indexes $(k_t^{(1)}, k_t^{(2)})$ as a longevity risk indicator which would fill in this gap.

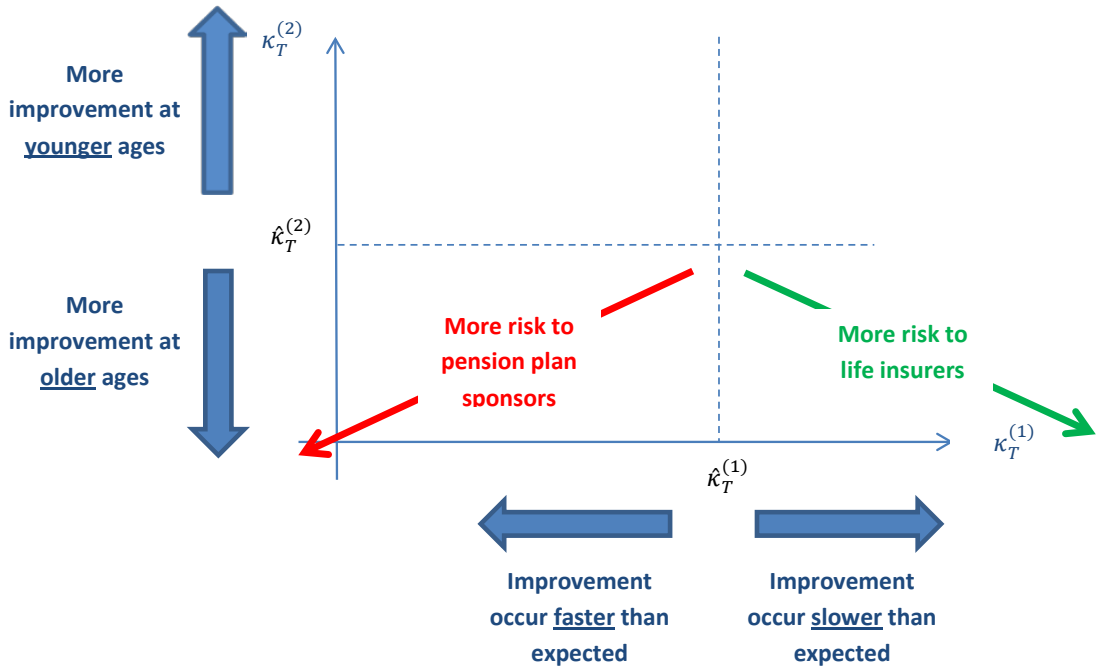
An Indicator of Longevity Risk

- It is a "simple" summary of a mortality pattern.
- The indicator contains only two numbers, $k_t^{(1)}$ and $k_t^{(2)}$, each of which is readily interpretable and they together tell how mortality rates at different ages change with time.
- It has the new-data invariant property. This property is important; because, as a proper indictator, we cannot allow new data to alter the index values of previous years. The logic is just the same as that we cannot alter the S&P 500 Index values on previous days.

A Graphical Risk Metric

- In the last section, we model the M5 mortality indexes $(k_t^{(1)}, k_t^{(2)})$ jointly. Here, we examine a joint view of them.

- Consider a Cartesian coordinate plane with $k_t^{(1)}$ as the **horizonal** axis and $k_t^{(2)}$ as the **vertical** axis, at a future time point $t = T$:



K1 and K2 Risks

- Future mortality rates are governed by $k_T^{(1)}$ and $k_T^{(2)}$, which are both random.
- We coin the uncertainty along the horizonal and vertical dimensions as **"K1 risk"** and **"K2 risk"**, respectively.
- The dotted lines show the best estimate (the MMSE forecast) of $k_T^{(1)}$ and $k_T^{(2)}$, respectively. These two lines divide the cartesian plane into four regions. We are the most interested in the two lower regions.

K1 and K2 Risks

- The lower left-hand region represents the situation when both $k_T^{(1)}$ and $k_T^{(2)}$ turn out to be lower than expected. To pension plan sponsors and annuity providers, this situation is the most undesirable, because the overall mortality improves more than expected and at the same time improvement at older ages is heavier.

- A mathematical argument: Under the CBD model, the logit-transformed death probability is given by $k_T^{(1)} + (x - \bar{x})k_T^{(2)}$. At older ages, i.e., when $x > \bar{x}$ (we use an age range of 40 to 90, which gives $\bar{x} = 65$), the resulting death probability is the lowest when both $k_T^{(1)}$ and $k_T^{(2)}$ are low.

K1 and K2 Risks

- The lower right-hand region represents the $k_T^{(1)}$ is higher than expected but $k_T^{(2)}$ is lower than expected. To life insurers selling term life insurances to younger people, this situation is the most undesirable, because mortality in general improves less than expected and improvement concentrates on older but not younger ages. The mathematical argument is similar to that for the previous case.

- This simple diagram partly explains why natural hedging often cannot work perfectly in practice. As an annuity provider, you can offset some "K1 risk" by acquiring a life insurance book. However, the life insurance book cannot offset (but indeed brings more) exposure to "K2 risk".

Joint Prediction Regions

- Conventionally, isolated (point-wise) prediction intervals (IPI) are used to quantify the uncertainty in future mortality indices.
- A pointwise interval only reflects uncertainty in ONE variable at ONE single time point.
- Joint Prediction Regions (JPR) is a joint simultaneous prediction region with coverage probability $0 < 1 - \alpha \leq 1$ for the l-ahead prediction of a vector time-series $\{Z_{T+l,m} \ \ (m = 1, \cdots, s)\}$.

$$\Pr(\mathbf{Z_T(l)} \in \mathbf{JPR}) = 1 - \alpha.$$

Joint Prediction Regions

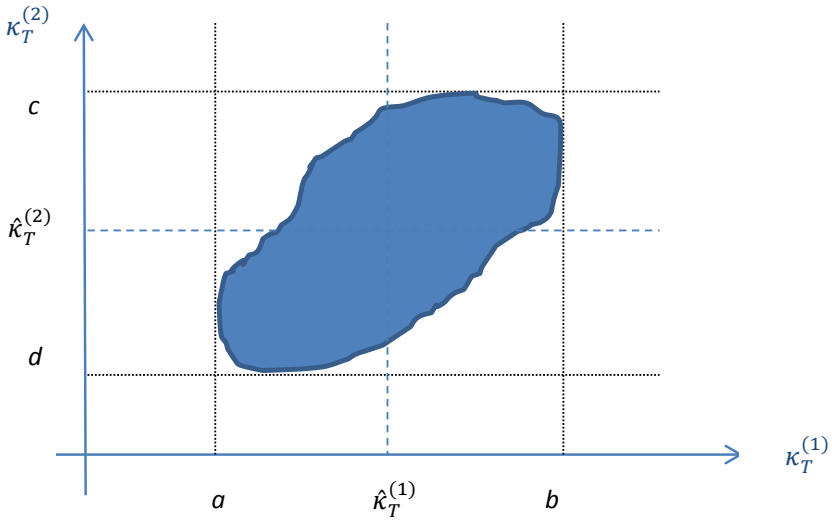
- Given that the association between $k_T^{(1)}$ and $k_T^{(2)}$ is of important to different financial institutions, there is a need to consider the joint prediction region for $k_T^{(1)}$ and $k_T^{(2)}$, rather than just the marginal prediction intervals for the two indexes.

- The joint prediction region can serve as a **graphical metric of longevity risk**. First of all, the area of a joint prediction region indicates the aggregate level of uncertainty (i.e., risk). Second of all, the shape of a joint prediction region indicates the **longevity risk profile** of a financial institution.

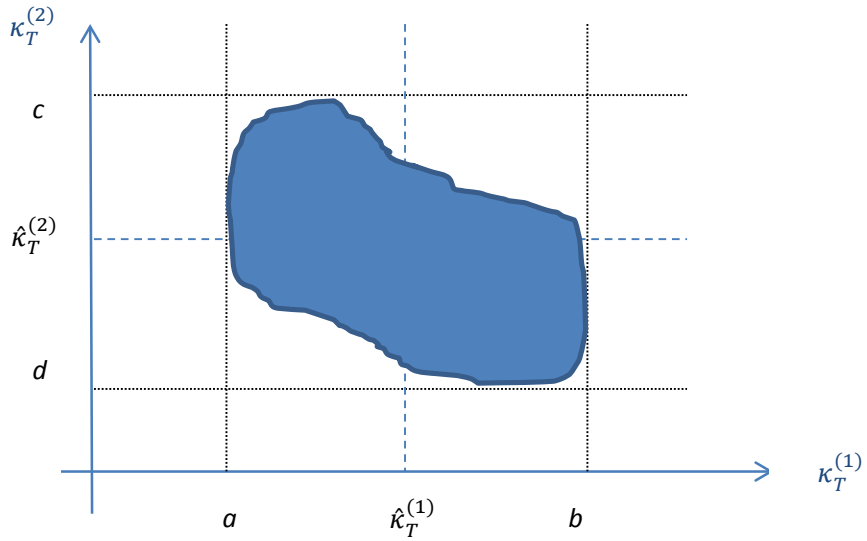
- Consider the following two examples:

Joint Prediction Regions

EXAMPLE 1



EXAMPLE 2



Joint Prediction Regions

- The marginal prediction intervals, (a, b) and (c, d), are the same for both examples.
- The areas of the above two joint prediction regions are similar, representing similar levels of aggregate uncertainty.
- However, to a specific financial institution, these two risk metrics are highly different. For a pension plan provider, Example 1 represents way more risk than Example 2; the opposite is true for a seller of term life insurance products.

Constructing Joint Prediction Regions

- There are different methods for constructing joint prediction regions.
- We propose a numerical method, which can be implemented as follows:
  - Calculate the $\hat{k}_T^{(1)}$ and $\hat{k}_T^{(2)}$, the MMSE forecasts of the CBD mortality indexes.
  - Calculate the $s_T^{(1)}$ and $s_T^{(2)}$, the standard error of the MMSE forecasts of the CBD mortality indexes.
  - From the estimated VARIMA model, simulate, say N, pairs of $k_T^{(1)}$ and $k_T^{(2)}$.

Constructing Joint Prediction Regions

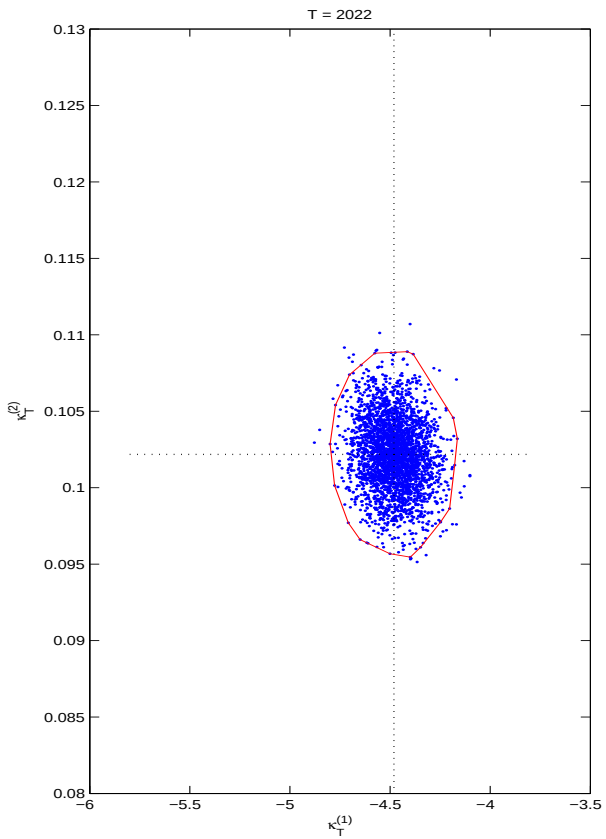
- For each simulated pair of $k_T^{(1)}$ and $k_T^{(2)}$, calculate its **weighted** distance to the MMSE forecast with the following formula:

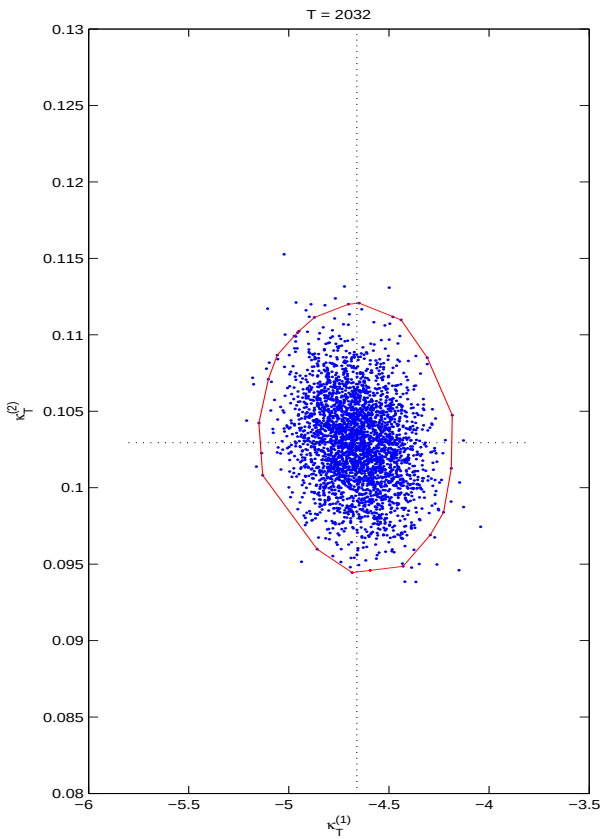
$$\sqrt{\left(\frac{k_T^{(1)} - \hat{k}_T^{(1)}}{s_T^{(1)}}\right)^2 + \left(\frac{k_T^{(2)} - \hat{k}_T^{(2)}}{s_T^{(2)}}\right)^2}.$$

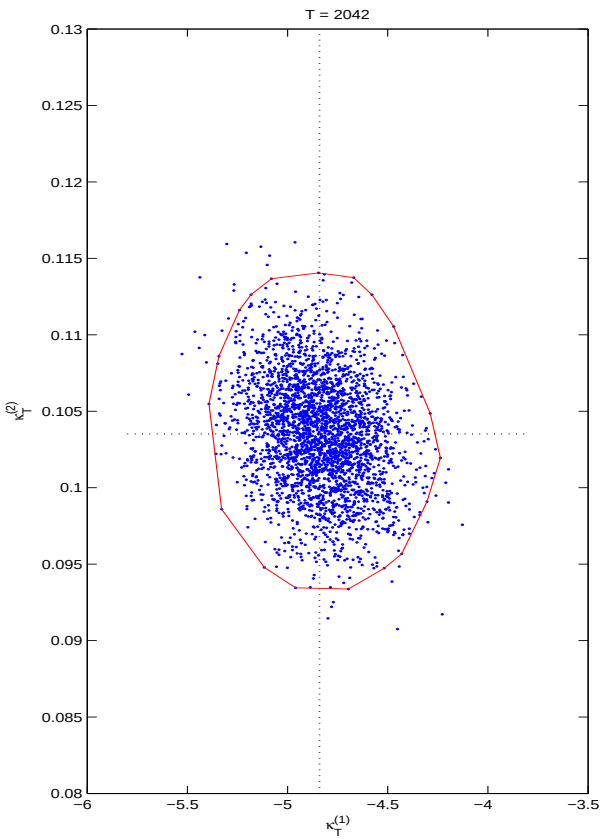
- Sort the N simulated pairs of $k_T^{(1)}$ and $k_T^{(2)}$ by their distances to the MMSE forecast.
- Pick the $\lceil (1-\alpha)N \rceil$ pairs with the shortest distances. Draw a convex hull to enclose these $\lceil (1-\alpha)N \rceil$ pairs of simulated pairs of $k_T^{(1)}$ and $k_T^{(2)}$. Geometrically speaking, the convex hull is the smallest convex set that contains the selected $\lceil (1-\alpha)N \rceil$ points.
- The convex hull drawn is a $100(1-\alpha)\%$ joint prediction region for $k_T^{(1)}$ and $k_T^{(2)}$.

5. Illustrative Examples

- **Example 1:** 99% JPR Results for English & Welch Males at $Year = 2022, 2032, 2042$; $N = 3000$, using VARIMA(5,1,0) model:







Example 1: Discussion

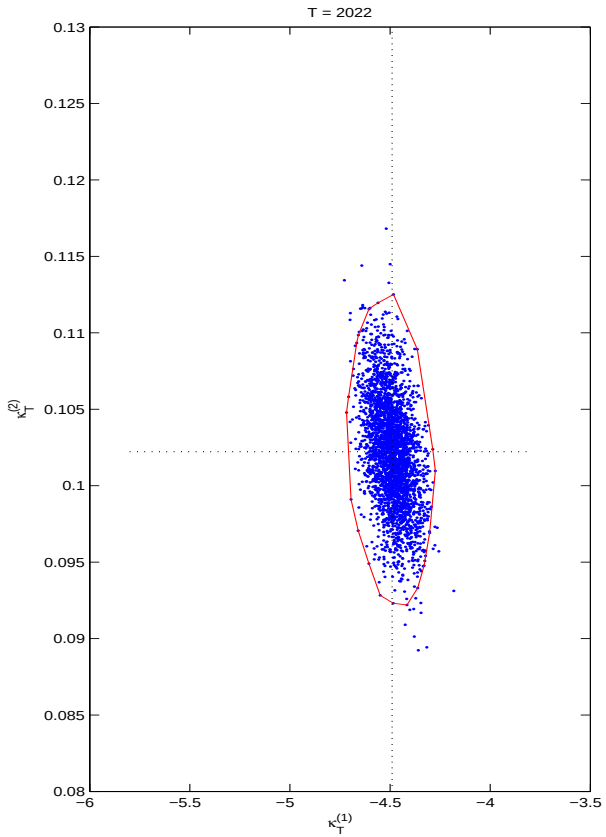
1. The area of the JPR represents the overall level of uncertainty. The JPR gets bigger over time, indicating uncertainty increases as we predict farther into the future.

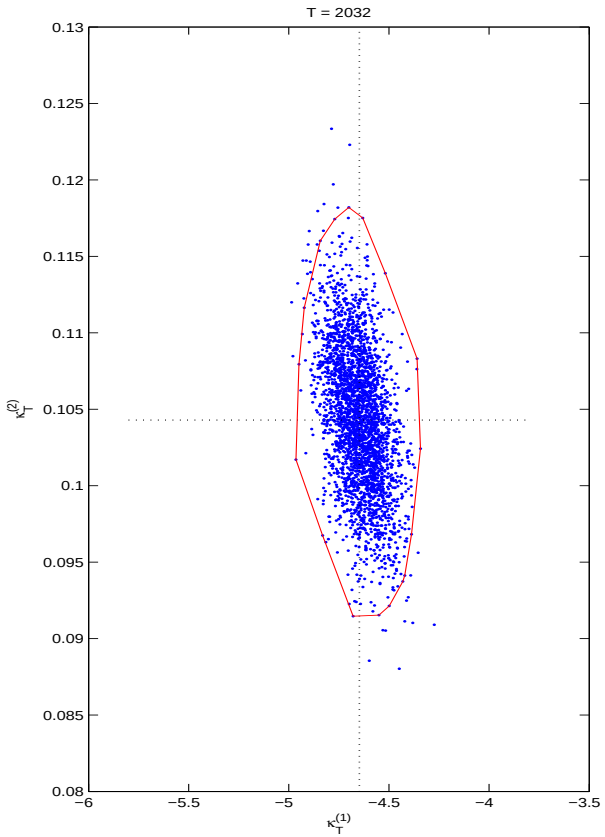
2. The dotted line shows the best estimate of $k_T^{(1)}$ and $k_T^{(2)}$, respectively. The vertical line ($k_T^{(1)}$) shifts leftwards, indicating that the overall level of mortality is reducing. On the other hand, the horizontal line ($k_T^{(2)}$) shifts upwards slowly, indicating that that mortality at younger ages (below $\bar{x} = 65$) is improving (slightly) faster than at older ages. Overall, the centroid of the JPR moves to the upper-left-hand corner over time.

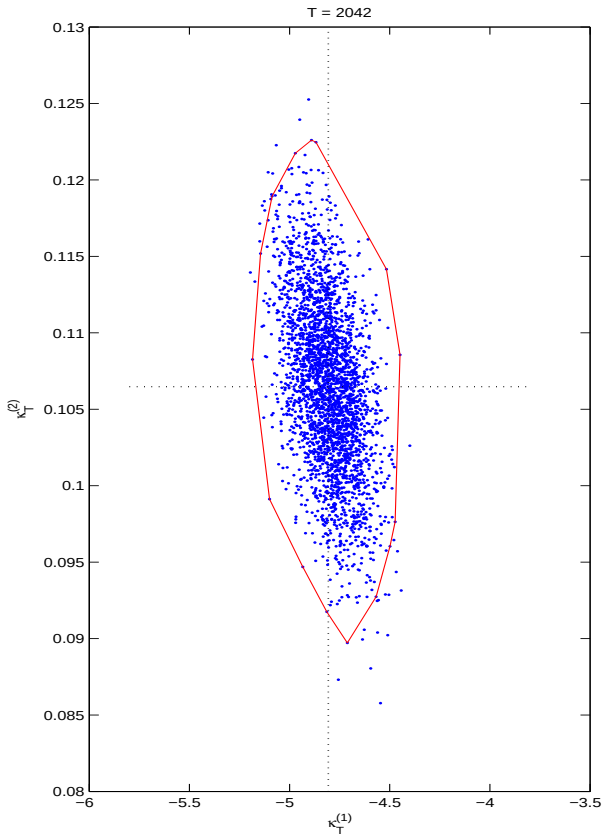
3. The tilt of the JPR is minimal. The lower quadrants are similar in size. We conclude that for this particular population, pension providers and life insurers are subject to similar levels of risk.

Example 2: Canadian Males

- **Example 2:** 99% JPR Results for Canadian Males at $Year = 2022, 2032, 2042$; $N = 3000$, using VARIMA(1,1,0) model:







Example 2: Discussion

1. The scales of axes in both EW and Canadian JPR graphs are the same, so that these two diagrams are comparable.

2. The JPRs for Canada are narrower and taller than those for EW. This means that Canadian males are subject to less K1 risk than EW males, but more K2 risk than EW males. In laymens terms, what this means is that for Canadians, there is less uncertainty associated with overall mortality improvements, but more uncertainty associated how mortality improvements are different among different age groups.

Example 2: Discussion (cont'd)

3. The JPRs for Canada are more tilted. Specifically, we observe that the region and the dots roughly form a diagonal, running from the upper-left-hand to the lower-right-hand corner.

4. Let us focus on the lower portion of the region. The lower left quadrant is much smaller than the lower right quadrant. A pattern like this means that in Canada, life insurers selling term life insurance are subject to relatively more amount of risk than pension plan providers.

Thank You!