Profit participation annuities: a business profitability analysis within a demographic risk sensitive approach

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MOTIVATION

New stress test for life insurance and need of general project aimed at:

- realizing the proper management policy, for optimizing the performance
- protecting the policyholders and pension beneficiaries
- preserving stability and transparency of the financial system

Two main risk drivers: ↗ financial risk

the core of an internal model ↘ demographic risk

The demographic risk

Betterment of the survival trend ↗ adult ages

↘ long duration

↓

underestimation of the future cash flows

business financial result:

the financial differential between
the total amounts resulting from
investments made by the insurer
and the cash flows at the end of the period,
consisting of the benefits payable and the accruals.

This result has a selfconsisting value with reference to the business profitability, and it is also an indicator of performance solvency.

Research focus

measure the ability to absorb
the impact of risk sources
preserving the competitiveness
and achieving a satisfactory performance

The contract

Participating policies,
in particular life annuities,
that participate in the insurer’s profits
on the basis of a participation rate
defined in the contractual scheme.

AIM OF THE RESEARCH

The study of performance indicators apt to measure the impact of the systematic demographic components on the financial results of a portfolio of life annuity participating policies, under different hypotheses for the participating quota and with stochastic assumptions for the accumulation and the discounting financial processes and for the survival description

MAIN NUMERICAL RESULTS

- Sensitivity of the portfolio performance to the survival projection
- Break even points
- Time of optimum performance

AGENDA

1) contractual profiles and mathematical structures

2) the internal valuation approach by means of accounting indexes

3) financial and demographic scenarios

1) numerical evidences

The contract: the Profit Participation Annuity

R_t financial result in the period [t, t + 1]

If R_{t+1}, net of the annual quota of the administrative expenses γ, is positive

↓

a bonus $\alpha(R_{t+1} - \gamma)$ is added to the provision allocated in t + 1

Practical implications of a correct ALM strategy

- the choice of the participation rate sustainable for the insurer and appealing for the policyholders, based on the weighted period results
- appropriate financial ratios and balance sheet indexes for performance valuations

STOCHASTIC PROCESSES AT TIME 0

Financial

Random movements of:

- the interest rates involved in provisions' estimate;
- the return on investment rates

Demographic

- Best estimate of the evolution in time of the accidental deviations of the number of deaths from its expected value, as well as of the systematic deviations of the mortality rates

- Filtrations at time t on the three stochastic components:

$$\Im' = \{F'(s)\}_{s\in\{0,1,\dots,t\}} \qquad \Im'' = \{F''(s)\}_{s\in\{0,1,\dots,t\}}$$

$$\aleph = \{X(s)\}_{s\in\{0,1,\dots,t\}}$$

- The overall information are summarized by the filtration

$$\Re = \{R(s)\}_{s\in\{0,1,\dots,t\}}$$

with $$R(s) = F'(s) \cup F''(s) \cup X(s)$$

Financial stochastic assumptions

Stochastic accumulation process

$$\{r(t,s), 0 \le t \le s\}$$

- CIR model

Stochastic discount process

$$\{v(t,s), 0 \le t \le s\}$$

- HJM model

Life annuity with deferment period T, premium payment until the time |, annual installments b_s

Stochastic provision at time t:

$$V_t = \sum_{i=t}^{\infty} \left(b_i 1_{(T \le i \le K(x))} - P_i 1_{(i<\tau)} \right) v(t,i)$$

Financial result of the (t+1)-th accounting period:

$$TR_{t+1} = \min\left[R_{t+1}, (1-\alpha) R_{t+1} + \alpha\gamma \right]$$

Return On Equity (ROE) index

how much the total amount collected by the insurer to manage a certain insurance business yields in a certain time interval

ROE = ratio of profits to equity

$$ROE_{t+1} = \frac{TR_{t+1}}{\sum_{j=0}^{r-1} P_j r(j,t)}$$

Demographic scenario

Stochastic proportional hazard model

$$B(x,t) = \mu(x+t)Y(t)$$

with $$dY(t) = \beta[1 - Y(t)]dt + \sigma\sqrt{Y(t)}dW(t)$$

$$Y^*(t) = Y(t) - 1$$

$$\hat{Y}^*(t) = \sum_{i=0}^{t-1} \phi^i \sigma_a \sqrt{1 + \frac{2\phi}{1+\phi} \hat{Y}^*_{t-i+1} a_{t-1}} \qquad t = 1,2,3,\ldots$$

a_t standard normal variables independent on $\{a_i / i \leq t-1\}$

$\phi > 0$ estimator of the drift rate

$\sigma_a > 0$ diffusion parameter

CIR centred process

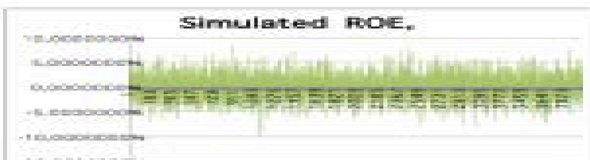
Numerical evidences

Portfolio assumptions

- 1000 immediate participating life annuities with 10 annual anticipated benefits
- single premium paid in 0
- age at issue 65
- pure premium calculated at 2%

Demographic sensitivity assumptions

- minimum projection: long term mean equal to 1
- medium projection: expected lifetime duration increases from 18 to 20
- maximum projection: expected lifetime duration increases of 22



An example of Simulated ROE Medium Projection

t	Q_ε (ε=99%)	Min Simulated ROE	Max Simulated ROE
2	-0.0219%,	-0.17%	0.15%
4	-0.33%	-2.1%	2.5%
6	-2.17%	-4.8%	5%
8	0.11%	-7.9%	8%
9	2.68%	-4%	4%

t	Q_ε (ε=99%)	Min Simulated ROE	Max Simulated ROE
2	-0.0037%	-0.11%	0.12%
4	1.47%	-2.9%	2.5%
6	0.48%	-4.2%	4.8%
8	3.36%	-8%	8.5%
9	1.18%	-10%	7%

t	Qe (e=99%)	Min Simulated ROE	Max Simulated ROE
2	-0.0301%	-0.175%	0.15%
4	0.18%	-1.1%	1%
6	1.15%	-3.4%	4%
8	3.44%	-15%	15%
9	4.21%	-15%	17%

E(ROE)	α=20%	α=40%	α=60%	α=80%
t=1	-3.4466%	-3.4466%	-3.4466%	-3.4466%
t=2	-3.2120%	-3.2120%	-3.2120%	-3.2120%
t=3	2.7579%	2.2307%	1.7034%	1.1762%
t=4	2.8352%	2.2841%	1.7331%	1.1821%
t=5	2.7953%	2.2502%	1.7051%	1.1600%
t=6	3.7018%	2.9156%	2.1294%	1.3433%
t=7	2.8303%	2.2589%	1.6874%	1.1160%
t=8	2.1137%	1.7190%	1.3243%	0.9296%
t=9	1.1903%	1.0255%	0.8607%	0.6959%

E(ROE)	α=20%	α=40%	α=60%	α=80%
t=1	-3.7374%	-3.7374%	-3.7374%	-3.7374%
t=2	-3.5290%	-3.5290%	-3.5290%	-3.5290%
t=3	2.7346%	2.2132%	1.6918%	1.1703%
t=4	2.7988%	2.2568%	1.7149%	1.1730%
t=5	2.7439%	2.2116%	1.6794%	1.1472%
t=6	3.6372%	2.8672%	2.0971%	1.3271%
t=7	2.7450%	2.1949%	1.6448%	1.0947%
t=8	2.0047%	1.6372%	1.2698%	0.9124%
t=9	1.0821%	0.9443%	0.8066%	0.6688%
t=10	1.0629%	1.7254%	0.6682%	0.6254%

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