Mortality Surface by Means of Continuous Time Cohort Models

Petar Jevtić, Elisa Luciano and Elena Vigna

Longevity Eight 2012,
Waterloo, Canada, 7-8 September 2012

Outline

Motivation

- **Insurance companies** and **pension funds** are exposed to **mortality risks**.
- The development of a **liquid** and **transparent** mortality-linked capital market is desired.
- Mortality-risk appraisal consisting in an **accurate**, yet **easy-to-handle description of human survivorship** is fundamental in this respect.
- A number of proposals have been put forward, some of which have great potential, but still without any consensus being reached on the best approach to mortality risk modeling.

Goals

We have developed our model with the following goals in mind:

1. Analytical tractability
2. Parsimoniousness
3. Fit to historical data
4. Null or low probability of negative intensities (specific to our model)
5. Possibility and ability of deterministic forecasting
6. Possibility and ability of stochastic forecasting
7. Possibility and ability of measuring correlation among different generations

A single generation

The standard uni-dimensional framework

Stochastic mortality of a given generation is described by means of a Cox process

- $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, where $\mathbb{P}$ is the real-world probability measure
- $\{\mathcal{F}_t : 0 \leq t \leq T\}$ satisfies the usual technical conditions
- $\mu(t, x)$ - the mortality intensity of an individual belonging to a given generation, initial age x, at calendar time t
- n - number of state processes
- $\boldsymbol{X}(t) = [X_1(t), ..X_n(t)]^T$ - the vector of state processes
- $R : D \to \mathbb{R},\ D \subset \mathbb{R}^n$

We define:

$$\mu(t) \stackrel{\text{def}}{=} R(\boldsymbol{X}(t))$$

Consequently, the survival probability from t to T, conditional on being alive at t is:

$$S(t, T) = \mathbb{E}_t\left[e^{-\int_t^T \mu(s)ds}\right] = \mathbb{E}_t\left[e^{-\int_t^T R(X(s))ds}\right]$$

A single-generation DPS framework

The Duffie, Pan and Singleton (2000) framework

If $d\mathbf{X}(t) = \boldsymbol{\lambda}(\mathbf{X}(t))dt + \boldsymbol{\sigma}(\mathbf{X}(t))d\mathbf{Z}(t)$, having

- $\boldsymbol{Z}$ a $(\mathcal{F}_t)$-standard Brownian motion in $\mathbb{R}^n$,
- $\boldsymbol{\lambda} : D \to \mathbb{R}^n$, $\boldsymbol{\sigma} : D \to \mathbb{R}^{n\times n}$, $\boldsymbol{\lambda}$, $\boldsymbol{\sigma}$, and $R : D \to \mathbb{R}$ are affine,
- $\boldsymbol{\lambda}(x) = \mathbf{K}_0 + \boldsymbol{K}_1 x$, for $\mathbf{K} = (\mathbf{K_0}, \mathbf{K_1}) \in \mathbb{R}^n \times \mathbb{R}^{n\times n}$,
- $(\boldsymbol{\sigma}(x)\boldsymbol{\sigma}(x)^T)_{ij} = (\boldsymbol{H}_0)_{ij} + (\boldsymbol{H}_1)_{ij} \cdot x$, for $\boldsymbol{H} = (\boldsymbol{H}_0, \boldsymbol{H}_1) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n\times n}$,
- $R(x) = r_0 + \boldsymbol{r}_1 x$, where $(r_0, \boldsymbol{r}_1) \in \mathbb{R} \times \mathbb{R}^n$.

we have that

$$\mathbb{E}[e^{-\int_t^T R(\mathbf{X}(s))ds} \mid \mathcal{F}_t] = e^{\alpha(t;T)+\beta(t;T)\cdot\mathbf{X}(t)},$$

where $\alpha(\cdot) : \mathbb{R}^+ \to \mathbb{R}^n$ and $\beta(\cdot) : \mathbb{R}^+ \to \mathbb{R}^n$ satisfy the complex-valued ODEs

$$\beta'(t;T) = \mathbf{r}_1 - \boldsymbol{K}_1^T \beta(t;T) - \beta(t;T)^T \boldsymbol{H}_1 \beta(t;T)/2,$$
$$\alpha'(t;T) = r_0 - \boldsymbol{K}_0 \beta(t;T) - \beta(t;T)^T \boldsymbol{H}_0 \beta(t;T)/2,$$

with boundary conditions $\alpha(T,T) = \beta(T,T) = 0$.

Transition to the entire mortality surface

Transitioning from single-generation to mortality surface

- We label each generation with a proper index $i \in \mathbb{I} \subset \mathbb{N}$,
- Each generation has its own mortality intensity,
- We assume that the state processes of each of the generations are driven by Brownian motions that have a correlation **unique** for that generation.
- In effect, **each generation** is assigned **its own correlation matrix**.

Given the n state processes driving the mortality intensity of generation i, we have:

$$d\mathbf{X}^i(t) = \boldsymbol{\lambda}(\mathbf{X}^i(t))dt + \boldsymbol{\sigma}(\mathbf{X}^i(t))d\mathbf{W}^i(t)$$

where

- $\boldsymbol{W}(t) = [W_1^i(t), W_2^i(t), \ldots W_n^i(t)]$
- $\boldsymbol{\rho}^i_{n\times n} = \{\rho^i_{lm}\}_{1\leq l,m\leq n}$ - instantaneous correlation matrix proper of generation i
- $\rho^i_{lm}dt = \langle dW_l^i(t), dW_m^i(t)\rangle$

Using the DPS framework for a mortality surface

In order to use the DPS framework in case of a mortality surface, we need to:

- Use the Cholesky decomposition of the correlation matrix ρ^i

$$\rho^i = \mathbf{H}^i(\mathbf{H}^i)^T, \text{ and}$$

- Transform $\boldsymbol{W}(t)$ to

$$d\mathbf{W}^i(t) = \mathbf{H}^i d\mathbf{Z}(t),$$

  where $\boldsymbol{Z}(t)$ is a vector of uncorrelated Brownian motions.

Finally, we obtain:

$$d\mathbf{X}^i(t) = \lambda(\mathbf{X}^i(t))dt + \sigma(\mathbf{X}^i(t))\mathbf{H}^i d\mathbf{Z}^i(t)$$

allowing us to make use of the DPS framework.

Additional assumptions

We make the following additional assumptions:

- For any generation i, each of its state processes follows an Ornstein-Uhlenbeck dynamic
$$dX_k^i(t) = \psi_k X_2^i dt + \sigma_k dW_k^i(t),\ \ k = \overline{1,n},$$
where $\psi_k \in \mathbb{R}, \sigma_k > 0$ and $W_1^i(t), W_2^i(t), \ldots, W_n^i(t)$ are correlated.
- Using a more compact form, we have:
$$d\boldsymbol{X}^i(t) = \boldsymbol{\Psi}\boldsymbol{X}^i(t)dt + \boldsymbol{\Sigma} d\mathbf{W}^i(t),$$
or
$$d\boldsymbol{X}^i(t) = \boldsymbol{\Psi}\boldsymbol{X}^i(t)dt + \boldsymbol{\Sigma}\mathbf{H}^i d\mathbf{Z}^i(t),$$
where $\boldsymbol{\Psi} = \text{diag}[\psi_1, \psi_2, ..., \psi_n]$, and $\boldsymbol{\Sigma} = \text{diag}[\sigma_1, \sigma_2, ..., \sigma_n]$.
- Finally, we set
$$R(\mathbf{X}^i(t)) = \mathbf{1} \cdot \mathbf{X}^i(t).$$

The general solution

- In the survivorship context, it is convenient to set the valuation time $t = 0$, and reason in terms of remaining life $\tau = T - t$. With this transformation, using the DPS framework, we need to solve n systems of the form:

$$\hat{\beta}'(\tau) = -\mathbf{1} + \mathbf{\Psi}\hat{\beta}(\tau)$$
$$(\hat{\alpha}^i)'(\tau) = \frac{1}{2}\hat{\beta}(\tau)^T \mathbf{\Sigma}\boldsymbol{\rho}^i\mathbf{\Sigma}^T\hat{\beta}(\tau)$$
$$(\hat{\alpha}^i)'(0) = \hat{\beta}'(0) = 0.$$

- Finally, we have

$$S^i(0,\tau) = \mathbb{E}\left[\exp\left(-\int_0^\tau \mathbf{1}\cdot\mathbf{X}^i(s)ds\right)\right] = e^{\hat{\alpha}^i(\tau)+\hat{\beta}(\tau)\cdot\mathbf{X}^i(0)}.$$

- The n solutions of the n systems are given by:

$$\hat{\beta}(\tau) = -\int_0^\tau e^{\mathbf{\Psi}(\tau-s)}\cdot\mathbf{1}ds$$
$$\hat{\alpha}^i(\tau) = \int_0^\tau \frac{1}{2}\hat{\beta}(s)^T\mathbf{\Sigma}\boldsymbol{\rho}^i\mathbf{\Sigma}^T\hat{\beta}(s)ds$$

Two models

Reminder: Previously, we have omitted the argument x for notational convenience.

Two models of different complexities

The General OU APC Model

- $\psi_1, \psi_2, ..., \psi_n, \sigma_1, \sigma_2, ..., \sigma_n$ depend on x
- $\hat{\alpha}^i_x(\tau), \hat{\beta}_x(\tau)$ depend on x
- $S^i_x(0,\tau) = e^{\hat{\alpha}^i_x(\tau)+\hat{\beta}_x(\tau)\cdot \mathbf{X}^i_x(0)}$
- $S^i_{x+t}(t,T) = e^{\alpha^i_{x+t}(T-t)+\beta_{x+t}(T-t)\cdot \mathbf{X}^i_{x+t}(t)}$

The Simple OU APC Model

- All of the coefficients are **not age-dependent**.
- $S^i(0,\tau) = e^{\hat{\alpha}^i(\tau)+\hat{\beta}(\tau)\cdot \mathbf{X}^i(0)}$

Onward, we will be examining the **Simple OU APC Model**.

The data set

An important question:

How many **factors** do we actually need for calibration?

Our calibration data set:

- **The male population of the United Kingdom**,
- Cohort death rates for life age $\mathbf{x} = \mathbf{40}$,
- We examine them until they have reached the age of **59** having $\tau = 1, \ldots, 19$
- The generations *i* span from **1900-1950**, with a **5-year increment**,
- **11 cohorts** in total.

$$\tilde{S}_x^i(0, \tau) = \prod_{s=1}^{\tau}(1 - q_i(x + s - 1, x + s))$$

The survival probability surface

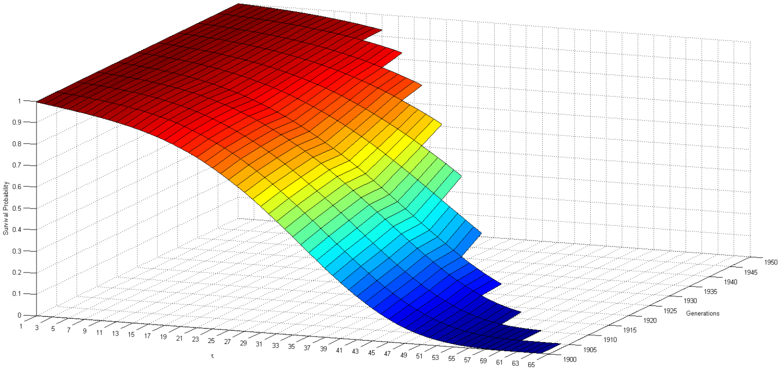


Figure 1: The survival probability surface representing the data set

The survival probability surface - the relevant segment

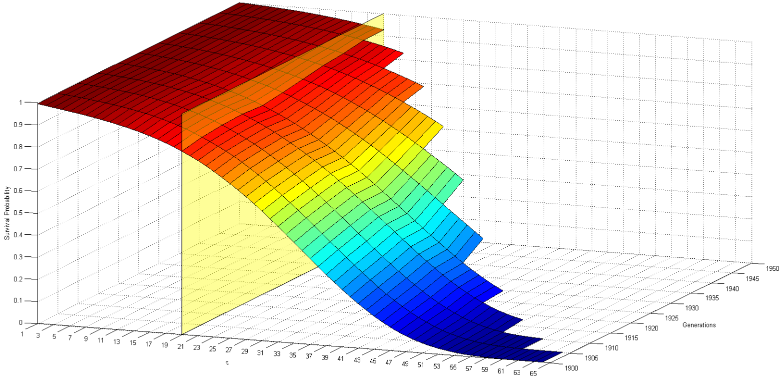


Figure 2: The relevant segment of the survival probability surface representing the data set

Principal Component Analysis

Application of the PCA

We use the observed survival probabilities to compute the corresponding average mortality intensity for generation i.

$$\bar{\mu}_x(0,\tau_j) = -\frac{1}{\tau_j}\log \tilde{S}_x^i(0,\tau_j) = -\frac{1}{\tau_j}\sum_{s=1}^{\tau_j}\log(1-q_i(x+s-1,x+s))$$

Results from the PCA:

- The **mean** and the **first** principal component account for **95**.**72**% of the data,
- The **mean**, the **first** and the **second** principal component account for **99**.**83**% of the data.

Conclusion

The obtained results indicate that having **two factors** is a rational initial choice.

The two-factor model

- Within the simple APC model, we have, for a given generation i

$$dX_1(t) = \psi_1 X_1 dt + \sigma_1 dZ_1(t)$$
$$dX_2^i(t) = \psi_2 X_2 dt + \sigma_2 \rho^i dZ_1(t) + \sigma_2 \sqrt{1-(\rho^i)^2} dZ_2(t)$$

- The mortality intensity of generation i is given by

$$\mu^i(t) = X_1(t) + X_2^i(t).$$

- By solving the ODEs, we obtain that:

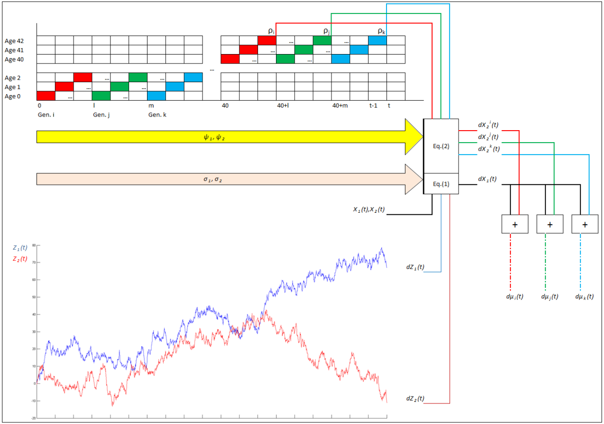
$$\hat{\beta}_j(\tau) = -\int_0^\tau e^{\psi_j(\tau-s)} ds = \frac{1}{\psi_j}\left(1 - e^{\psi_j \tau}\right), \; j \in \{1,2\}$$

$$\hat{\alpha}^i(\tau) = \sum_{j=1}^{2} \frac{\sigma_j^2}{2\psi_j^3}\left(\psi_j \tau - 2e^{\psi_j \tau} + \frac{1}{2}e^{2\psi_j \tau} + \frac{3}{2}\right)$$
$$+ \frac{\rho^i \sigma_1 \sigma_2}{\psi_1 \psi_2}\left(\tau - \frac{e^{\psi_1 \tau}}{\psi_1} - \frac{e^{\psi_2 \tau}}{\psi_2} + \frac{e^{(\psi_1+\psi_2)\tau}}{\psi_1+\psi_2} + \frac{\psi_1^2 + \psi_1\psi_2 + \psi_2^2}{\psi_1\psi_2(\psi_1+\psi_2)}\right).$$

Visual representation of the model - An example with three generations

Main characteristics:

- every generation characterized by its own ρ



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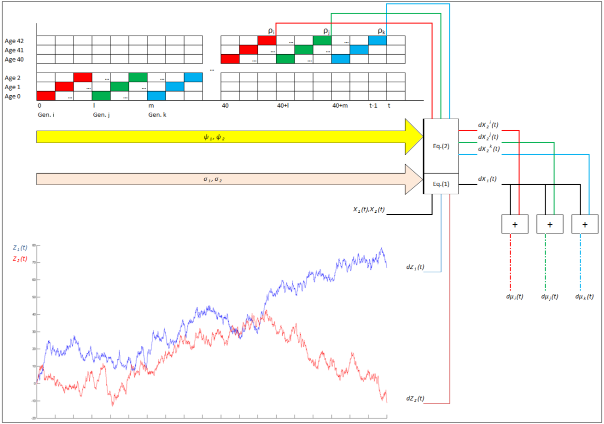

$$dX_1(t) = \psi_1 X_1 dt + \sigma_1 dZ_1(t)$$

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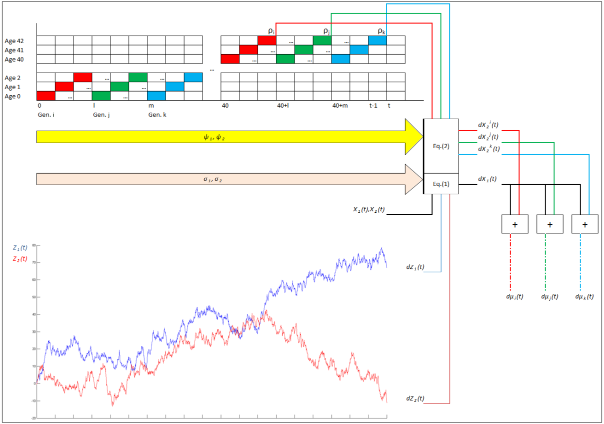
Visual representation of the model - An example with three generations

Main characteristics:

- every generation characterized by its own ρ
- $\psi_1, \psi_2, \sigma_1, \sigma_2$ same for all generations
- $Z_1(t), Z_2(t)$ are orthogonal BMs



$$dX_1(t) = \psi_1 X_1 dt + \sigma_1 dZ_1(t)$$
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Theoretical constraint

The specified model for the mortality intensity of one generation is known in the interest-rate domain as the **two-factor Gaussian model**, or **G2++** (Brigo and Mercurio, 2006). Each intensity is Gaussian, and can become negative with a positive probability:

$$Pr(\mu^i(\tau) < 0) = \Phi\left(-\frac{E(\mu^i(\tau))}{\sqrt{Var(\mu^i(\tau))}}\right)$$

where $\Phi(\cdot)$ is the **CDF** of the **standard normal distribution**,

$$E(\mu^i(\tau)) = f^i(0,\tau) + \frac{\sigma_1^2}{2\psi_1^2}\left(1 - e^{\psi_1\tau}\right)^2 + \frac{\sigma_2^2}{2\psi_2^2}\left(1 - e^{\psi_2\tau}\right)^2 + \rho^i\frac{\sigma_1\sigma_2}{\psi_1\psi_2}\left(1 - e^{\psi_1\tau}\right)\left(1 - e^{\psi_2\tau}\right),$$

$$Var(\mu^i(\tau)) = -\frac{\sigma_1^2}{2\psi_1}\left(1 - e^{2\psi_1\tau}\right) - \frac{\sigma_2^2}{2\psi_2}\left(1 - e^{2\psi_2\tau}\right) - 2\rho^i\frac{\sigma_1\sigma_2}{\psi_1+\psi_2}\left(1 - e^{(\psi_1+\psi_2)\tau}\right),$$

and $f^i(0,\tau)$ is the **forward mortality intensity** for the instant τ:

$$f^i(0,\tau) = -\frac{\partial \log S^i(0,\tau)}{\partial\tau} = e^{\psi_1\tau}X_1(0) + e^{\psi_2\tau}X_2^i(0)$$
$$-\sum_{j=1}^{2}\frac{\sigma_j^2}{2\psi_j^3}\left(\psi_j - 2\psi_j e^{\psi_j\tau} + \psi_j e^{2\psi_j\tau}\right) - \frac{\rho^i\sigma_1\sigma_2}{\psi_1\psi_2}\left(1 - e^{\psi_1\tau} - e^{\psi_2\tau} + e^{(\psi_1+\psi_2)\tau}\right)$$

Correlations

Benefits of the model

The model enables the derivation of formulas for **instantaneous correlations** among intensities of different generations.

- For the generation i the instantaneous mortality intensities follows the SDE:

$$d\mu^i(t) = [\psi_1 X_1(t) + \psi_2 X_2^i(t)]dt + (\sigma_1 + \rho^i \sigma_2) dZ_1(t) + \sigma_2 \sqrt{1 - (\rho^i)^2} dZ_2(t)$$

- **Instantaneous correlation** between $\mu^i(\cdot)$ and $\mu^j(\cdot)$ is equal to

$$\text{Corr}[d\mu(t_i), d\mu(t_j)] = \frac{(\sigma_1 + \rho^i \sigma_2)(\sigma_1 + \rho^j \sigma_2) + \sigma_2^2 \sqrt{(1 - (\rho^i)^2)(1 - (\rho^j)^2)}}{\sqrt{(\sigma_1 + \sigma_2 \rho^i)^2 + \sigma_2^2 (1 - (\rho^i)^2)} \sqrt{(\sigma_1 + \sigma_2 \rho^j)^2 + \sigma_2^2 (1 - (\rho^j)^2)}}$$

Simulation

Benefits of the model

The model enables us to make **stochastic forecasting** of the survival probability curve in an **arbitrary future time** $p > 0$.

- Simple two-factor model – no dependence on age.
- When viewed from time 0, survival curve at time $p > 0$, for a head in generation i, is the random object:

$$S^i(p,\tau) = e^{\alpha^i(\tau-p)+\beta(\tau-p)\cdot \mathbf{X}^i(p)}.$$

- Given the calibrated parameters, we simulate $X_1(p)$ and $X_2(p)$:

$$X_1(t) = \exp(\psi_1 t)X_1(0) + \sigma_1 \exp(\psi_1 t)\sqrt{\frac{1}{2\psi_1}(1-\exp(-2\psi_1 t))}\ Z_1$$

$$X_2(t) = \exp(\psi_2 t)X_2(0) + \sigma_2\rho^i \exp(\psi_2 t)\sqrt{\frac{1}{2\psi_2}(1-\exp(-2\psi_2 t))}\ Z_1$$

$$+\ \sigma_2\sqrt{1-(\rho^i)^2}\exp(\psi_2 t)\sqrt{\frac{1}{2\psi_2}(1-\exp(-2\psi_2 t))}\ Z_2$$

Parameters

Parameters

Initial values of factors are calibrated, because they cannot be observed!

Fixing the number of relevant factors to two results in:

- four parameters common to all generations: $[\psi_1, \psi_2, \sigma_1, \sigma_2]$
- three parameters specific to each generation: $[\rho^i, X_1^i(0), X_2^i(0)]$
- given $k = 11$ generations, we have in total: $4 + 3k = 37$ parameters

We collect them in:

$$\boldsymbol{\theta} = [\psi_1, \psi_2, \sigma_1, \sigma_2, \rho^1, \rho^2, ..., \rho^k, X_1^1(0), X_1^2(0), ..., X_1^k(0), X_2^1(0), X_2^2(0), ..., X_2^k(0)].$$

Given the meaning, we restrict to:

$$\Theta = \left\{ \psi_1, \psi_2 \in [-1, 1],\ \sigma_1, \sigma_2 \in [0, 1],\ \boldsymbol{\rho} \in [-1, 1]^k, \boldsymbol{X}_1(0) \in [-1, 1]^k \right\}.$$

Parameters

Controlling for low probability of negative mortality intensity

Given an a priori decision of $\Pr(\mu^i < 0) \leq 1\%$, for each generation i we choose $X_2^i(0), i \in \{1, ..., k\}$ during calibration.

- If we treat $\Pr(\mu^i < 0)$ as one more parameter, then given the equations describing the theoretical constraints, and all of the other parameters, we can find $X_2^i(0), i \in 1, 2, ..., k$ for any given τ.
- We need it for τ between 1 and the extreme age, which is 69.

For each generation i we choose $X_2^i(0)$ such that the constraint is satisfied for all τ.

Error (or cost) definition

Error (or cost) definition

We minimize mean square error between the actual and estimated parameters with the mean computed across $k = 11$ generations and durations $\tau = 19$.

$$\theta^* = \arg\min_{\theta \in \Theta} \sqrt{\frac{1}{k} \sum_{i=1}^{k} \sum_{j=1}^{\tau} (\tilde{S}^i(j) - S^i(j;\theta))^2}$$

Differential evolution - Our use of the algorithm

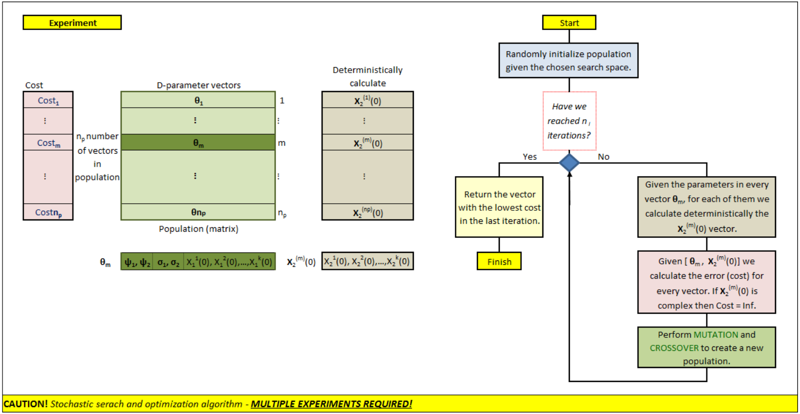

Figure 3: The Differential Evolution algorithm

Differential evolution - What have we done?

- 30 experiments, with $n_I = 100000$ iterations per experiment.
- Grid Computing Platform of The Wharton School, University of Pennsylvania
- Each experiment conducted on a single 2.5 GHz core with 4GB RAM
- Average duration of an experiment: 24h

Cost and Parameters

Table 1: Calibration results

	Cost	0.000620565995355	
	ψ_1	0.000000740897643	
	ψ_2	0.094424069684062	
	σ_1	0.000810431271431	
	σ_2	0.000255756355983	
	ρ	$X_1(0)$	$X_2(0)$
1900	0.999999999999999	0.001880353247158	0.002828417849011
⋮	⋮	⋮	⋮
1925	-0.250316766642870	-0.000433647930847	0.002851663487249
⋮	⋮	⋮	⋮
1950	-0.999999999999999	0.000351991785546	0.001355915840057

Probability of negative intensities

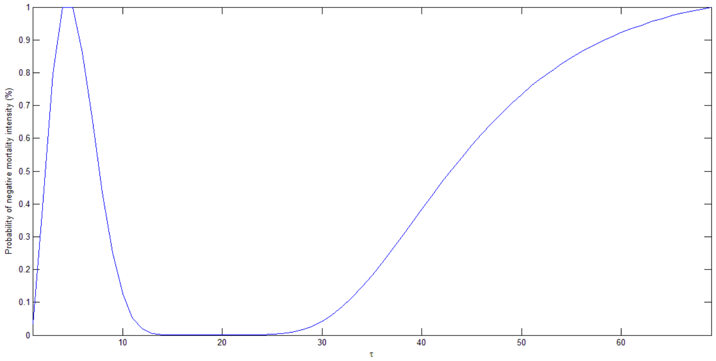

Figure 4: Probability of negative mortality intensity for the generation 1950

Residuals

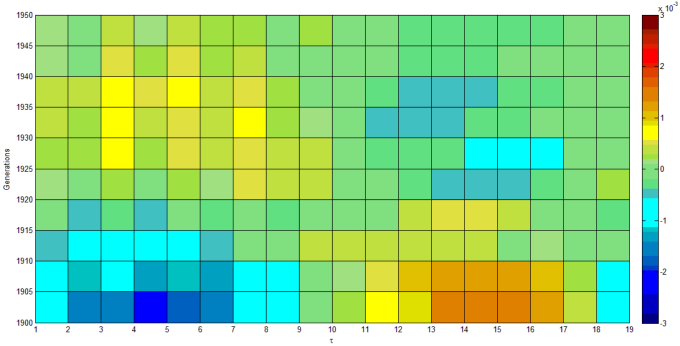

Figure 5: Calibration residuals plot

Correlation

Table 2: Correlation table (%)

	1900	1905	1910	1915	1920	1925	1930	1935	1940	1945	1950
1900	100.00										
1905	96.01	100.00									
1910	94.97	99.93	100.00								
1915	94.89	99.92	99.99	100.00							
1920	95.15	99.95	99.99	99.99	100.00						
1925	94.91	99.93	99.99	100.00	99.99	100.00					
1930	94.96	99.93	100.00	99.99	99.99	99.99	100.00				
1935	95.84	99.99	99.95	99.94	99.97	99.95	99.95	100.00			
1940	96.93	99.93	99.75	99.73	99.79	99.73	99.75	99.91	100.00		
1945	99.61	98.10	97.35	97.29	97.49	97.31	97.35	97.98	98.72	100.00	
1950	100.00	96.01	94.97	94.89	95.15	94.91	94.96	95.84	96.93	99.61	100.00

Percentage Absolute Relative Error

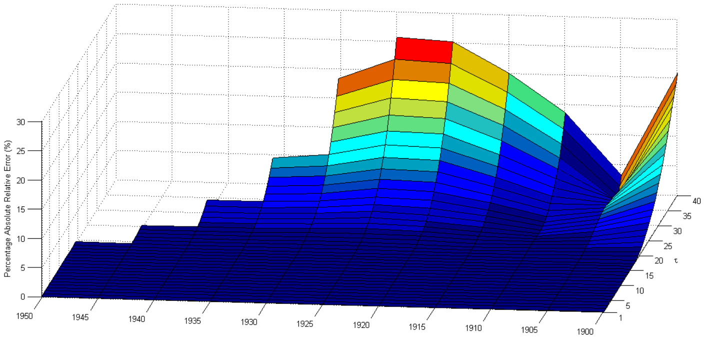

Figure 6: Percentage Absolute Relative Error

Forecast

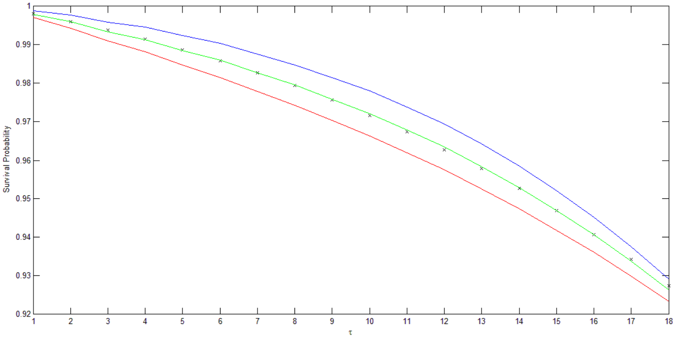

Figure 7: Survival probability curve at p=1, $S(1,\tau)$, given the mean and the 90% confidence interval

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- We control the probability of negative mortality intensities.

Conclusion

- Our model is analytically tractable and parsimonious.
- Calibration by means of **Differential Evolution** optimization algorithm produces a fit as good as $6 \cdot 10^{-4}$, and yields robust and stable parameters.
- We control the probability of negative mortality intensities.
- Both in-sample and out-of-sample **deterministic forecasts** have been examined.
  - **In-sample** errors **up to age 59** are very small.
  - **Out-of-sample** errors remain small **at least until age 65**.

Conclusion

- The **ex-post in-sample performance** of **stochastic forecasts** is very satisfactory. We anticipate that the increase in the error at later ages for out-of-sample forecast would probably be rectified with the introduction of a **third factor**, and will pursue this extension after further empirical investigation.

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- The resulting longevity intensity model extends the G2++ interest-rate model, as the factors here have different weights for each generation. By doing so, we allow for the calculation of **imperfect correlations** of mortality intensities across generations.

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- The resulting longevity intensity model extends the G2++ interest-rate model, as the factors here have different weights for each generation. By doing so, we allow for the calculation of **imperfect correlations** of mortality intensities across generations.
- The **calibration** to the data concerning UK males confirms that correlations across generations are **smaller than one**.

Conclusion

- The **ex-post in-sample performance** of **stochastic forecasts** is very satisfactory. We anticipate that the increase in the error at later ages for out-of-sample forecast would probably be rectified with the introduction of a **third factor**, and will pursue this extension after further empirical investigation.
- The resulting longevity intensity model extends the G2++ interest-rate model, as the factors here have different weights for each generation. By doing so, we allow for the calculation of **imperfect correlations** of mortality intensities across generations.
- The **calibration** to the data concerning UK males confirms that correlations across generations are **smaller than one**.
- The possibility of **capturing this correlation**, thanks to a generation-based model, together with the use of the **Differential Evolution** algorithm, which permits an efficient calibration, are the major contributions of this work.

Thank you!

Thank you for your attention!