Mortality Accelerations
Modelling Improvements in Spain

— Andrew Hunt and Giulia Padovan —

— September 2019 —

PACIFIC LIFE RE

LONGEVITY 15
WASHINGTON DC

— Overview —

Topics covered in this presentation

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Introduction

“The difference between theory and practice is that, in theory, there is no difference, but in practice, there is” – Anon

This talk discusses a new mortality model (the first in a new family of models potentially) that we developed for modelling mortality in Spain.

However, what it is really about is the process of innovation and model development working as a research actuary working in industry, as opposed to as an academic researcher.

- Section 01 -

Understanding the Data

01 Understanding the Data / 02 APCI Model / 03 APCA Model / 04 Conclusions

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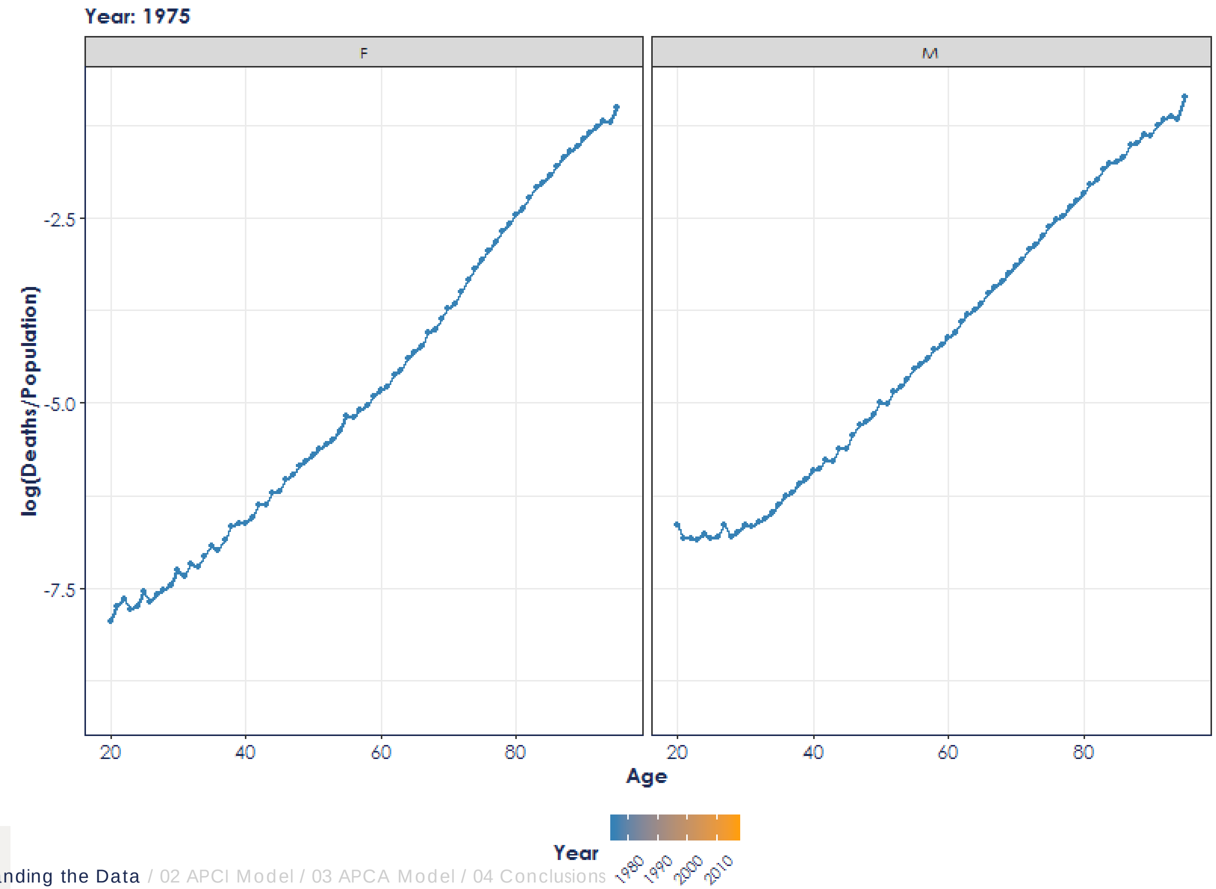
Understanding the Data

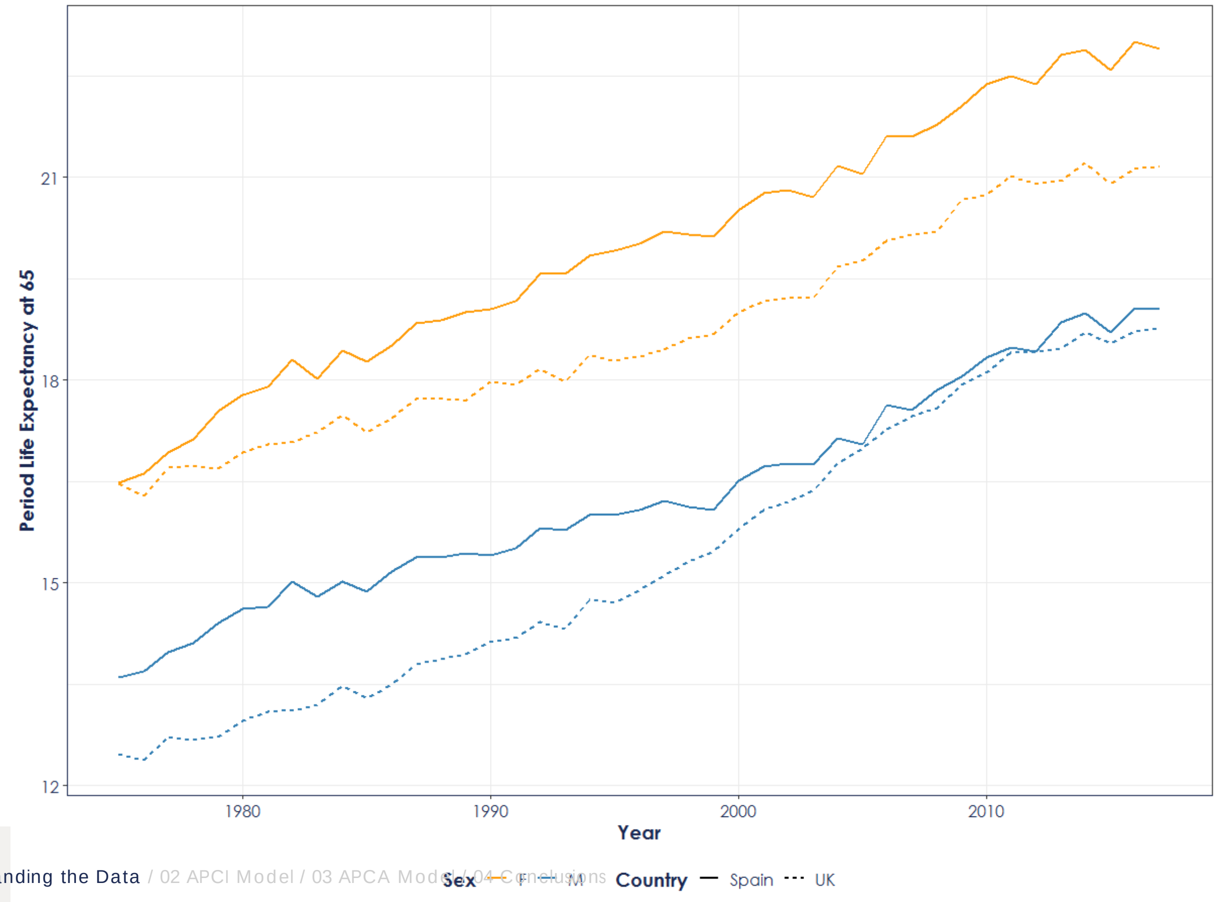
The first step in developing a trend assumption in a new country is to obtain and understand the available data.

Spanish data from the Institute Nacional de Estadistica (INE) is available for 1975-2017, ages 20-95, which we have used for this analysis.

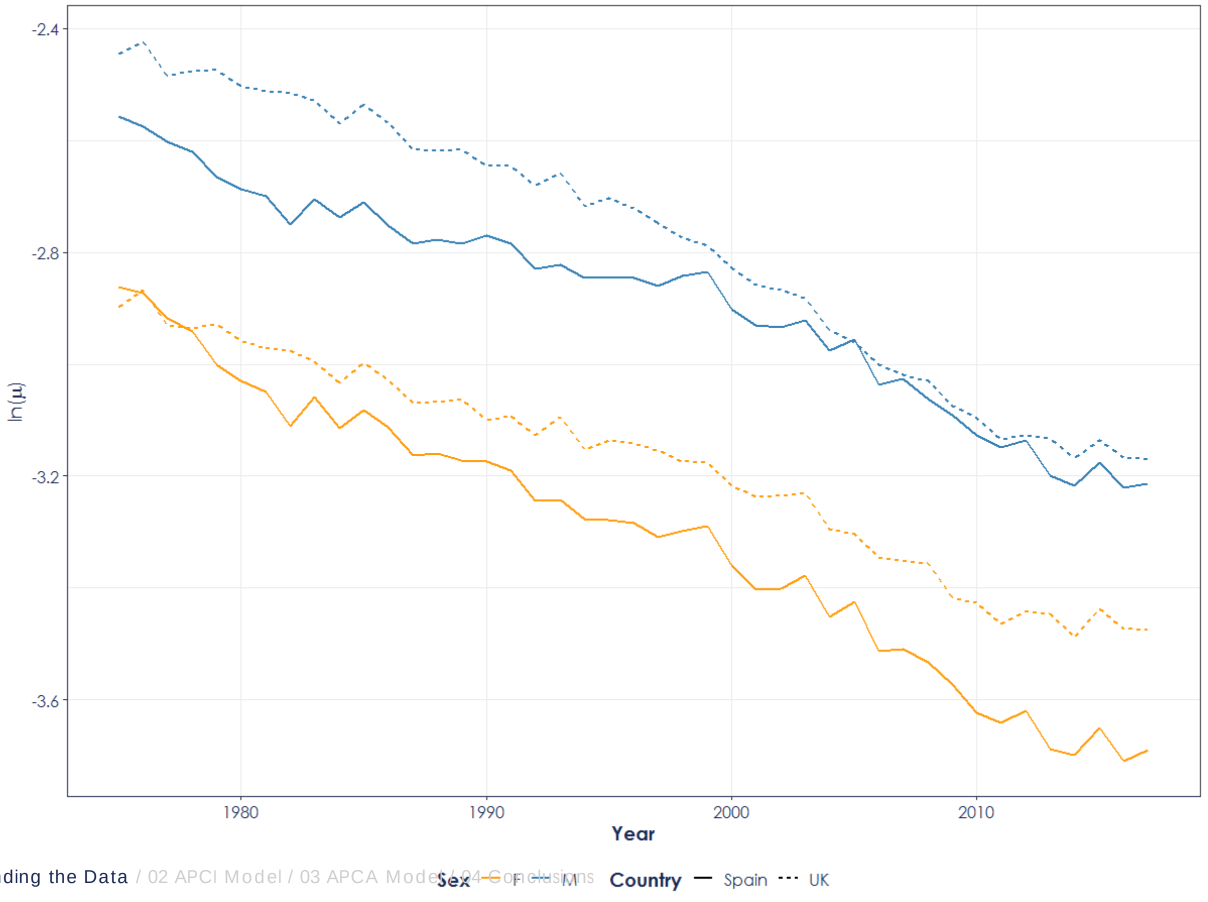
Understanding the Data



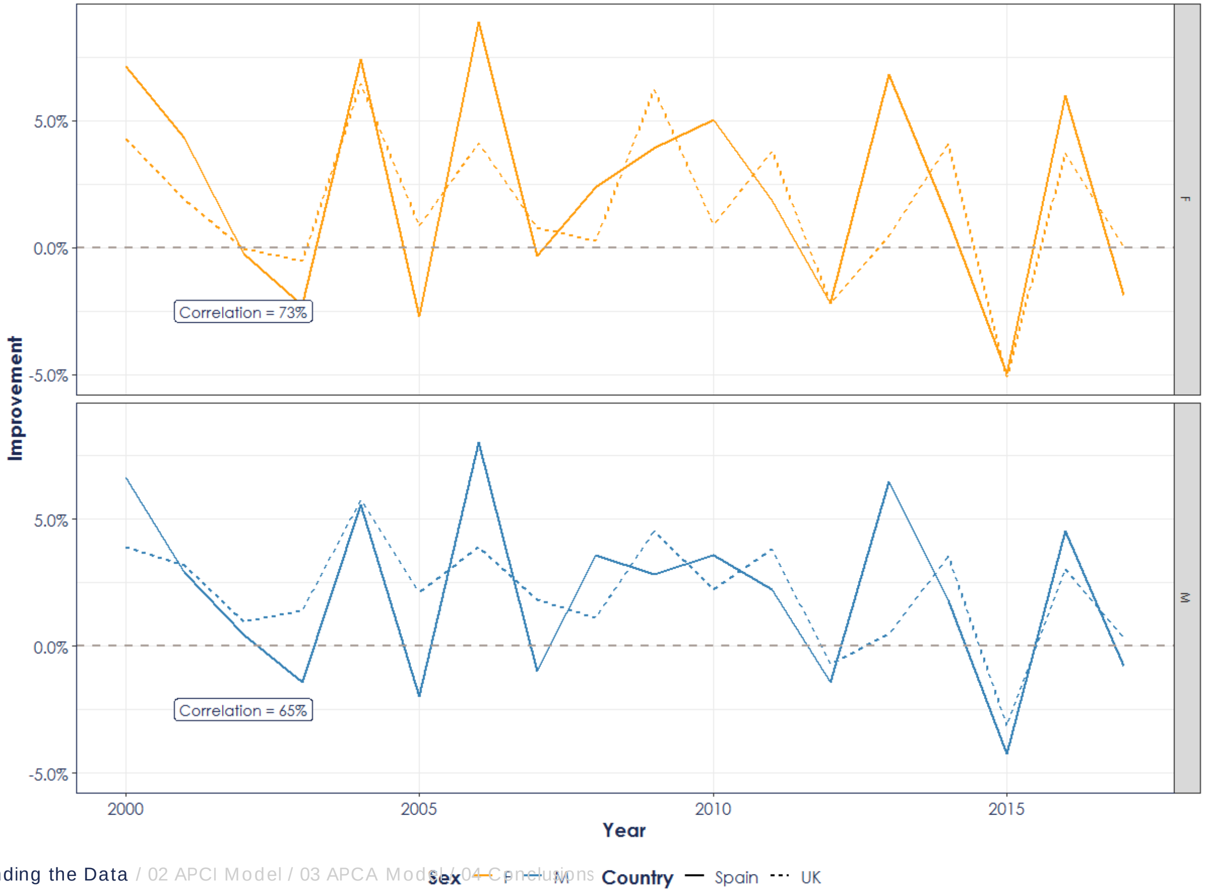
Understanding the Data



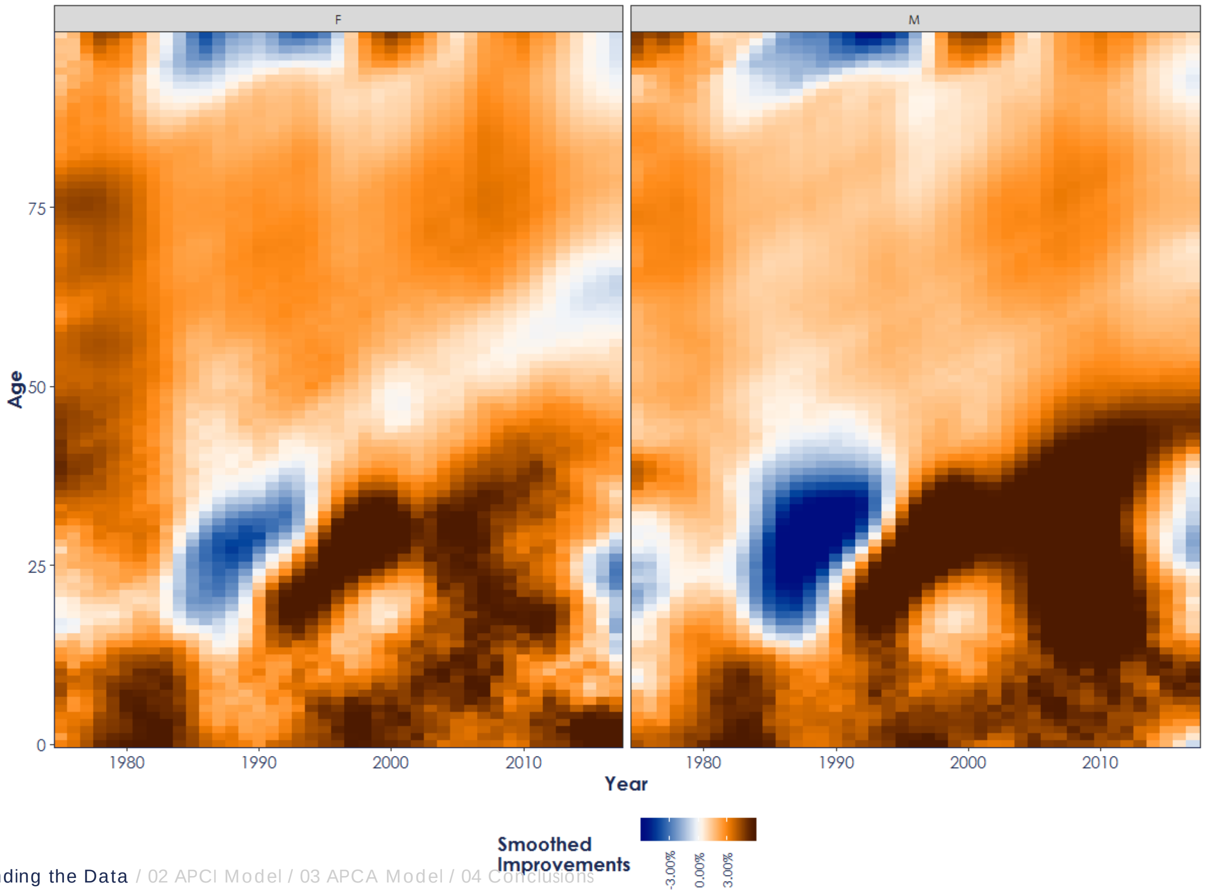
Understanding the Data



Understanding the Data



Understanding the Data



- Section 02 -

APCI Model

01 Understanding the Data / 02 APCI Model / 03 APCA Model / 04 Conclusions

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APCI Model

The next step is to fit a standard model to historic mortality rates and use this to make projections into the future.

For this, we use the APCI model

$$\ln \mu_{x,t} = \alpha_x + \beta_x(t - \bar{t}) + \kappa_t + \gamma_{t-x}$$

Age shape of mortality rates

Age shape of improvement rates

Period effects

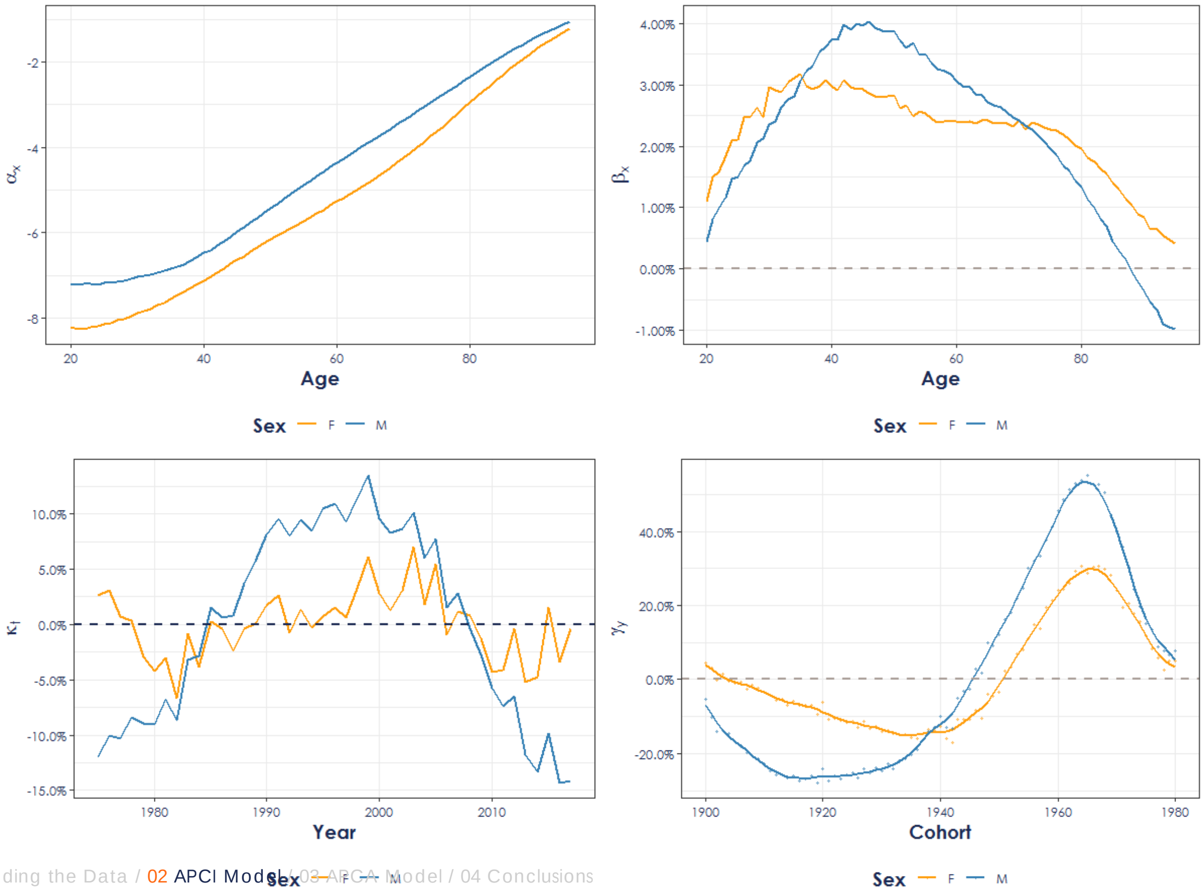
Cohort effects

See CMI WP 91, Richards (2017) and Hunt and Villegas (2017) for more details.

To project, we use:

- ARIMA(1,1,0) process for period parameters
- Bayesian cohort approach – Hunt and Blake (2015a)

APCI Model


α_x
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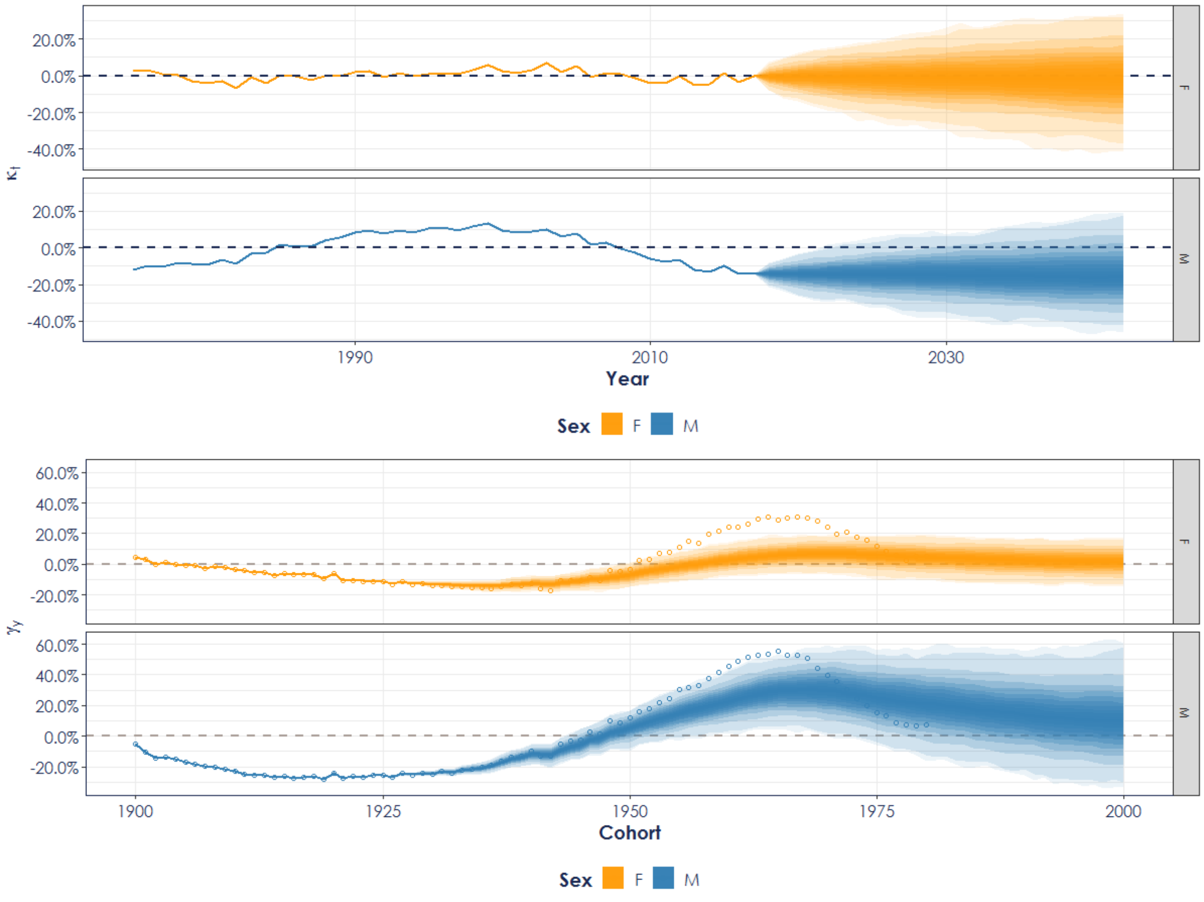
Sex F M

APCI Model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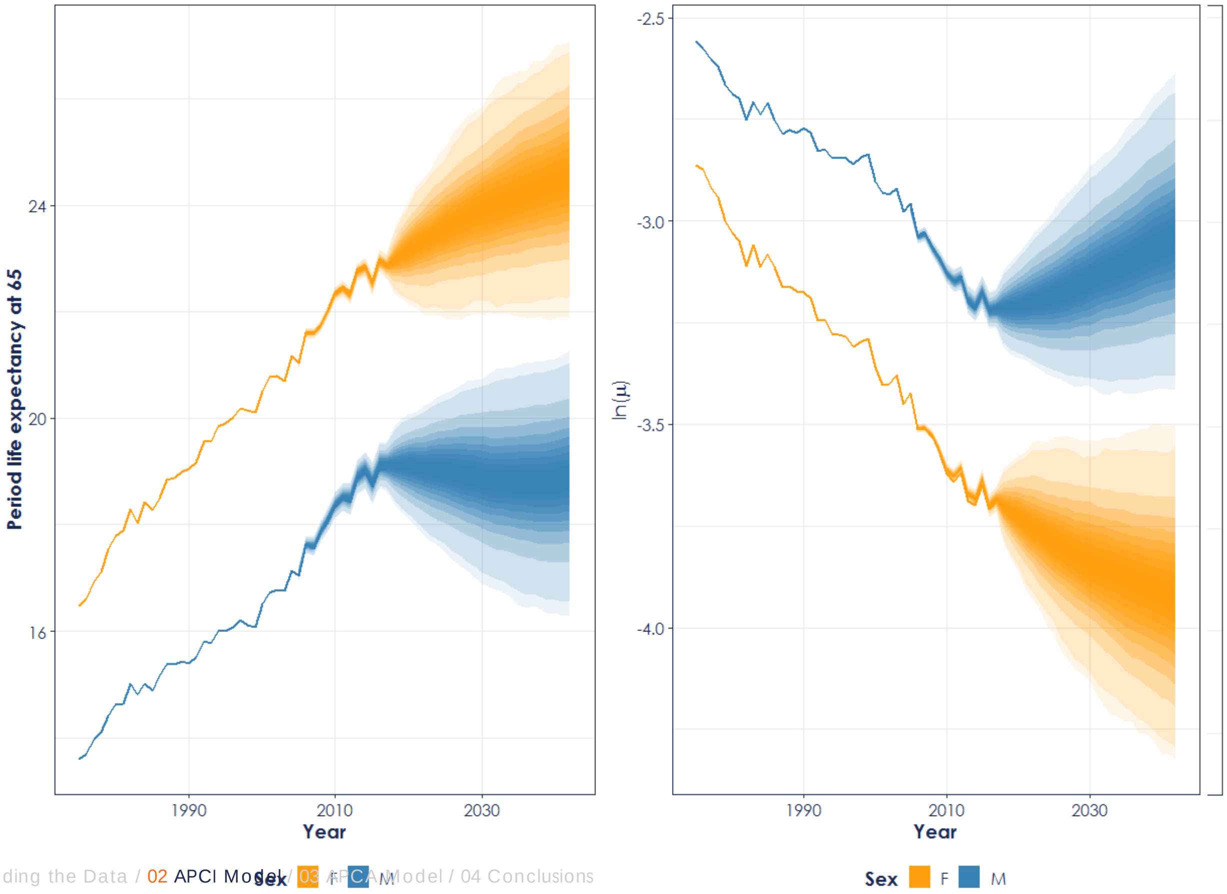
APCI Model


Period life expectancy at 65
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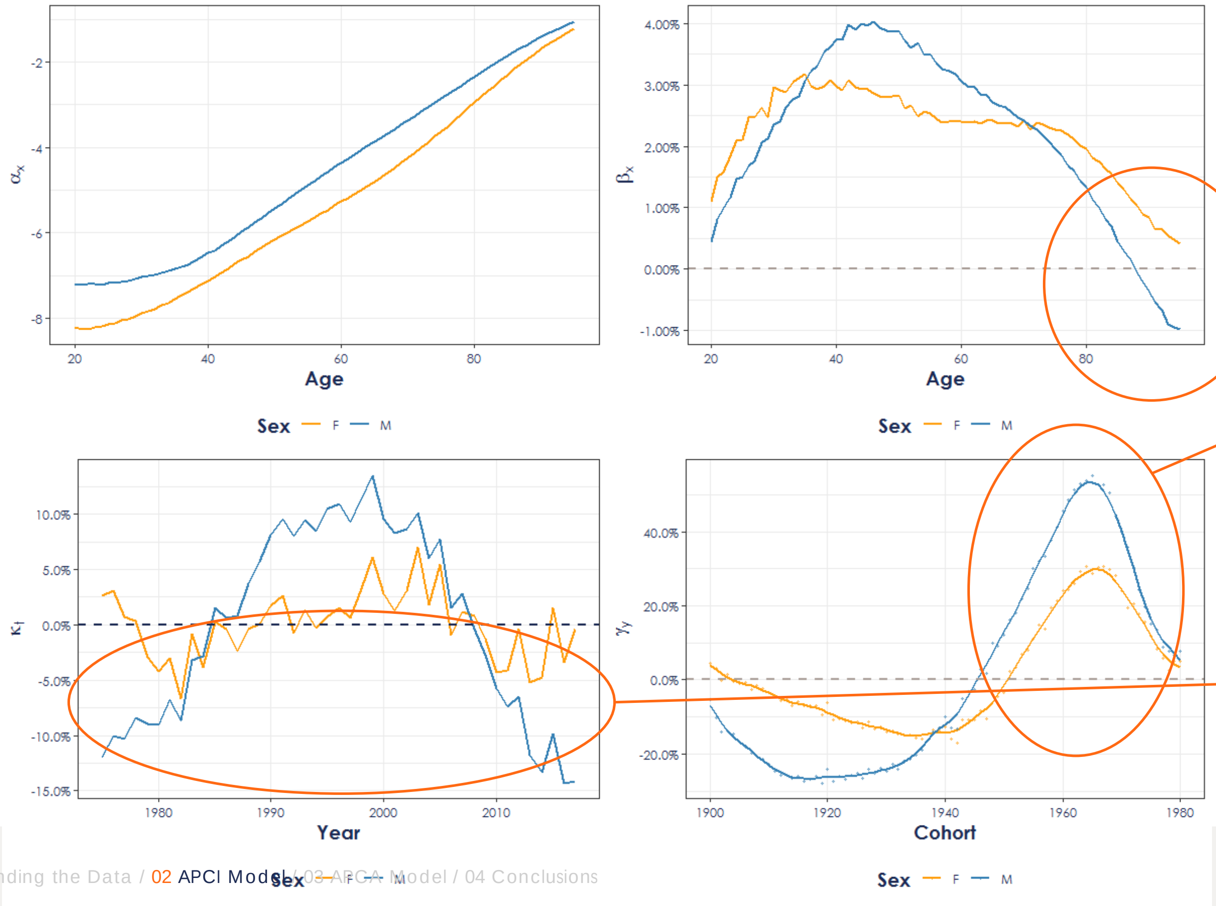
APCI Model

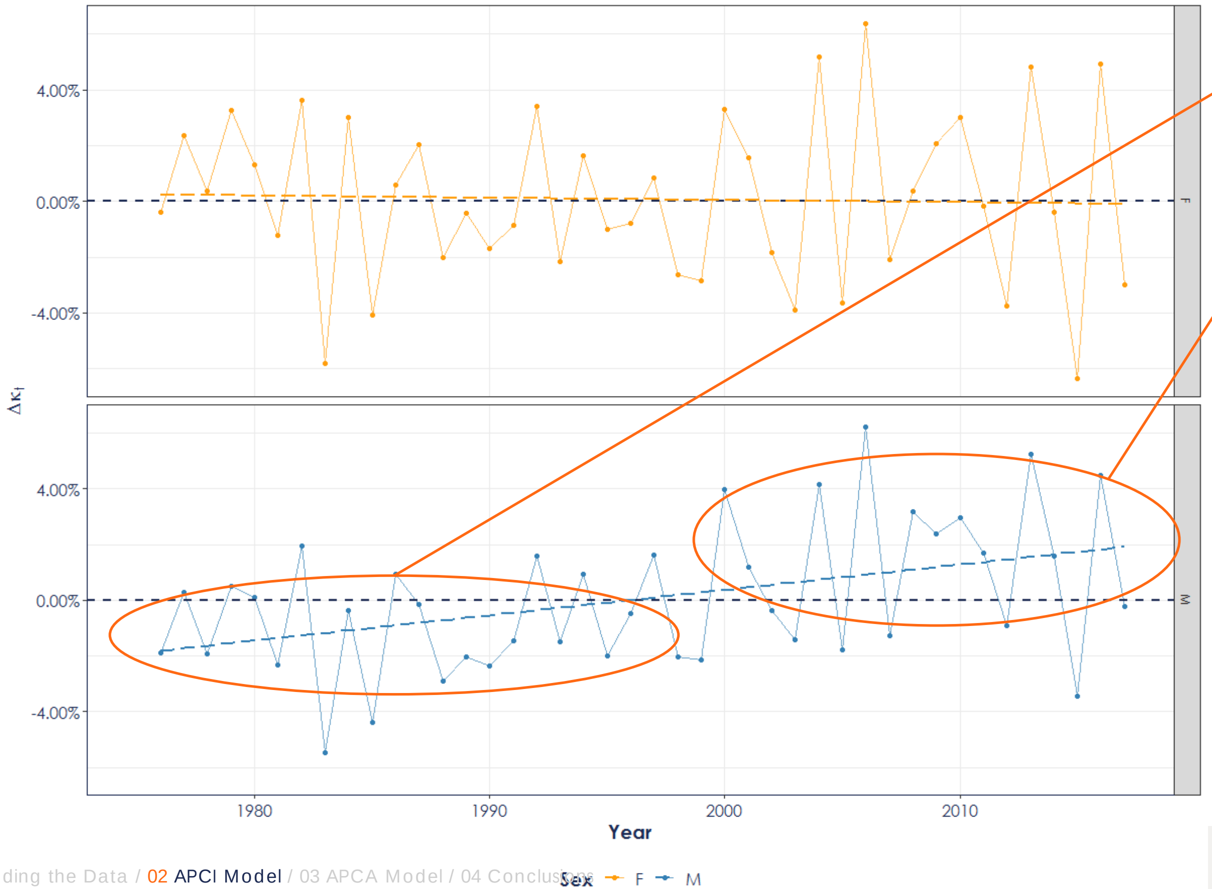


Negative improvements at older ages? Not the case in the crude mortality rates

Just plain weird!

High degree of curvature in the period parameters – quadratic trends?

APCI Model



Period improvements slower than average at beginning of data

Period improvements faster than average at end of data

Male improvements have accelerated over the period of the data, which is being captured (partially) by the period term

- Section 03 -

APCA Model

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APCA Model

Male parameters from APCI model are showing multiple issues that are giving implausible projections

- Cohort parameters are usually canary down the mineshaft in terms of warning of other issues going wrong in the modelling

Two approaches:

1. Bodge a solution – fit API model without cohort effects as first stage, offset parameters and estimate cohort effects separately as second stage
2. Develop a new model that fixes the underlying issue

APCA Model

What is the underlying issue?

- Quadratic trends in κ_t implies that average improvements are too fast in first half of the data but too slow in the second (improvements have accelerated?)
- Weird pattern to γ_y because any structure in the model that can't be captured by age/period terms (e.g., an age shape to accelerations?) dumped into cohort parameters as "garbage" term
- Excessive cohort improvements need to be offset when these cohort reach older ages in the data, giving negative β_x at high ages, which are inconsistent with raw data.

Solution – we possibly need to include accelerating mortality rates in the modelling

APCA Model

Introducing the age/period/cohort accelerations (APCA) model

$$\ln \mu_{x,t} = \alpha_x + \beta_x^{(1)}(t-\bar{t}) + 0.5\beta_x^{(2)}\left(\sigma_t - (t-\bar{t})^{\mathbf{2}}\right) + \kappa_t + \gamma_{t-x}$$

Why accelerations? Because if improvement rates are

$$r_{x,t} = -\ln\left(\frac{\mu_{x,t}}{\mu_{x,t-1}}\right) = -\beta_x^{(1)} + \beta_x^{(2)}\left((t-\bar{t}) - 0.5\right) - \Delta\kappa_t - \Delta\gamma_{t-x}$$

Then changes in improvements (accelerations) are

$$\Delta r_{x,t} = \beta_x^{(2)} - \Delta^2\kappa_t - \Delta^2\gamma_{t-x}$$

i.e., and age/period/cohort model on mortality accelerations, with $\boldsymbol{\beta_x^{(2)}}$ giving the shape of accelerations in mortality across ages.

APCA Model

New model = New invariant transformations = New identifiability constraints

See Hunt and Blakes (2015b)

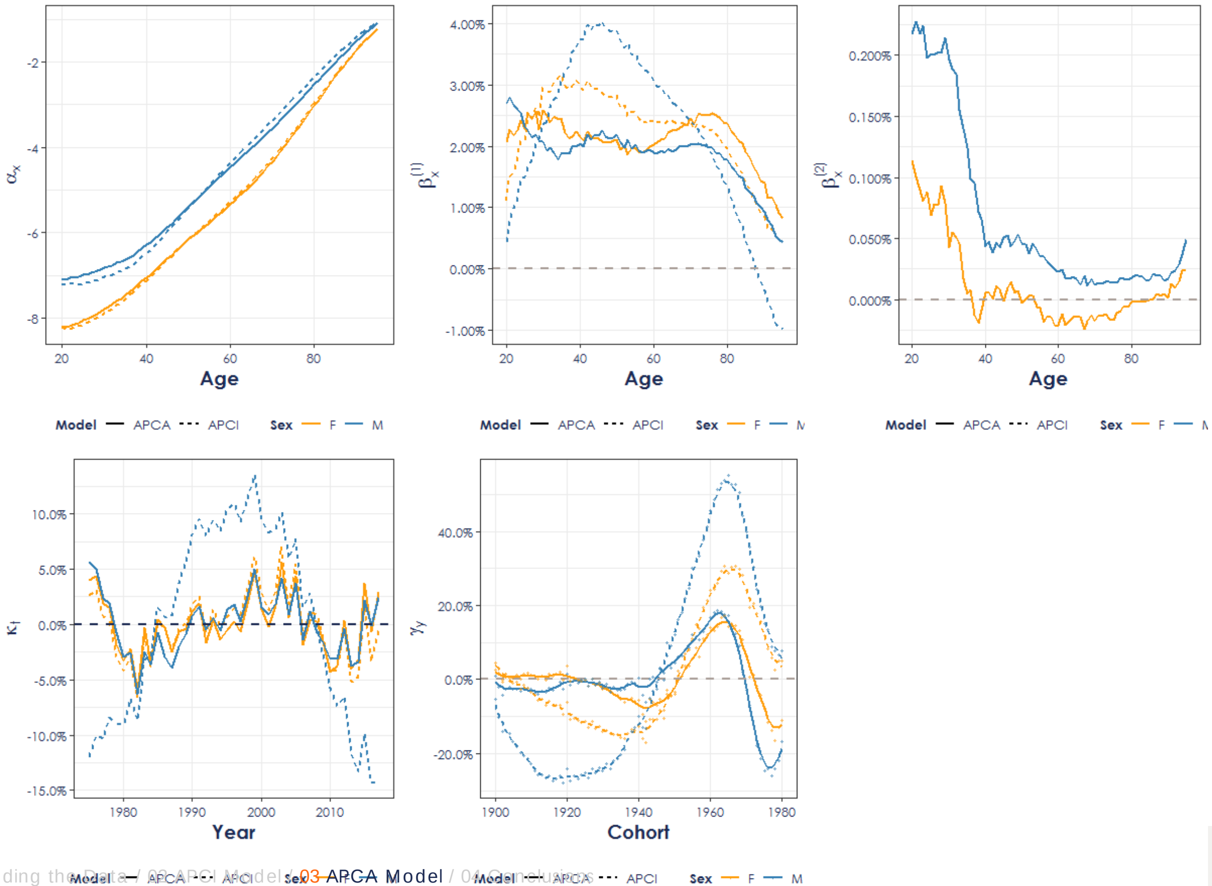
$$\left\{\alpha_x, \beta_x^{(1)}, \beta_x^{(2)}, \kappa_t, \gamma_y\right\} \rightarrow \left\{\hat{\alpha}_x, \hat{\beta}_x^{(1)}, \hat{\beta}_x^{(2)}, \hat{\kappa}_t, \hat{\gamma}_y\right\} =$$
$$\{\alpha_x + D(x-\bar{x})^3 - 3D\sigma_t, \qquad \beta_x^{(1)} + 3D(x-\bar{x})^2,$$
$$\beta_x^2 - 6D(x-\bar{x}), \qquad \kappa_t + D(t-\bar{t})^3, \qquad \gamma_y - D(y-\bar{y})^3\}$$

$$\sum_t \kappa_t = \sum_t \kappa_t(t-\bar{t}) = \sum_t \kappa_t(\sigma_t - (t-\bar{t})^2) = 0$$

$$\sum_y \gamma_y = \sum_t \gamma_y(y-\bar{y}) = \sum_t \gamma_y(\mathrm{y}-\bar{y})^2 = \sum_t \gamma_y(\mathrm{y}-\bar{y})^3 = 0$$

Which allows us to remove the quadratic trends in $\boldsymbol{\kappa_t}$ and some of the weird shapes in $\boldsymbol{\gamma_y}$

APCA Model


αx
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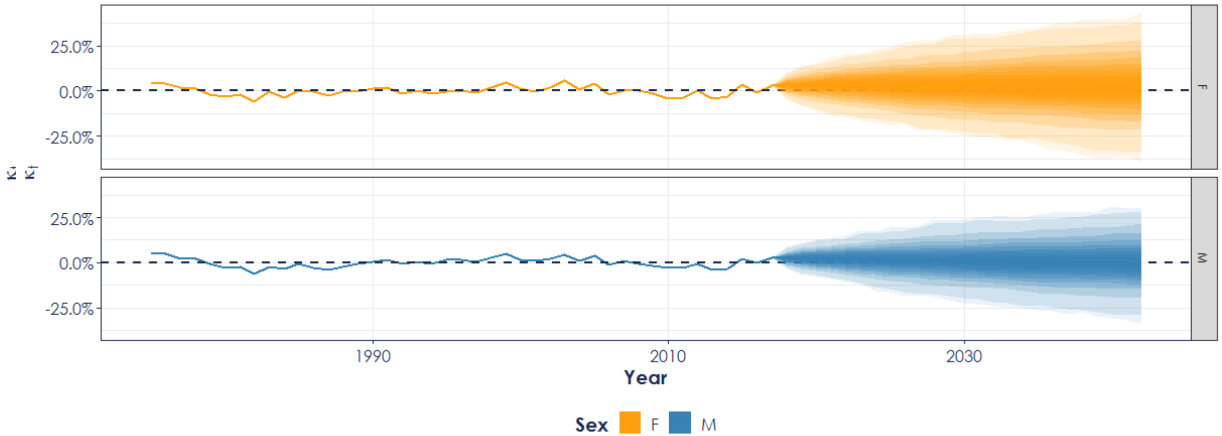
APCA Model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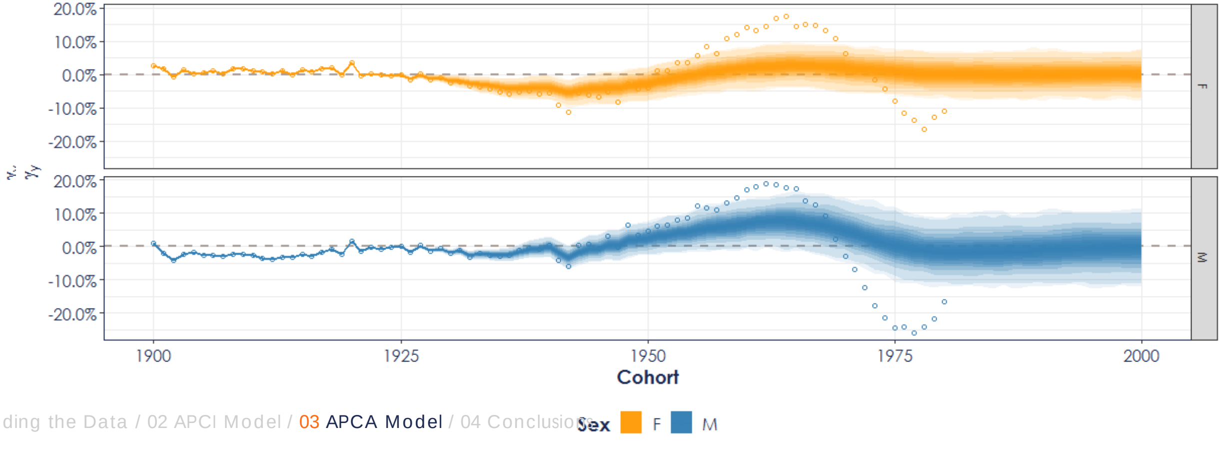
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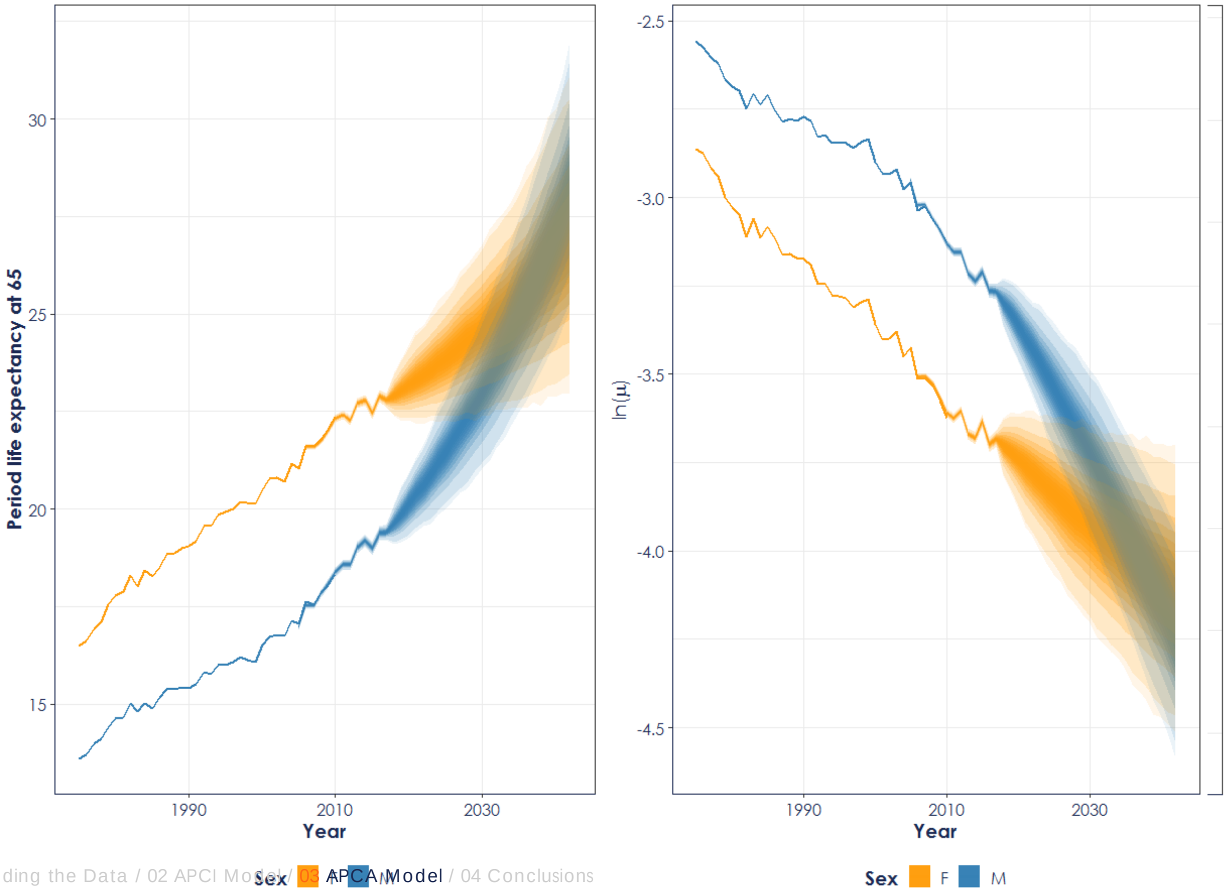
APCA Model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Sex
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M

APCA Model

So, we have solved one problem, but caused another

- Male life expectancy has gone from being highly likely to stagnate in short term to being predicted to catch up with female life expectancy by 2035
- However, simple straight line extrapolation of life expectancies would also lead to catch-up

Accelerations have been observed in past, likely to persist for some time in future, but not indefinitely

- In practice, most people use combination of modelling for short- and medium-term improvements, but use a subjective long-term assumption, avoiding this sort of issue

- Section 04 -

Conclusions

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Families of Mortality Models

APCA model fits into family of generalised age/period/cohort mortality models – Hunt and Blake (2015c) and Villegas et al (2018)

Model of	Predictor structure $\eta_{x,t} =$
Mortality rates	$\alpha_x + \sum_i \beta_x^{(i)} \kappa_t^{(i)} + \gamma_{t-x}$
Improvement rates	$\alpha_x + \beta_x (t - \bar{t}) + \sum_i \beta_x^{(i)} \kappa_t^{(i)} + \gamma_{t-x}$
Acceleration rates	$\alpha_x + \beta_x^{(1)} (t - \bar{t}) + 0.5 \beta_x^{(2)} \left(\sigma_t - (t - \bar{t})^2\right) + \sum_i \beta_x^{(i)} \kappa_t^{(i)} + \gamma_{t-x}$

Can be modelled as GLMs and GNMs (future version of StMoMo?)

Beyond this lie "jerks" (rates of changes of accelerations) and "jounces" (rates of changes of jerks)

– However, law of diminishing returns kicks in far before this

Practical Lessons

In practice, development of new methods or mortality models is driven by specific situations, rather than general principles

Strong preference for models with terms that can be interpreted (demographic significance)

- Model parameters tell stories that should be meaningful in terms of mortality or improvement rates

Model-based projections are the starting point, not the destination

- Parameters will be modified, prior views will be allowed for subjectively, adjustments will be made to account for socio-economic selection or narratives around cause-of-death drivers, etc
- However, these will be easier to apply using a model with demographically significant parameters
- Model-based projections should be dominant in short-run, where recent past is best guide to near-future

References

Continuous Mortality Investigation (2016): Working Paper 91 - CMI Mortality Projections Model consultation – technical paper

Hunt and Blake (2015a): A Bayesian approach to modelling and projecting cohort effects

Hunt and Blake (2015b): Identifiability in age/period/cohort mortality models

Hunt and Blake (2015c): On the structure and classification of mortality models

Richards, Currie, Kleinow and Ritchie (2017): A stochastic implementation of the APCI model for mortality projections

Villegas, Kaishev and Millossovich (2018): StMoMo: An R Package for stochastic mortality modeling