Longevity 13 Conference

Modeling and Forecasting Age-at-death Distributions

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Motivation

- **Background:**
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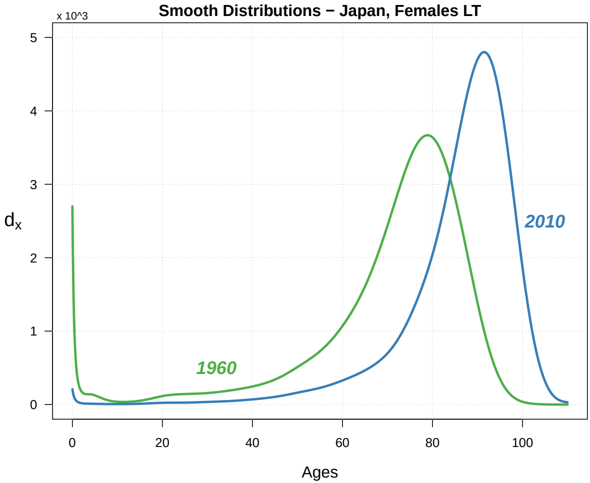
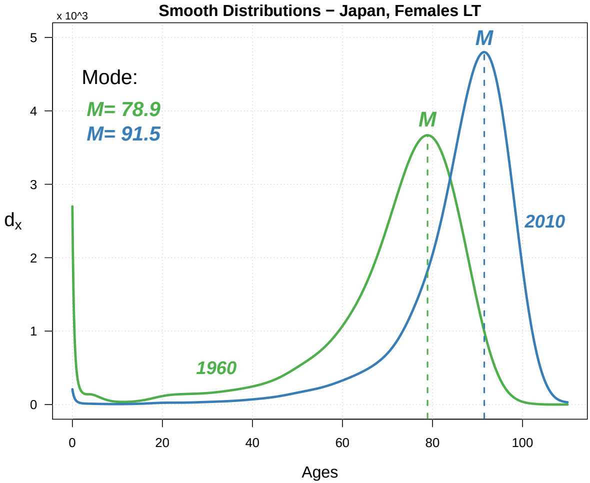
  - age-at-death distributions are very informative, yet neglected for modeling and forecasting
- **Research question:** model and forecast mortality by studying changes in age-at-death distributions

Age-at-death Distributions



Summary measures:

Age-at-death Distributions



Summary measures:

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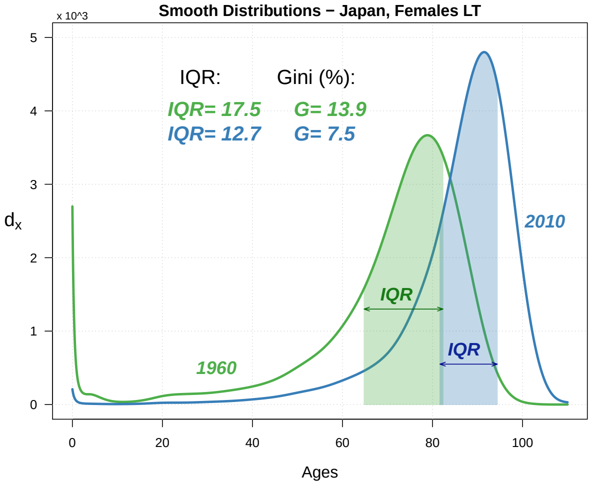
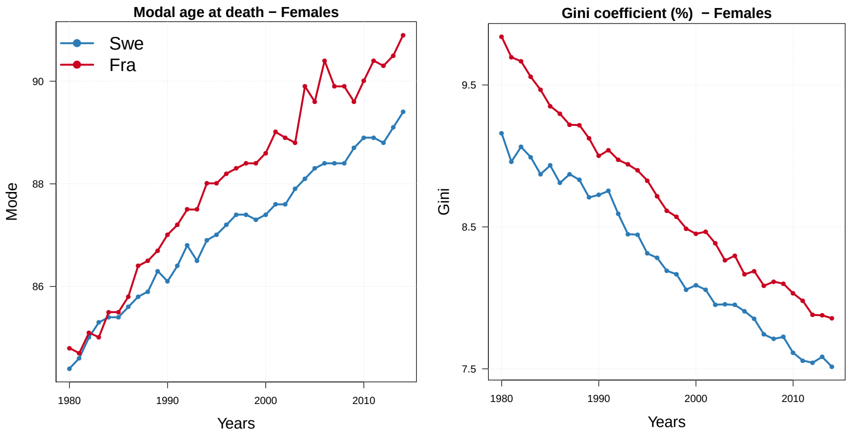


Summary measures:

- Longevity = Modal age at death
- Lifespan Inequality = (Relative) variability of death distribution

Longevity & Lifespan Inequality

Joint trends studied extensively, but hard to disentangle age-specific contributions



The STAD Model

Notation:

- x: age
- $f(x)$: standard distribution
- $g(x)$: observed distribution
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- $g(x)$ conforms to $f(x)$ on the warped axis, i.e. $g(x) = f(t(x))$
- $t(\cdot)$ is a **segmented function** of the difference in modal ages and the change in the variability before and after M:

$$t(x; s, b_L, b_U) = \begin{cases} M^f + b_L\,(x - s - M^f) & \text{if } x \leq M^g \\ M^f + b_U\,(x - s - M^f) & \text{if } x > M^g \end{cases}$$

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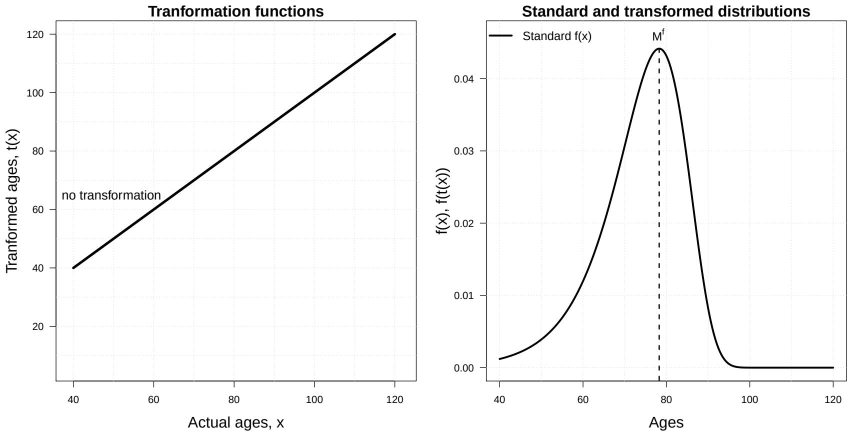
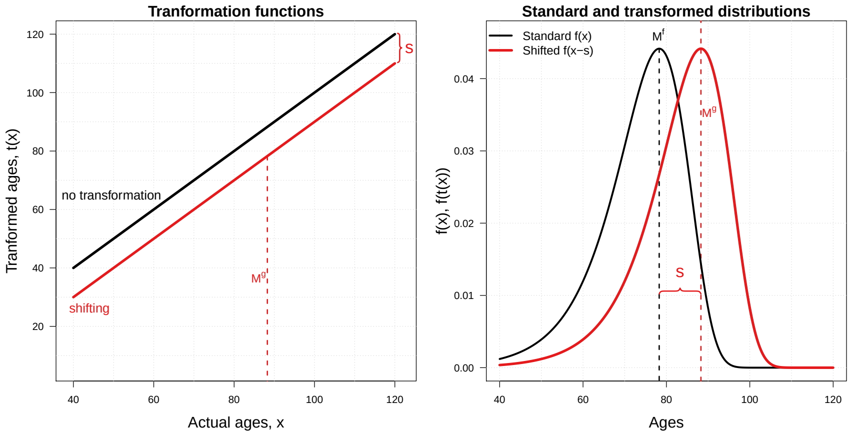


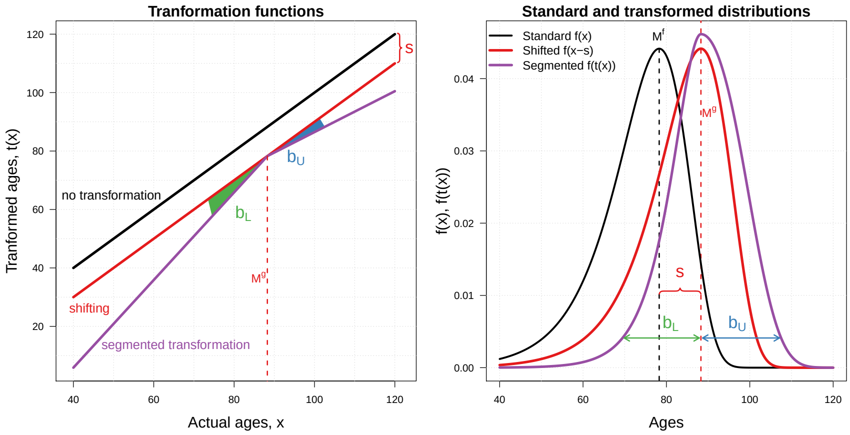
The STAD Model

$s = M^g - M^f$ is the difference between the M of $g(x)$ and $f(x)$ (*shifting* dynamic of mortality)



The STAD Model

b_L and b_U measure the change in lifespan variability of $f(x - s)$ before and after M^g (*compression* dynamic of mortality)



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- **Landmark registration:** alignment of observed densities to the mode of the first distribution

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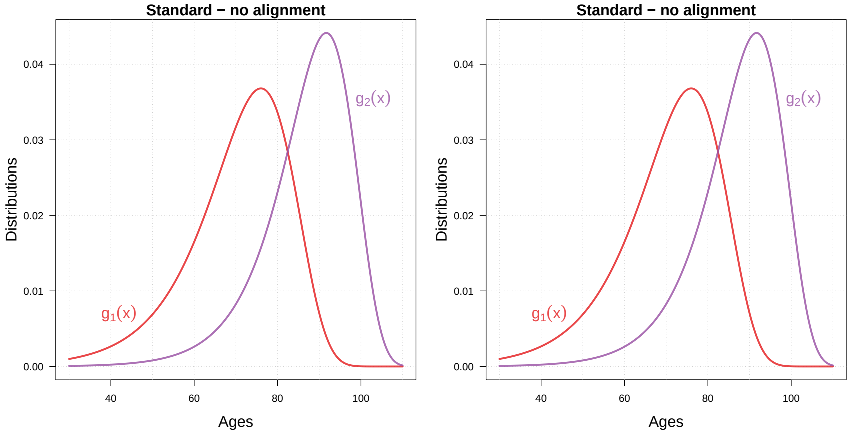

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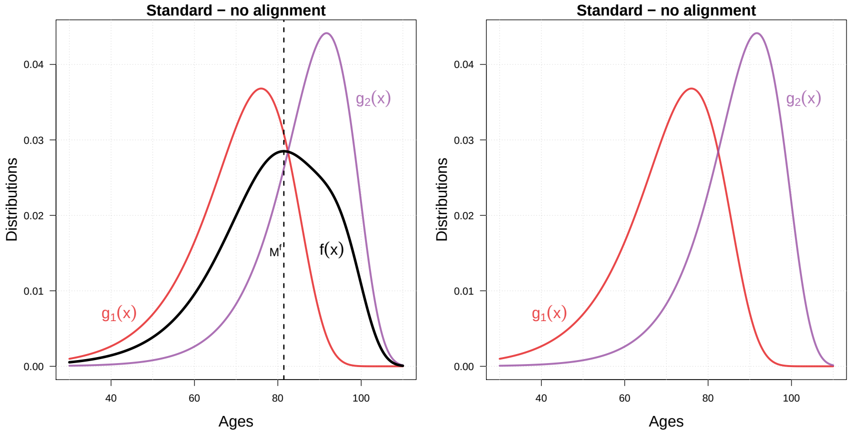

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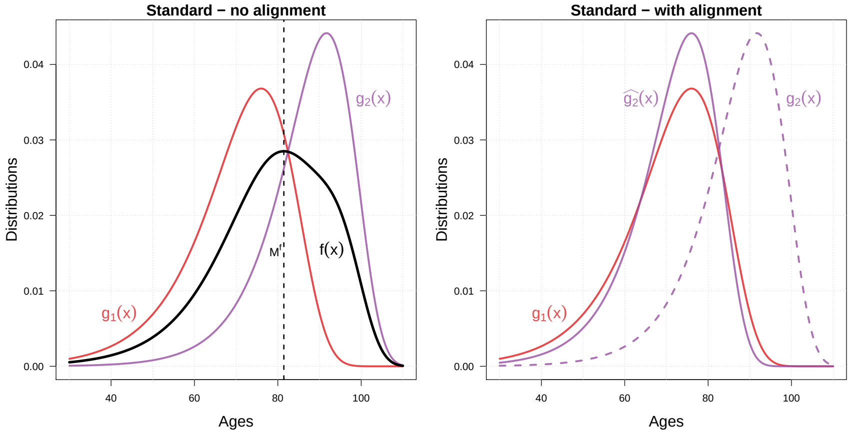

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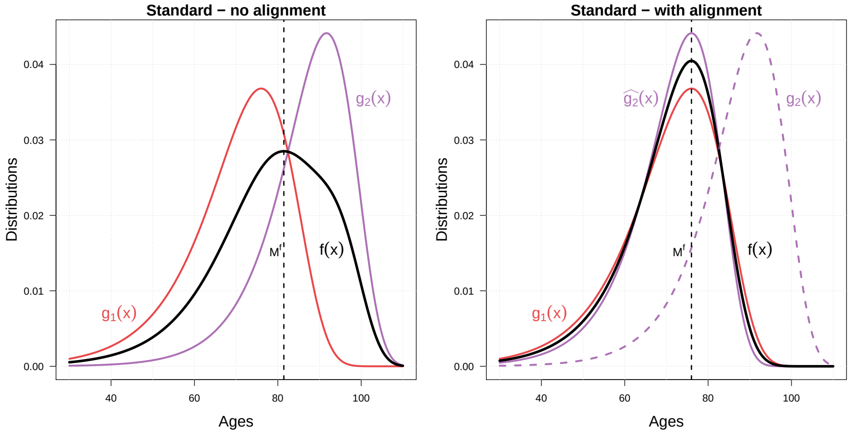

Application to observed data

- **Smoothing:**
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- **Estimation:** for each year, b_L and b_U estimated by maximum likelihood from the assumption:

$$D_x \sim \mathrm{Poisson}\left(E_x\, \mu_x\right)$$

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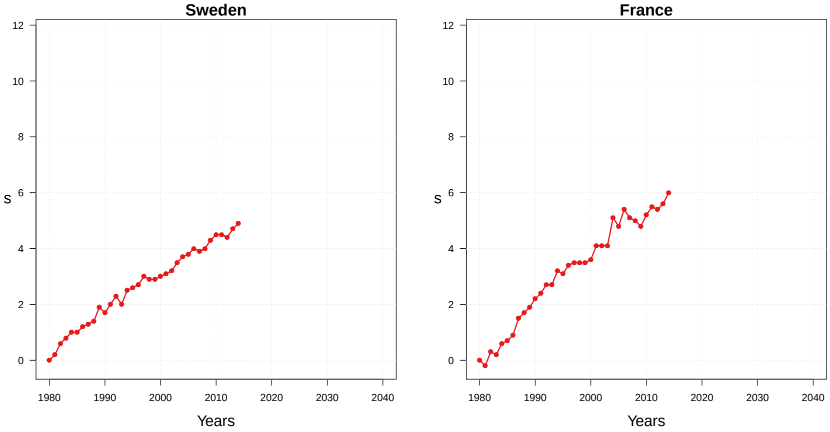
- **Data:** observed death counts and exposure times for females aged 30+ during 1980-2014 in Sweden and France (retrieved from the Human Mortality Database)

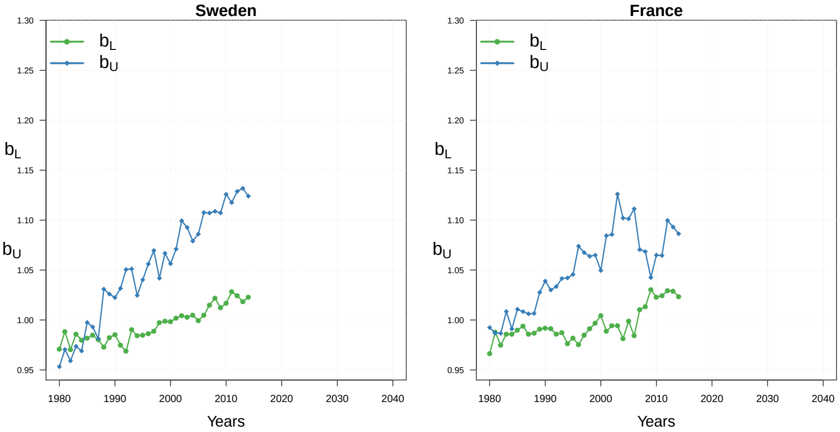
Estimation

Shifting parameter s:



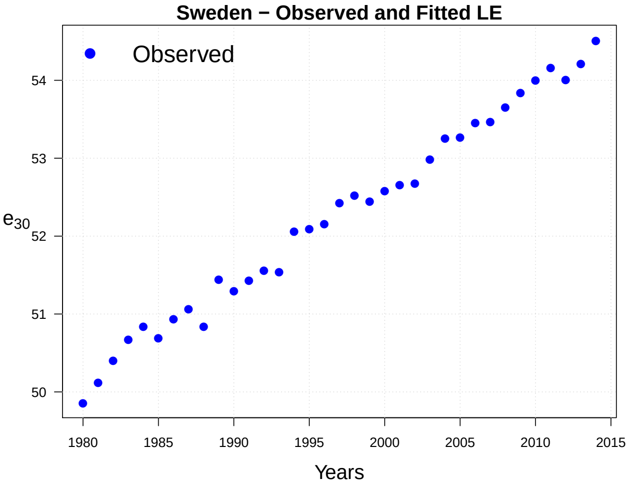
Estimation

Compression/expansion parameters b_L and b_U:



Observed vs Fitted Data

Good performance in terms of goodness-of-fit:



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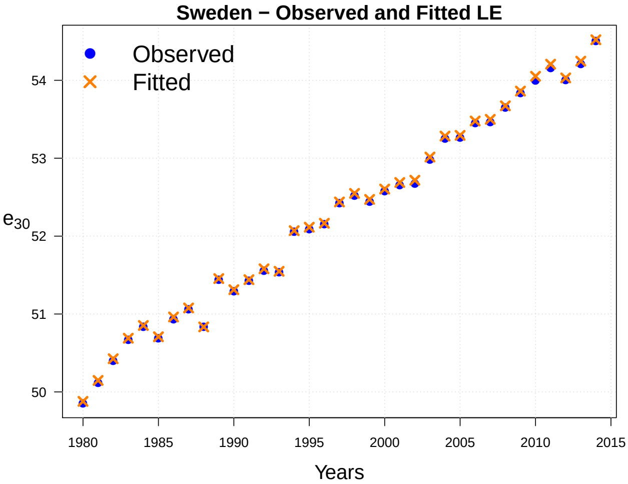

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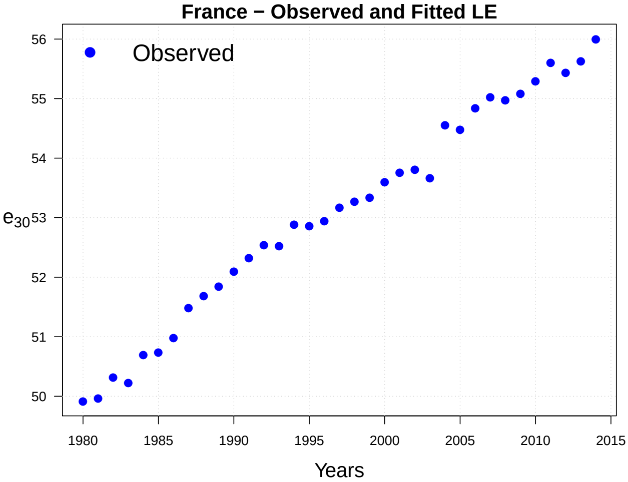

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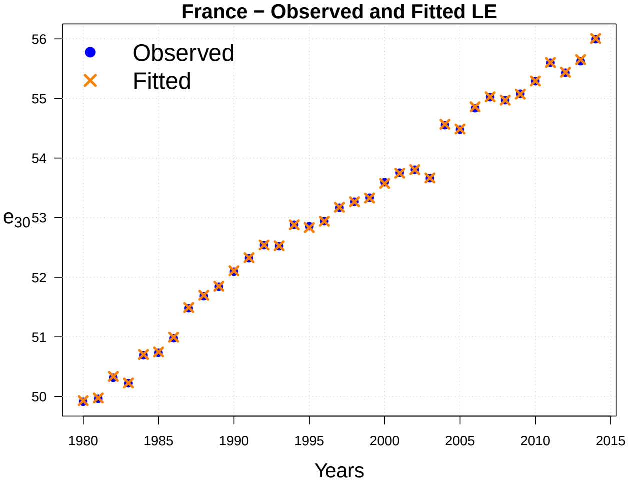
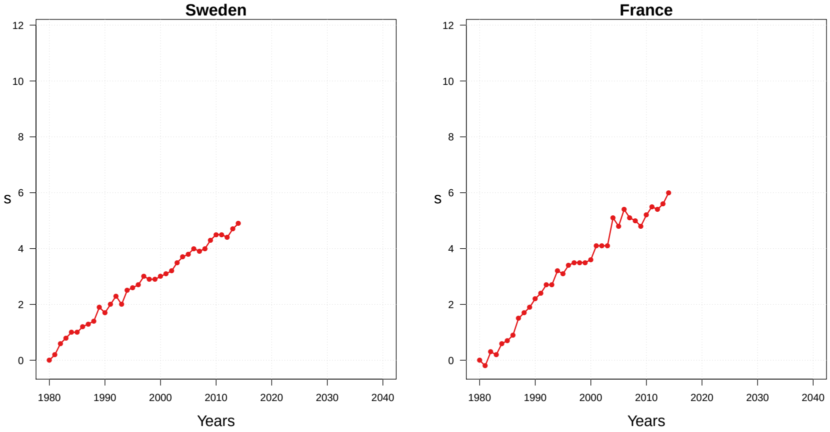


Forecasting with univariate ARIMA model

Shifting parameter *s* forecast with 80% C.I.:



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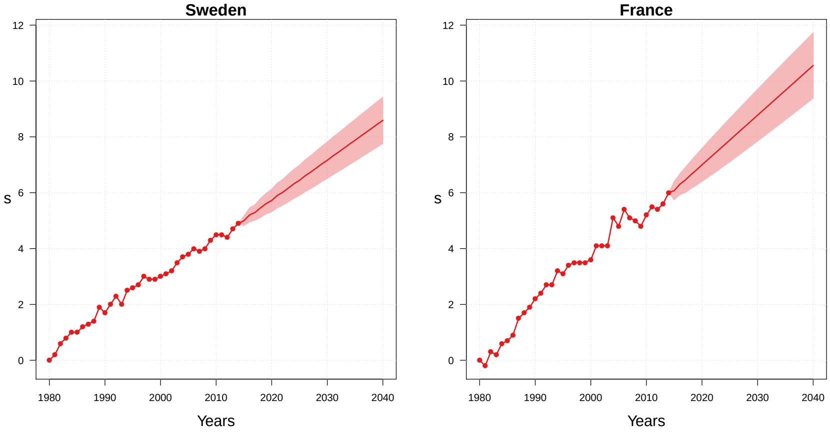

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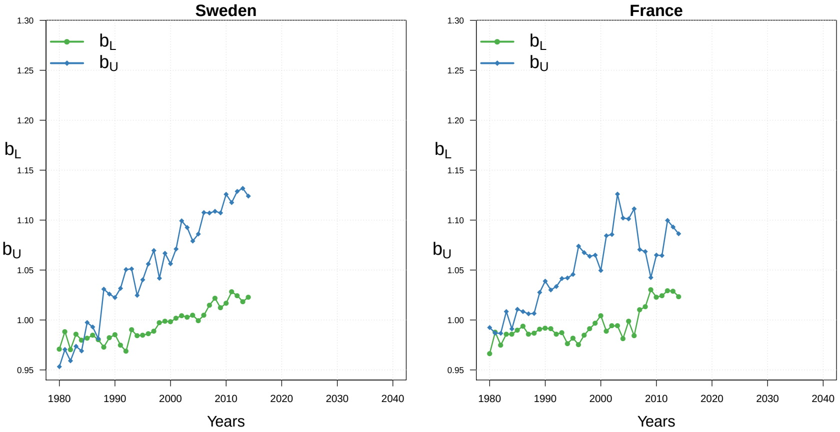

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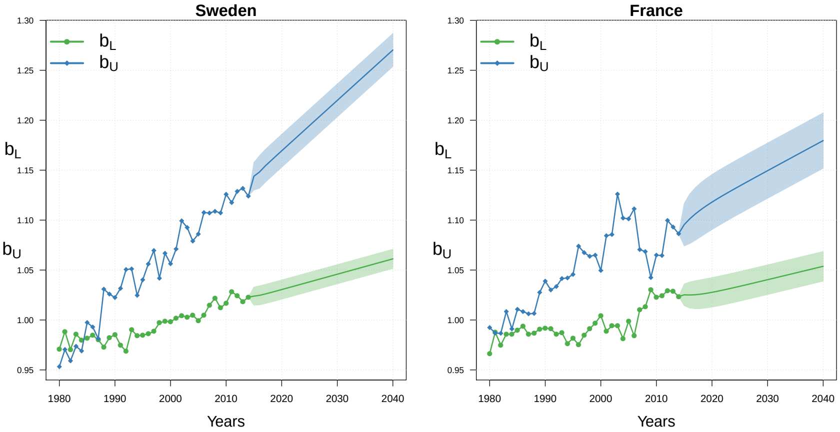

Forecasting - e_{30}

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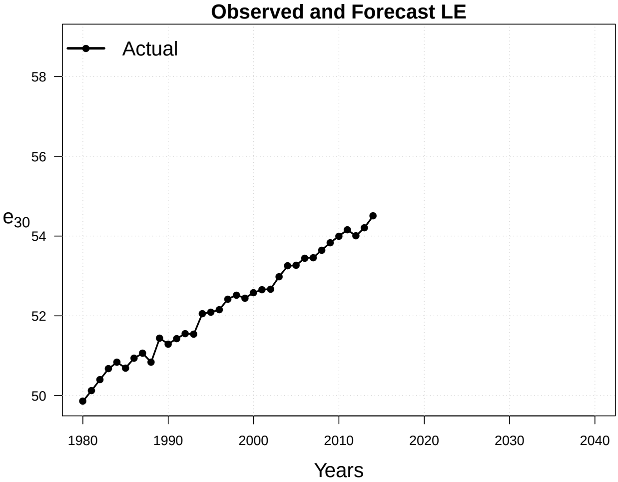

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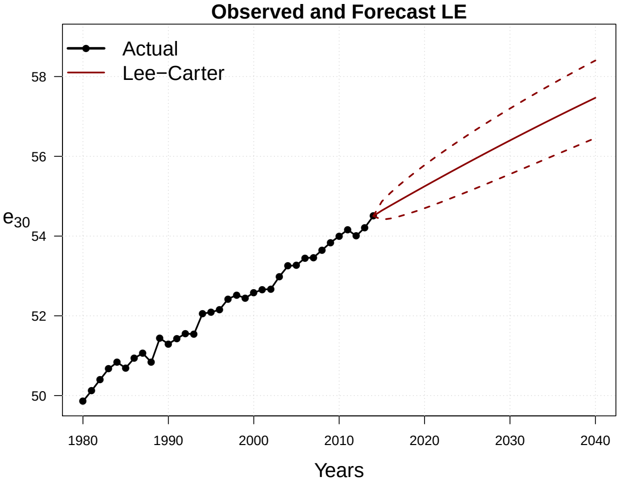

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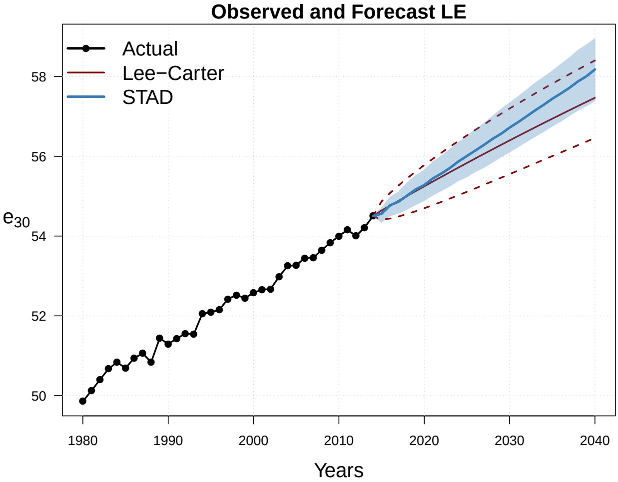

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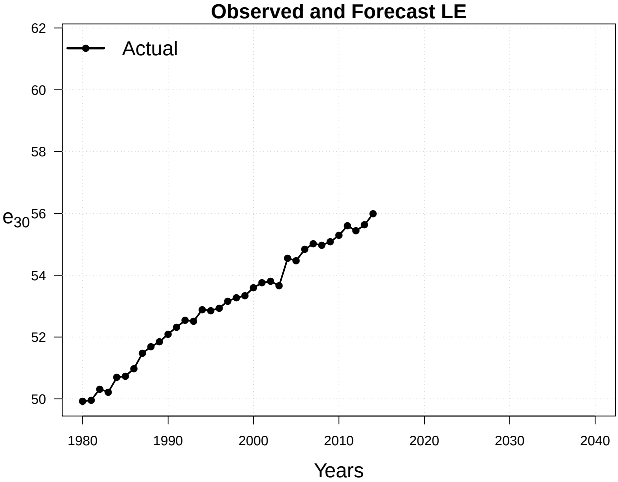

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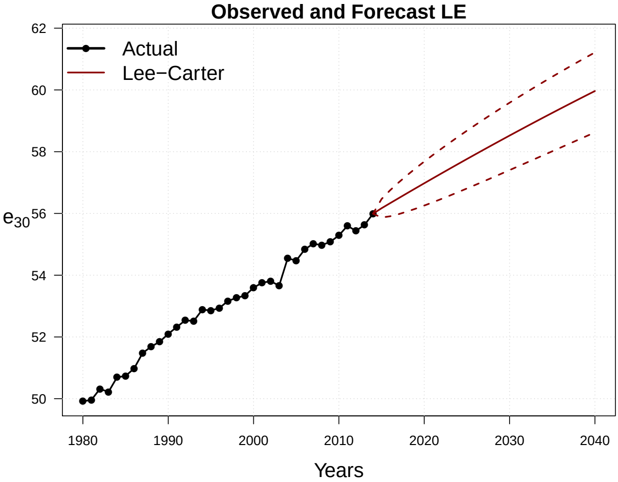

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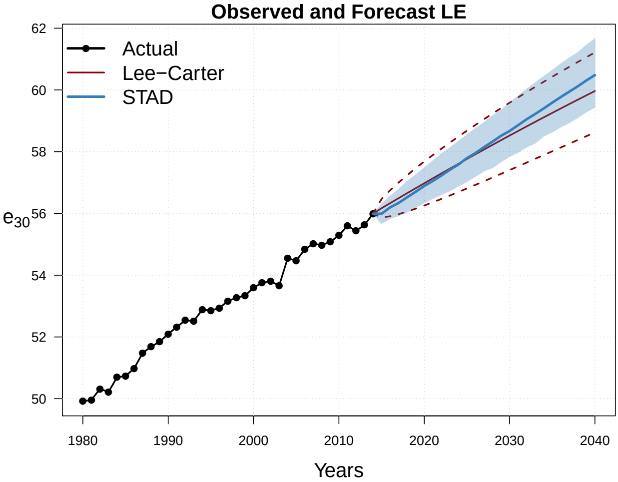

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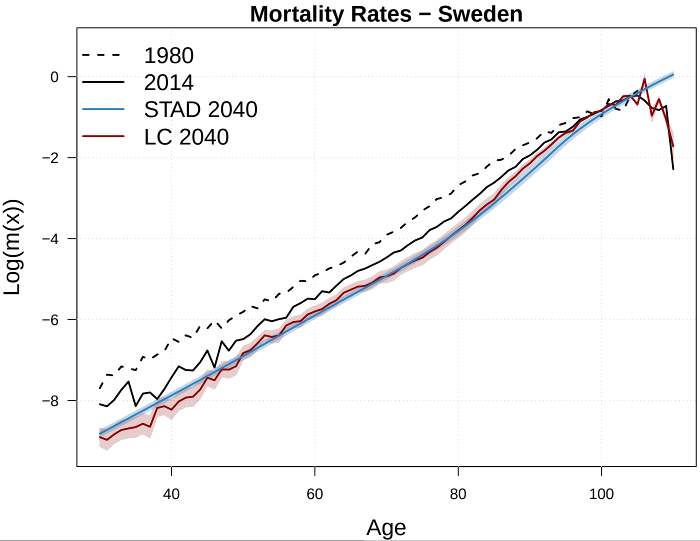

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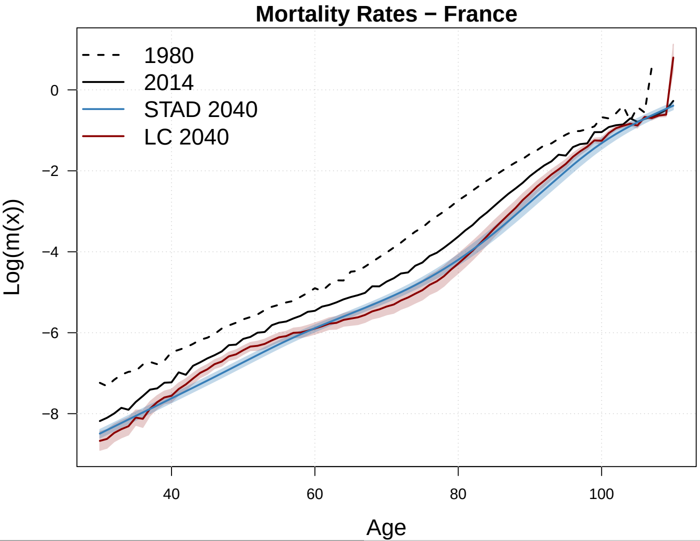

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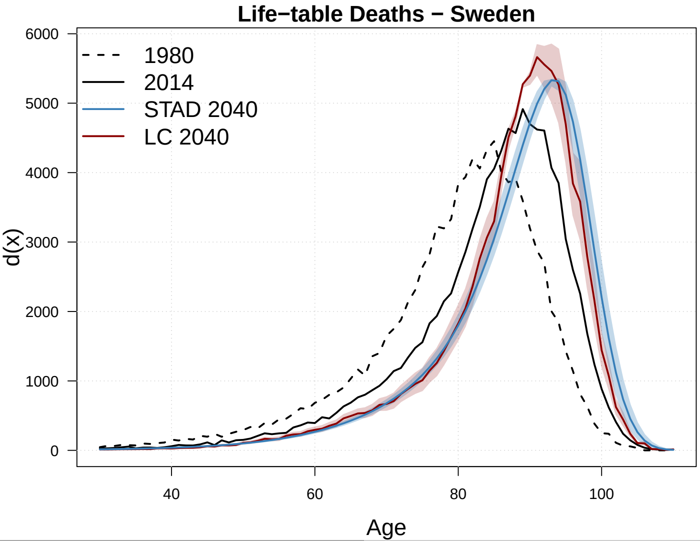

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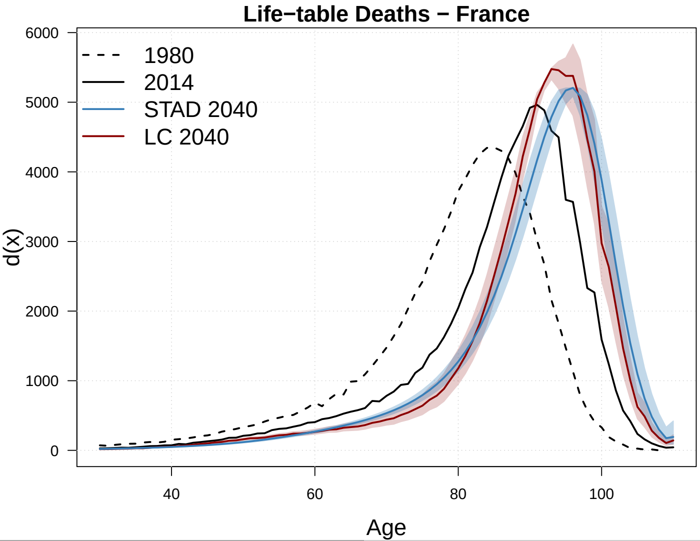

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Future work

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Thanks for your attention.

Comments and/or questions?

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