It Takes Two: Why Mortality Trend Modeling is more than modeling one Mortality Trend

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- Johannes Schupp
- September 13, 2019
- Longevity 15 Conference

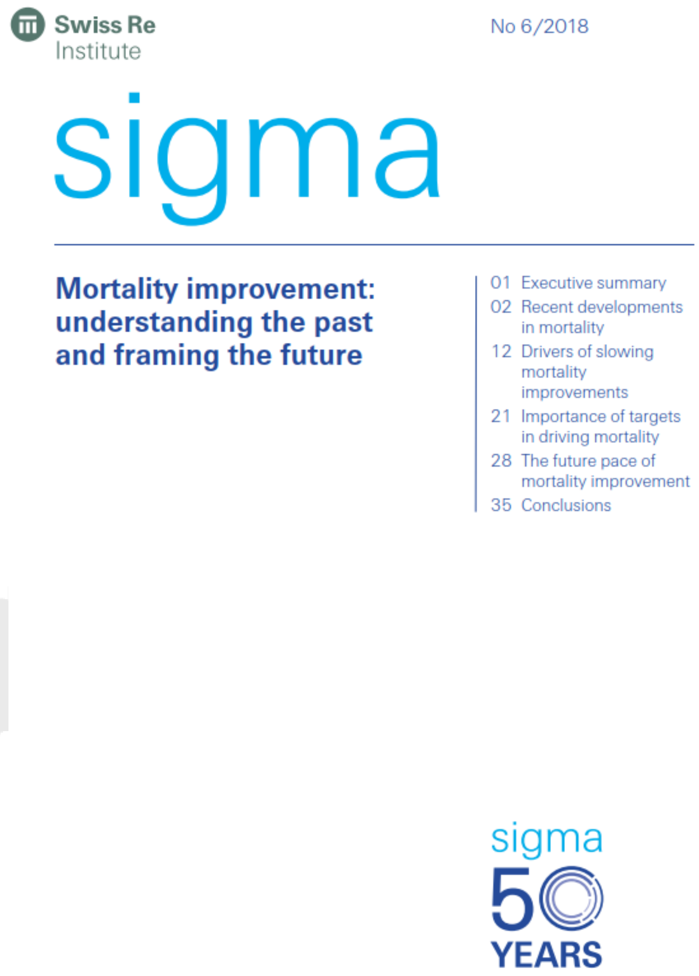
Agenda

- **Introduction: Why two mortality trends?**
  - Actual Mortality Trend (AMT)
  - Estimated Mortality Trend (EMT)
  - Some examples
- A combined model for AMT & EMT
  - AMT component
  - EMT component
- Applications
- Conclusion

Introduction: Why two mortality trends?

Highlights of recent SwissRe sigma study on „Mortality Improvements“

- “Mortality improvement has slowed unusually in the US, UK, Germany, the Netherlands and Taiwan.”
- “… but the recent slowdown is typically not statistically significant.”
- “Extrapolating future mortality trends solely from recent experience can be misleading unless we believe there has been a structural break.”
- “The ability to distinguish between shifts in the underlying mortality trend and short-term variability is crucial because a change in mortality trend is an aggregate risk that cannot be easily diversified away nor perfectly hedged.”


Swiss Re Institute

No 6/2018

sigma

Mortality improvement: understanding the past and framing the future

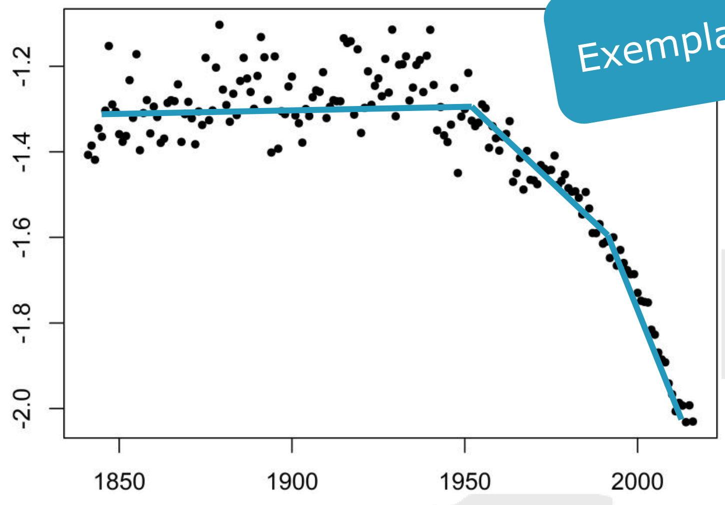
01 Executive summary
02 Recent developments in mortality
12 Drivers of slowing mortality improvements
21 Importance of targets in driving mortality
28 The future pace of mortality improvement
35 Conclusions

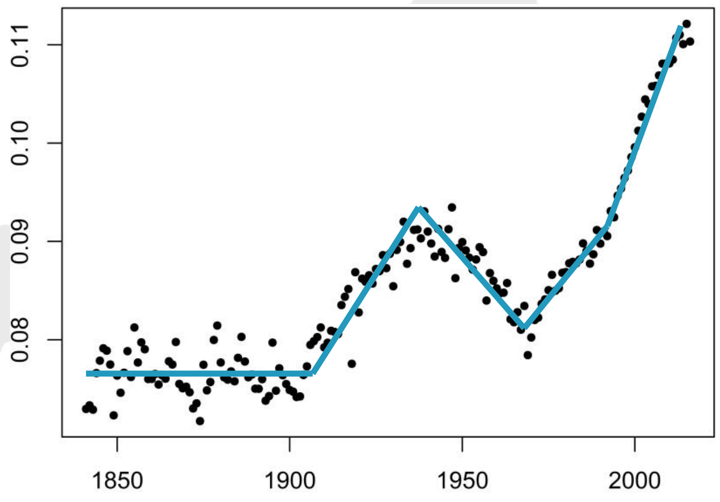
sigma 50 YEARS

Introduction: Why two mortality trends?

- two parameter processes (Cairns et al. (2006))
  - $\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^1 + \kappa_t^2 \cdot (x - \bar{x})$
  - parameters calibrated for English and Welsh males older than 60
- classical simulation approach: Random Walk with drift

Exemplary trends





- historical trend changed once in a while
  - only a piecewise linear trend with random changes in the trends slope
  - random fluctuation around the prevailing trend
- In principle, our approach can be applied to any changing mortality trend model.

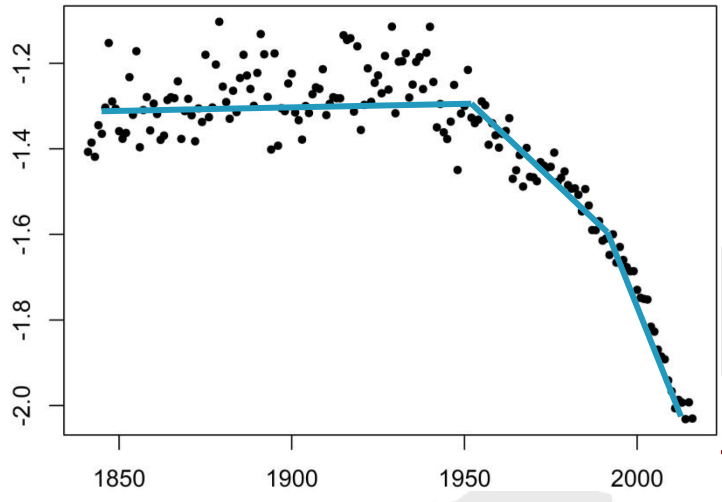
Introduction: Why two mortality trends?

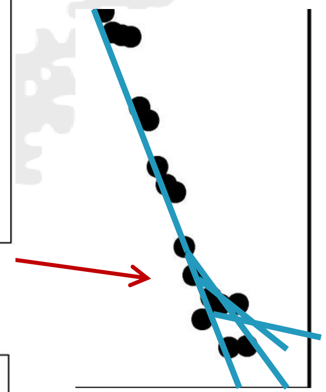
Actual Mortality Trend (AMT)

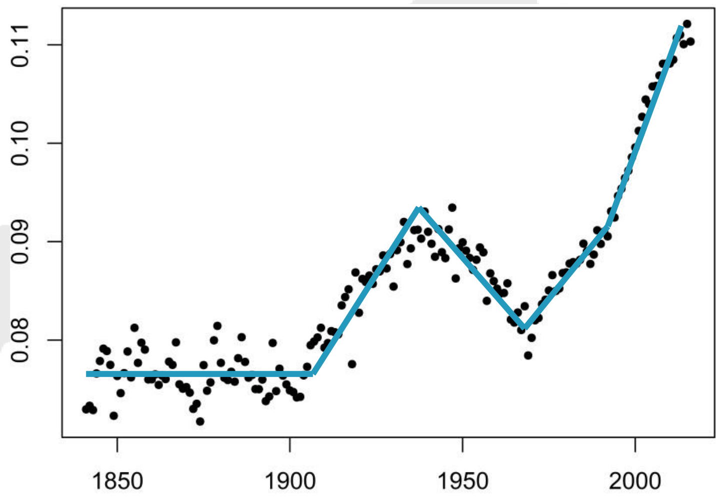
- The AMT describes realized mortality trends.
  - core of most existing mortality models
  - Time and magnitude of changes in the AMT and the error structure around the trend process need to be modeled.
- We have an idea of the historical AMT but it's not fully observable!
- We can't always distinguish between a recent trend change and "normal" random fluctuation around the prevailing trend.

→ possible undetected trend change in the recent years

- unknown current value and slope of the AMT
- **one model for the AMT**







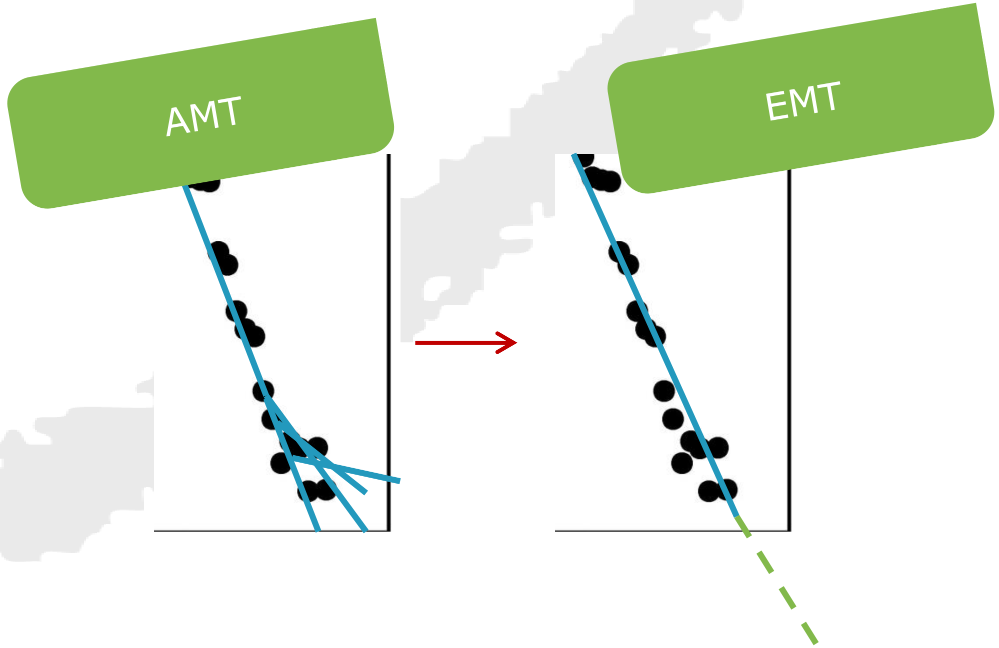
Introduction: Why two mortality trends?

Estimated Mortality Trend (EMT)

- The EMT describes the expectation of an actuary/demographer about the AMT, i.e. the current slope and value of the mortality trend at some point in time.
- based on most recent historical, observed mortality evolution and updated as soon as new observations become available
- The EMT is the basis for mortality projections, (generational) mortality tables, reserves, etc.
- **one model for the EMT**

"… but the recent slowdown is typically not statistically significant."

"Extrapolating future mortality trends solely from recent experience can be misleading unless we believe there has been a structural break."



Why two mortality trends?

Some examples

Why another trend?

- Requirement for AMT and/or EMT depends on application:
  - reserves for a portfolio → EMT today
  - capital for a portfolio run-off → AMT over the run-off
  - reserves for a portfolio after 10 years → AMT over the 10 years, EMT after 10 years
  - payout of a mortality derivative → AMT up to maturity, EMT at maturity
  - analyze the hedge effectiveness of the previous derivative → EMT at maturity, AMT beyond

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A Combined model for AMT/EMT

AMT component

- continuous piecewise linear trend, with random changes in the slope and random fluctuation around the trend
- AMT model specification:
  - Model the trend process with random noise → $\kappa_t = \hat{\kappa}_t + \epsilon_t;\ \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.
  - Extrapolate the most recent actual mortality trend → $\hat{\kappa}_t = \hat{\kappa}_{t-1} + \hat{d}_t$.
  - In every year, there is a possible change in the mortality trend with probability p.

    $$\rightarrow \hat{d}_t = \begin{Bmatrix} \hat{d}_{t-1} & \text{with probability } 1-p \\ \hat{d}_{t-1} + \lambda_t & \text{with probability } p \end{Bmatrix}$$
  - in the case of a trend change → $\lambda_t = M_t \cdot S_t$
    - with absolute magnitude of the trend change $M_t \sim \mathcal{LN}(\mu, \sigma^2)$
    - sign of the trend change S_t bernoulli distributed with values -1, 1 each with probability $\frac{1}{2}$

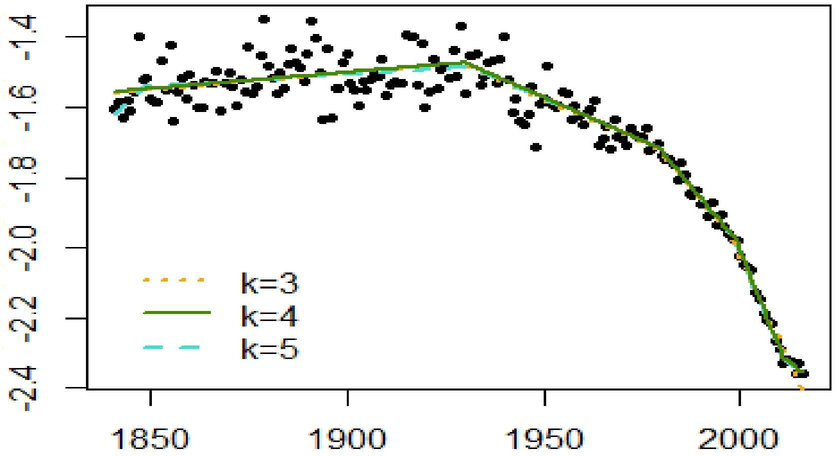
parameters to be estimated for projections:

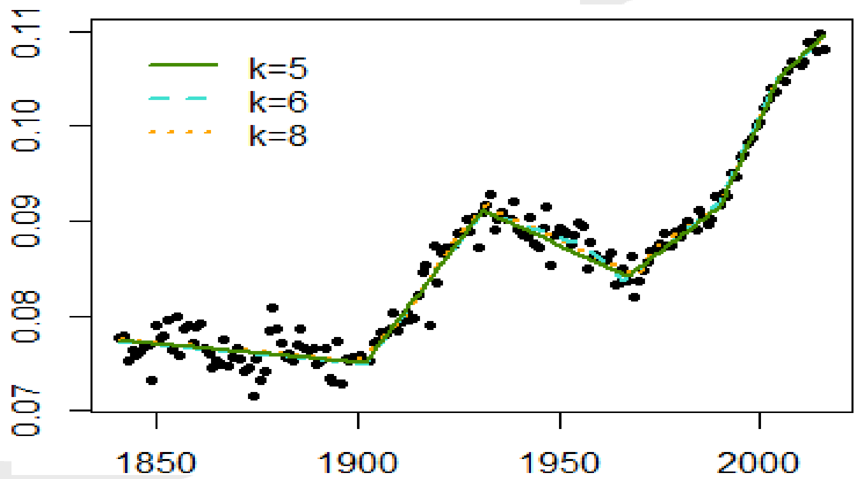
$p, \sigma_\epsilon^2, \mu, \sigma^2, \hat{d}_n, \hat{\kappa}_n$

A Combined model for AMT/EMT

AMT component

Idea: Use historic trends to estimate the parameters $p, \sigma_\epsilon^2, \mu, \sigma^2, \hat{d}_n, \hat{\kappa}_n$.





For details on the calibration we refer to Börger and Schupp (2018) and Schupp (2019). Parameter uncertainty is included.

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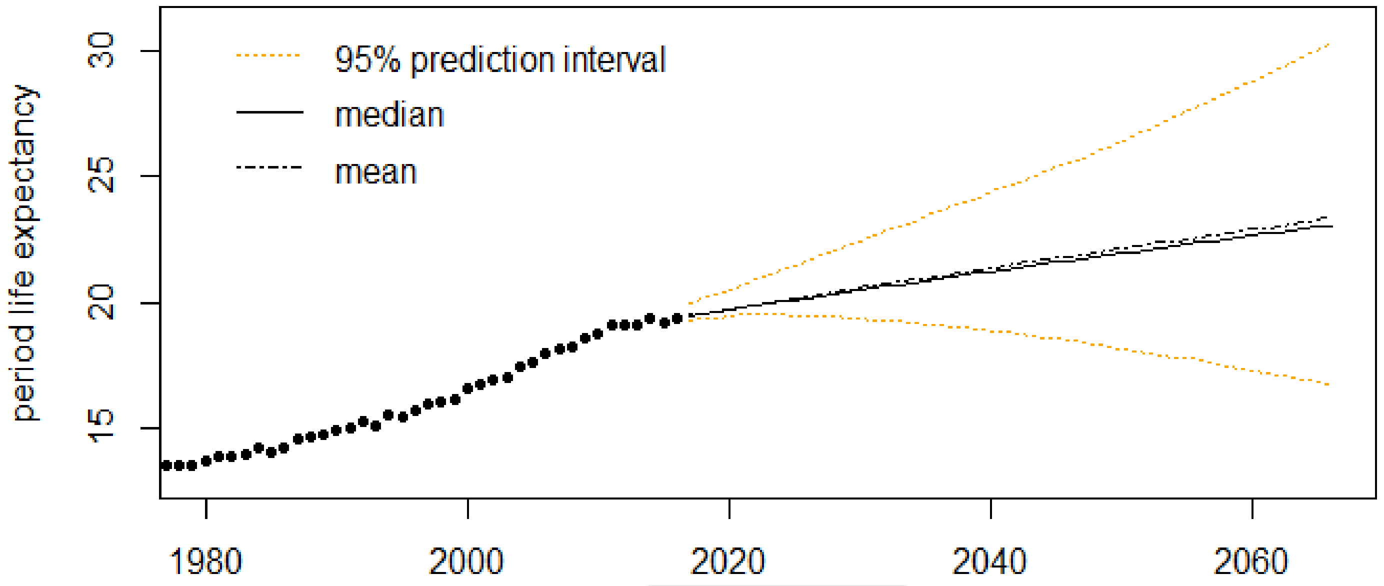
A Combined model for AMT/EMT

AMT component

Period life expectancies for 65-year old males in England and Wales



Ongoing of recent improvements and also slowdown of mortality improvements incorporated

A Combined model for AMT/EMT

EMT component

- We don't know today's AMT, but we want a model to estimate it: $\mathbb{E}(\hat{d}_t) = \tilde{d}_t$ and $\mathbb{E}(\hat{\kappa}_t) = \tilde{\kappa}_t$.
- Calculation of EMT is complex as all potential evolutions of unknown AMT need to be incorporated. Especially, in a simulation this is not feasible.
  - path-dependent calculation of the EMT
  - path-dependent recalibration of whole AMT

- piecewise linear trend process with symmetric changes in the AMT

  → Calibrate the EMT with a weighted linear regression on most recent data.

- How many years should be included in the regression?
  - too many → delayed reaction of EMT on trend changes in the AMT
  - too little → EMT is vulnerable to random noise in the AMT

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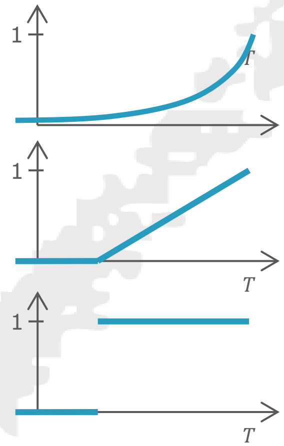
A Combined model for AMT/EMT

EMT component

- Higher influence of most recent data in the estimation of the regression.
  - weighted exponential regression in year T : $w_{exp}(t,T) = \frac{1}{\left(1+{}^{1}/_{h_{exp}}\right)^{T-t}}, t \leq T.$
  - weighted linear regression in year T : $w_{lin}(t,T) = \max\left(0; 1 - \frac{1}{h_{lin}}(T-t)\right), t \leq T,$
  - weighted constant regression in year T : $w_{const}(t,T) = \begin{cases} 1 & , if\ T - h_{const} < t \leq T \\ 0 & , if\ t \leq T - h_{const} \end{cases}.$



Calibration of the weighting parameters

- calibrate the AMT model
- Simulate the future evolution of the AMT 100.000 times to avoid dependencies on fixed historical trends.
- EMT calibration
  - After T=40 years, calculate the optimal weighting parameters based on:
    - EMTs cohort life expectancy of 65-year old males $\tilde{e}_{65,T}$ close to AMTs $\hat{e}_{65,t_{\omega}}$

A Combined model for AMT/EMT

EMT component – results

- EMT calibration
  - After T=40 years, calculate the optimal weighting parameters based on:
    - EMTs cohort life expectancy of 65-year old males $\tilde{e}_{65,T}$ close to AMTs $\hat{e}_{65,t_\omega}$

weights	constant	linear	exponential	fixed period
$h_{\cdot}^{(1)}$	10.0	11.0	2.2	30
$h_{\cdot}^{(2)}$	8.0	10.5	2.1	30
mse	1.59	1.56	1.58	3.04
$P(\tilde{e}_{65,T} < 95\% \cdot \hat{e}_{65,t_\omega})$	8.7%	8.5%	8.7%	15.0%
$P(\tilde{e}_{65,T} > 105\% \cdot \hat{e}_{65,t_\omega})$	7.3%	7.1%	7.0%	11.1%

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Applications
Overview

- 1. Hedge Effectiveness of a Value Hedge
- 2. SCR for Longevity Risk

- common assumptions
  - deterministic and constant interest rate 2%
  - annuitants'/pensioners' mortality rates are exactly as for males in England and Wales
  - portfolios are large enough → no unsystematic mortality risk
  - For the EMT's, we use linear weighting based on life expectancy optimization.

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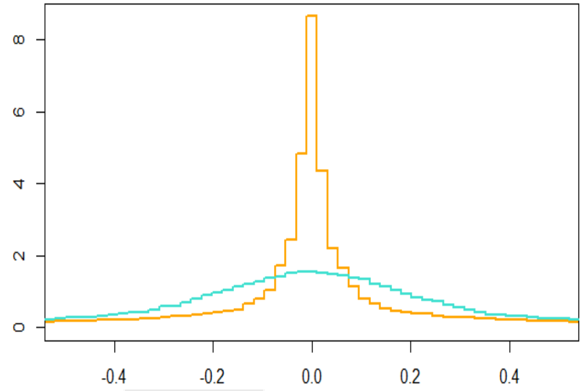
Applications

Example 1

Hedge Effectiveness of a Value Hedge



- pension fund with members aged 45 in t_0
- Hedge provider offers value hedge when they retire at $T = 20 + t_0$.
- If necessary, hedge fills up fund's liabilities at expiry.
- two Risks:
  - AMT changes after T
  - AMT assumption at T is inaccurate
- Unfortunately, the pension fund's trustees do not distinguish between AMT and EMT. They assume, that the current AMT is observable. Thus, they think their remaining risk is
  - $PV_T\left(pension\ payouts \mid AMT_{t_\omega}\right) - PV_T(pension\ payouts \mid AMT_T)$ (yellow).
  - hedge effectiveness: $1 - \frac{Var(Risk\ after\ hedge)}{Var(without\ hedge)} = \mathbf{92{,}1\%}$
- However, the actual risk is
  - $PV_T\left(pension\ payouts \mid AMT_{t_\omega}\right) - PV_T(pension\ payouts \mid EMT_T)$ (blue).
  - true hedge effectiveness: $1 - \frac{Var(Risk\ after\ hedge)}{Var(without\ hedge)} = \mathbf{87{,}2\%}$

hedge looks better than it is

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Applications

Example 2

SCR for Longevity Risk

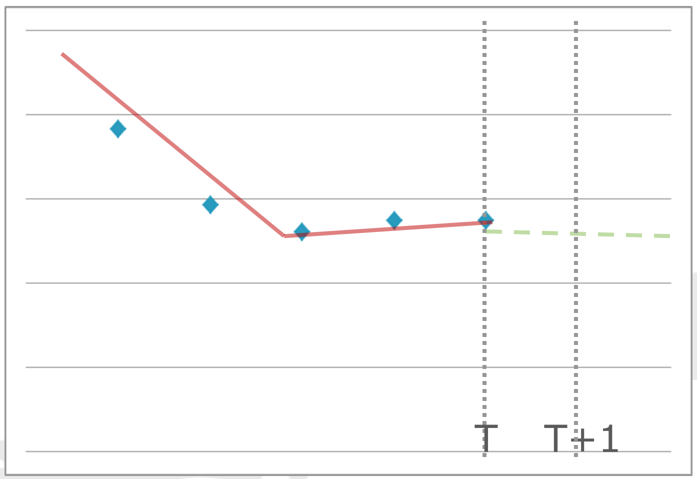
- Consider a portfolio of 75-year old annuitants at $T = 20 + t_0$ (no costs, no premiums).
- Insurer with internal model calculates the SCR as the 99,5% percentile of (see Börger (2010)):
- $\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$
  - CF_{T+1} actual cashflow
    - realized mortality evolution over 1-year horizon
    - →AMT component
  - BEL_t best-estimate of liabilities
    - influence of the additional one year observation
    - →EMT component with optimal weighting

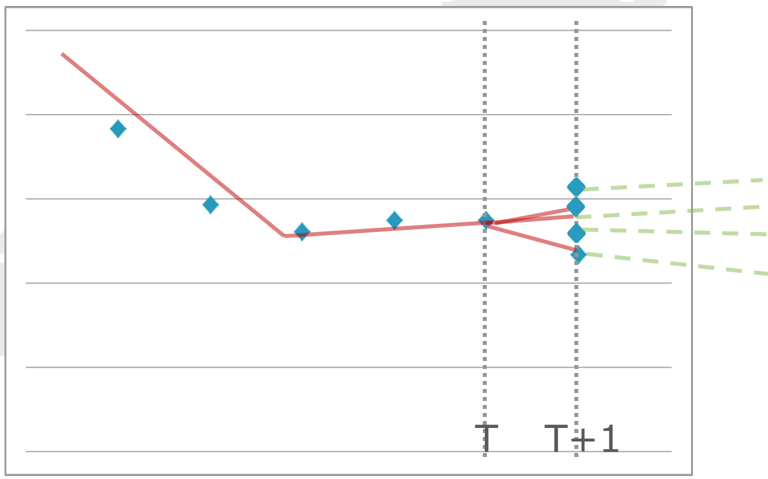
Applications
Example 2

SCR for Longevity Risk – continued

- $\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$
- estimate AMT up to T 10.000 times (outer paths)
  - for each simulated AMT, simulate 10.000 inner 1-year paths
  - estimate 99.5% precentile of ΔBEL_{T+1} based on the 10.000 inner paths
- illustration: one outer path

$$\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$$





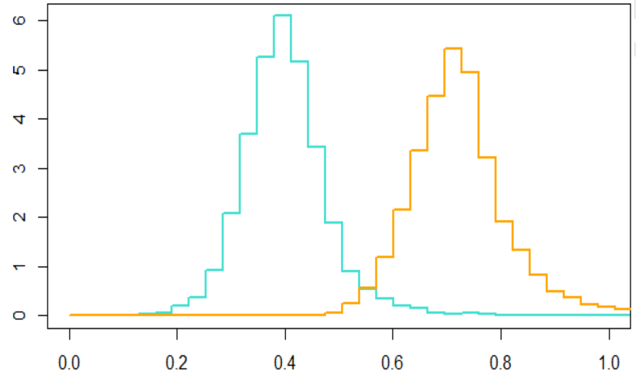
Applications

Example 2

SCR for Longevity Risk – continued

- If the insurer falsely assumes the AMT to be known, he would calculate the BEL_t based on the AMT.

→The SCR would be on average **0.73** (yellow).



- Instead, if the insurer recognizes the AMT to be unknown,

→the SCR would be on average **0.40** (blue).

- If the AMT is assumed to be known, the longevity risk would be **overestimated** in this example!
- Why?
  - AMT exhibits rather massive trend changes in one year
  - Annual changes in EMT are not that strong as the EMT does not pick up trend changes immediately

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Conclusion

- Two trends need to be distinguished and modeled.
  - The actual mortality trend (AMT) is the prevailing, unobservable mortality trend.
  - The estimated mortality trend (EMT) is the estimate of the AMT.
- The trend to consider depends on the question in view.
- The AMT is modeled as a continuous and piecewise linear trend with random changes in the trend’s slope.
- Choice of EMT approach is crucial in many practical situations.
  - A weighted regression approach seems reasonable.
  - Optimal regression weights can be determined in a practical setting.

- If the AMT is wrongfully assumed observable, risk is significantly misestimated in all our examples – sometimes underestimated, sometimes overestimated.

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