Downside Risk Management of a Defined Benefit Plan Considering Longevity Basis Risk

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Existing Literature on Pension Downside Risk Management

- Capital market risk and longevity risk in defined benefit plans
- Control total pension cost
  - Delong et al. (2008); Josa-Fombellida and Rincon-Zapatero (2004); Cox et al. (2011); and others
  - Downside risk management: Maurer et al. (2009)
- Control pension underfunding
  - Haberman (1997); Haberman et al. (2000); Owadally and Habermana (2004); Habermana and Sung (2005); Chang et al. (2003); Kouwenberg (2001); and others
  - Downside risk management: Bogentoft et al. (2001)

These papers do not control downside risk arising from extreme underfunding and excessive total pension cost at the same time.

Outline

- We propose an optimization model by imposing two conditional value at risk (CVaR) constraints to control tail risk related to pension funding status and total pension cost.
- We investigate optimal longevity risk hedge ratios with basis risk.
  - Basis risk arises from the mismatch between a plan's actual longevity risk and the risk of a reference population underlying a hedging instrument.
  - Two longevity risk hedging strategies: the ground-up hedging strategy and the excess-risk hedging strategy.
  - The excess-risk hedging strategy is much more vulnerable to longevity basis risk.

Basic Framework

- The pension underfunding/surplus at time t, UL_t:

$$UL_t = PBO_t - PA_t - C \tag{1}$$

- Total underfunding liability TUL before retirement T

$$TUL = \sum_{t=1}^{T} \frac{UL_t}{(1+\rho)^t}$$

- Total pension cost TPC (Maurer, Mitchell and Rogalla, 2009)

$$TPC = \sum_{t=1}^{T} \frac{C + SC_t(1+\psi_1) - W_t(1-\psi_2)}{(1+\rho)^t},$$

where ρ is the valuation rate. The constants ψ_1 and ψ_2 are penalty factors on supplementary contributions SC_t and withdrawals W_t respectively.

Two-Population Mortality Model

Li and Lee (2005)'s Two-Population Mortality Model

$$\begin{aligned} \ln q(x,t) &= s(x) + B(x)K(t) + b(x)k(t) + \epsilon(x,t) \\ \ln q'(x,t) &= s'(x) + B(x)K(t) + b'(x)k'(t) + \epsilon'(x,t). \end{aligned} \tag{2}$$

- The mortality common risk factor:

$$K(t) = g + K(t-1) + \sigma_K e(t), \quad e(t) \sim N(0,1). \tag{3}$$

- The country-specific mortality risk factors:

$$\begin{aligned} k(t) &= r_0 + r_1 k(t-1) + \sigma_k e_1(t), \quad e_1(t) \sim N(0,1) \\ k'(t) &= r'_0 + r'_1 k'(t-1) + \sigma'_k e_2(t), \quad e_2(t) \sim N(0,1). \end{aligned} \tag{4}$$

Objective Function and Optimization Problem

$$
\begin{aligned}
\underset{w,C}{\text{Minimize}} \quad & \mathrm{E}\left[\sum_{t=1}^{T}\left(\frac{UL_t}{(1+\rho)^t}\right)^2\right] \\
\text{subject to} \quad & \mathrm{E}(TUL) = 0 \\
& \mathrm{CVaR}_{\alpha_{\mathrm{TPC}}}(TPC) \le \tau \\
& \mathrm{CVaR}_{\alpha_{\mathrm{TUL}}}(TUL) \le \zeta \\
& 0 \le w_i \le 1, \qquad\qquad i = 1, 2, \ldots, n \\
& \sum_{i=1}^{n} w_i = 1 \\
& C \ge 0,
\end{aligned}
\tag{5}
$$

where the constants ζ and τ are the pre-specified parameters reflecting the plan's downside risk tolerance.

Example Assumptions

- A US cohort joins the plan at age $x_0 = 45$ at $t = 0$.
- They will retire at $T = 20$ at age $x = 65$.
- The initial pension fund $M = \$5$ million at $t = 0$
- Annual retirement benefit of $B = \$10$ million
- The pension funds are invested equally in three assets:
  - S&P 500 index;
  - Merrill Lynch corporate bond index;
  - 3-month T-bill.
- The plan now makes a normal contribution of $C = \$2.5$ million annually.
- Pension valuation rate $\rho = 0.08$
- Penalty factors on supplementary contributions and withdrawals are both equal to $\psi_1 = \psi_2 = 0.2$

Optimization Results without Hedging—Example 1

Table: Initial and Optimal Pension Strategies without Hedging Given $\zeta = 45.86$ and $\tau = 34.56$

	w_1	w_2	w_3	C	J	$\text{CVaR}_{95\%}$ (TUL)	$\text{CVaR}_{95\%}$ (TPC)
Initial	1/3	1/3	1/3	2.50	1119	45.86	34.56
Optimal	0.14	0.56	0.30	2.65	1019	36.63	34.56

J is the value of the objective function without hedging.

Optimization Results without Hedging—Example 2

$$\zeta = z_1 \times \text{CVaR}_{95\%}(TUL)_0 = z_1 \times 45.86$$
$$\tau = z_2 \times \text{CVaR}_{95\%}(TPC)_0 = z_2 \times 34.56$$

$\zeta = 44.71$ and $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$)

w_1	w_2	w_3	C	J	$\text{CVaR}_{95\%}(TUL)$	$\text{CVaR}_{95\%}(TPC)$
0.20	0.62	0.18	2.46	1081	42.91	33.70

J is the value of the objective function without hedging.

Two Pension Longevity Risk Hedging Strategies

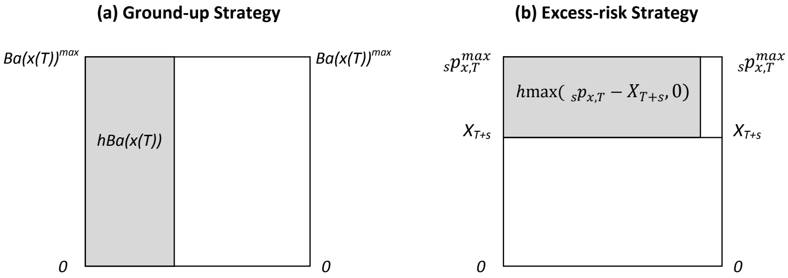

Figure: Two pension longevity risk hedging strategies: the ground-up hedging strategy (on the left) and the excess-risk hedging strategy with $s = 1, 2, \ldots$ (on the right)

Ground-Up Hedging Strategy with Basis Risk

- The ground-up hedging strategy is subject to a transaction cost factor δ^G and a basis risk penalty factor γ^G
- The US DB plan hedges with a longevity security whose payoffs are based on the UK population mortality experience.
- The upper limits of the two CVaR constraints:

$$\begin{aligned} \zeta &= z_1 \times \text{CVaR}_{95\%}(TUL)_0 = 0.975 \times 45.86 = 44.71, \\ \tau &= z_2 \times \text{CVaR}_{95\%}(TPC)_0 = 0.975 \times 34.56 = 33.70. \end{aligned} \tag{6}$$

Ground-Up Hedging Strategy with Basis Risk

Table: Optimal Ground-up Hedging Strategies with Longevity Basis Risk Given $\zeta = 44.71$ and $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$)

δ^G	0		0.05		0.07	
γ^G	0	0.1	0	0.1	0	0.1
C^G	2.68	2.68	2.58	2.58	2.49	2.48
w_1^G	0.13	0.13	0.15	0.15	0.18	0.18
w_2^G	0.55	0.54	0.64	0.64	0.64	0.64
w_3^G	0.32	0.33	0.21	0.21	0.18	0.18
h^G	18.5%	18.5%	17.6%	17.6%	6.0%	5.7%
J^G	993	993	1057	1057	1079	1079

h^G is the longevity risk hedging ratio and J^G is the value of the objective function with the ground-up strategy.

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Excess-Risk Hedging Strategy with Basis Risk

- The US DB plan hedges with a longevity security whose payoffs are based on the UK population mortality experience.
- A series of exercise prices ${}_s\bar{p}'_{x,T} + \sigma_{p'_{x,T}}$ at time $T+s, s = 1, 2, \ldots.$
- The excess-risk hedging strategy is subject to a transaction cost factor δ^E and a basis risk penalty factor γ^E
- The upper limits of the two CVaR constraints:

$$\begin{aligned} \zeta &= z_1 \times \text{CVaR}_{95\%}(TUL)_0 = 0.975 \times 45.86 = 44.71, \\ \tau &= z_2 \times \text{CVaR}_{95\%}(TPC)_0 = 0.975 \times 34.56 = 33.70. \end{aligned} \tag{7}$$

Excess-Risk Hedging Strategy with Basis Risk

Table: Optimal Excess-risk Hedging Strategies with Longevity Basis Risk Given $\zeta = 44.71$, $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$) and Strike Level ${}_s\bar{p}'_{x,T} + \sigma_{p'_{x,T}}$

δ^E	0		0.05		0.1	
γ^E	0	0.1	0	0.1	0	0.1
C^E	2.46	2.46	2.46	2.46	2.46	2.46
w_1^E	0.17	0.17	0.16	0.17	0.17	0.18
w_2^E	0.67	0.67	0.68	0.67	0.66	0.65
w_3^E	0.16	0.16	0.16	0.16	0.17	0.17
h^E	84.8%	4.2%	54.9%	1.1%	39.1%	0.7%
J^E	1078	1079	1078	1079	1079	1079

h^E is the longevity risk hedging ratio and J^E is the value of the objective function with the excess-risk strategy.

Excess-Risk Hedging Strategy with Basis Risk

Table: Optimal Excess-risk Hedging Strategies with Longevity Basis Risk Given $\zeta = 44.71$, $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$) and Strike Level ${}_s\bar{p}'_{x,T} + \sigma_{p'_{x,T}}$

δ^E	0		0.05		0.1	
γ^E	0	0.1	0	0.1	0	0.1
C^E	2.46	2.46	2.46	2.46	2.46	2.46
w_1^E	0.17	0.17	0.16	0.17	0.17	0.18
w_2^E	0.67	0.67	0.68	0.67	0.66	0.65
w_3^E	0.16	0.16	0.16	0.16	0.17	0.17
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h^E is the longevity risk hedging ratio and J^E is the value of the objective function with the excess-risk strategy.

Basis Risk vs. No Basis Risk—Ground-up Hedging

Table: Optimal Ground-up Hedging Ratios Given $\zeta = 44.71$, $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$) and $\gamma^G = 0$

δ^G	0	0.05	0.07	0.1
No Basis Risk h^G	18.5%	17.6%	6.4%	0.0%
Basis Risk h^G	18.5%	17.6%	6.0%	0.0%

h^G is the longevity risk hedging ratio with the ground-up strategy.

Basis Risk vs. No Basis Risk—Excess-risk Hedging

Table: Optimal Excess-risk Hedging Ratios Given $\zeta = 44.71$, $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$), Strike Level ${}_s\bar{p}'_{x,T} + \sigma_{p'_{x,T}}$ and $\gamma^E = 0$

δ^E	0	0.05	0.1	0.15
No Basis Risk h^E	100%	100%	99.9%	96.3%
Basis Risk h^E	84.8%	54.9%	39.1%	20.1%

h^E is the longevity risk hedging ratio with the excess-risk strategy.

Basis Risk vs. No Basis Risk—Excess-risk Hedging

Table: Optimal Excess-risk Hedging Ratios Given $\zeta = 44.71$, $\tau = 33.70$ (i.e. $z_1 = z_2 = 0.975$), Strike Level ${}_s\bar{p}'_{x,T} + \sigma_{p'_{x,T}}$ and $\gamma^E = 0$

δ^E	0	0.05	0.1	0.15
No Basis Risk h^E	100%	100%	99.9%	96.3%
Basis Risk h^E	84.8%	54.9%	39.1%	20.1%

h^E is the longevity risk hedging ratio with the excess-risk strategy.

Why the Excess-risk Hedging Strategy is So Sensitive to Basis Risk?

- A good hedging strategy: a contract whose mortality dynamic is highly correlated with that to be hedged.
- Effectiveness of excess-risk hedging

$$\mathrm{Corr}\Big(\sum_{s=1}^{\infty} v^s \max\left[{}_s p_{x,T} - ({}_s\bar{p}_{x,T} + \sigma_{p_{x,T}}), 0\right], \sum_{s=1}^{\infty} v^s \max\left[{}_s p'_{x,T} - ({}_s\bar{p}'_{x,T} + \sigma_{p'_{x,T}}), 0\right]\Big) = 0.02$$

- Effectiveness of ground-up hedging

$$\mathrm{Corr}(a(x(T)), a'(x(T))) = 0.97$$

Conclusion

- This paper proposes a model to identify the optimal contribution, asset allocation and longevity risk hedging strategies subject to two CVaR constraints on underfunding and total pension cost for a DB pension plan.
- We investigate how sensitive a hedging strategy is to longevity basis risk.
  - We compare two longevity risk hedging strategies—the ground-up hedging strategies and the excess-risk hedging strategy.
  - The excess-risk hedging strategy is much more sensitive to longevity basis risk than the ground-up hedging strategy.