Optimisation in Life–Saving Models of Mortality

Jim Oeppen[1] José Manuel Aburto[1] Jesús–Adrián Álvarez[1]
Marie–Pier Bergeron Boucher[1]

[1]Interdisciplinary Centre on Population Dynamics
University of Southern Denmark

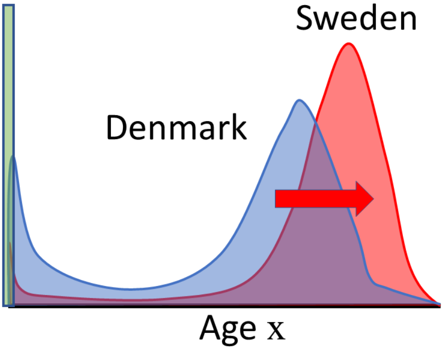
Longevity 16

Marienlyst, Denmark, 13th August, 2021

Outline

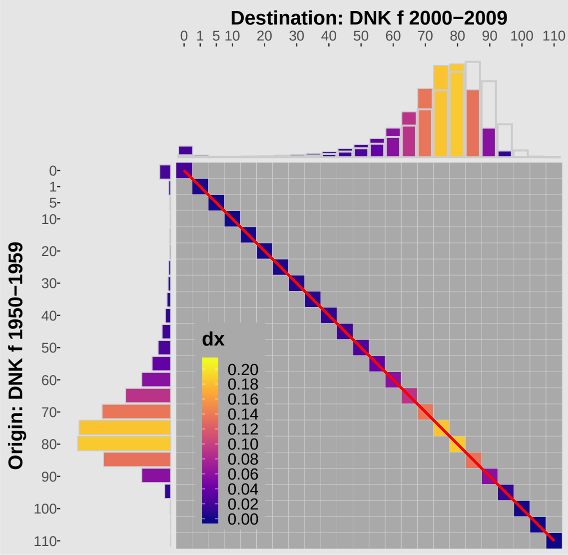
1. **The Life–Table as Earth–Moving**
2. The Cost of Life–Saving
3. Optimisation Algorithm
4. Individual Variability
5. Forecasting

The Problem



Wasserstein Distance (EMD) $W_1=|e_0^S - e_0^D|$

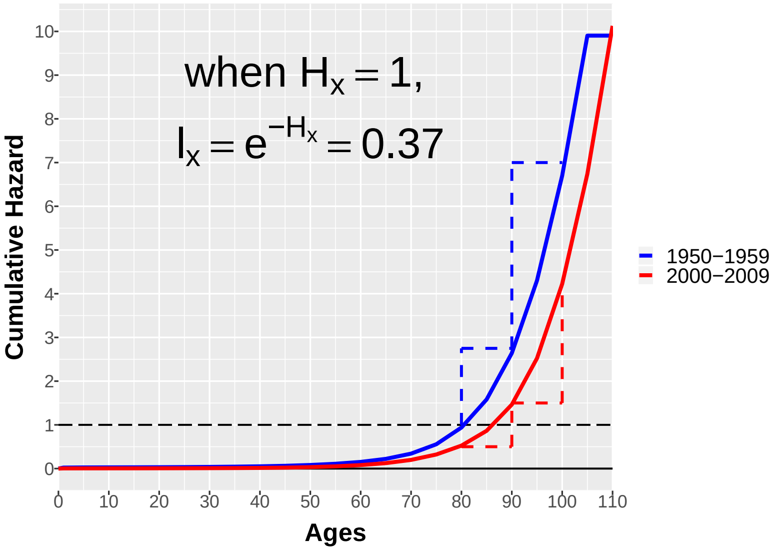
Transport Plan before Transitions: target in grey



Outline

1. The Life–Table as Earth–Moving
2. **The Cost of Life–Saving**
3. Optimisation Algorithm
4. Individual Variability
5. Forecasting

Denmark: female



Work done in the Life Table

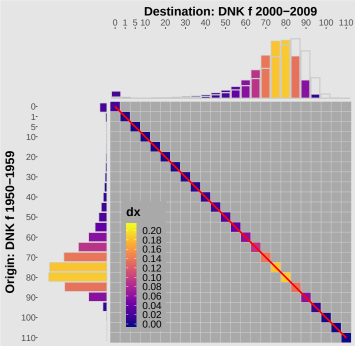
Denmark: female, 2010–2016

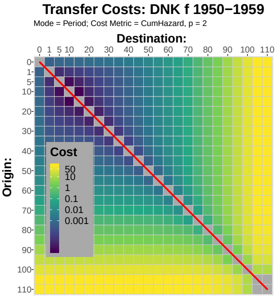
$$e_0 = \sum_{x=0}^{\omega} d_x \cdot x = 82 \quad \sum_{x=0}^{\omega} d_x \cdot H_x = 1$$

Average lifetime
or age at death
Fatal dose of Age = 82

Average Hazard survived
or killed by.
Fatal dose of Hazard = 1

Transport Plan and Cost Matrix





Outline

1 The Life–Table as Earth–Moving

2 The Cost of Life–Saving

3 **Optimisation Algorithm**

4 Individual Variability

5 Forecasting

Balanced Optimal Transportation Problem

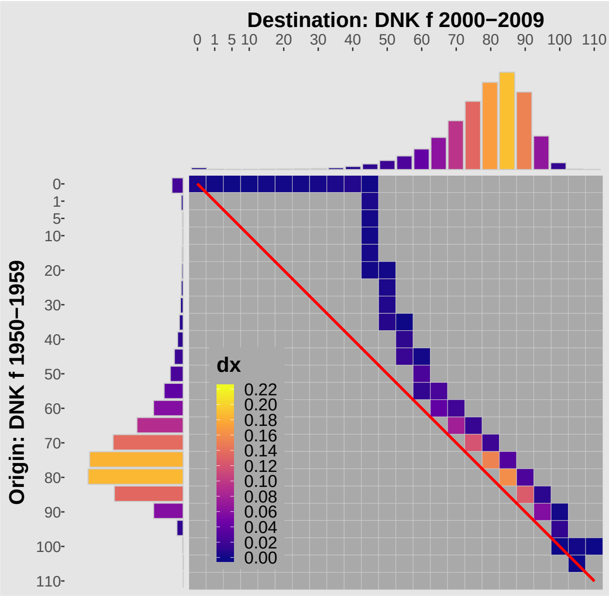
$$\text{minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \tag{1}$$

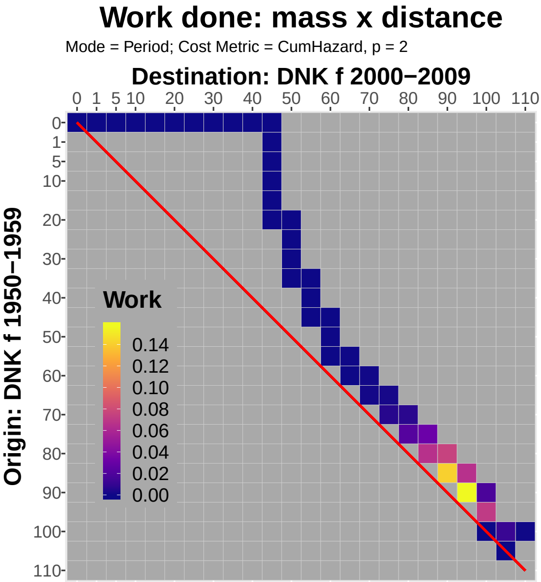
$$\text{subject to} \sum_{j=1}^{n} x_{ij} = o_i, \; \sum_{i=1}^{m} x_{ij} = d_j \tag{2}$$

$$\sum_{i=1}^{m} o_i = \sum_{j=1}^{n} d_j = \text{Life Table Radix} \tag{3}$$

$$x_{ij} \geq 0 \tag{4}$$

This is a linear program with $m \times n$ decision variables x_{ij}, $n + m$ functional constraints, and $m \times n$ non–negativity constraints.


Destination: DNK f 2000–2009
0 1 5 10 20 30 40 50 60 70 80 90 100 110
Origin: DNK f 1950–1959
0- 1- 5- 10- 20- 30- 40- 50- 60- 70- 80- 90- 100- 110-
dx
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00


Work done: mass x distance
Mode = Period; Cost Metric = CumHazard, p = 2
Destination: DNK f 2000−2009
0 1 5 10 20 30 40 50 60 70 80 90 100 110
Origin: DNK f 1950−1959
0 1 5 10 20 30 40 50 60 70 80 90 100 110
Work
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00

Outline

1 The Life–Table as Earth–Moving

2 The Cost of Life–Saving

3 Optimisation Algorithm

4 **Individual Variability**

5 Forecasting

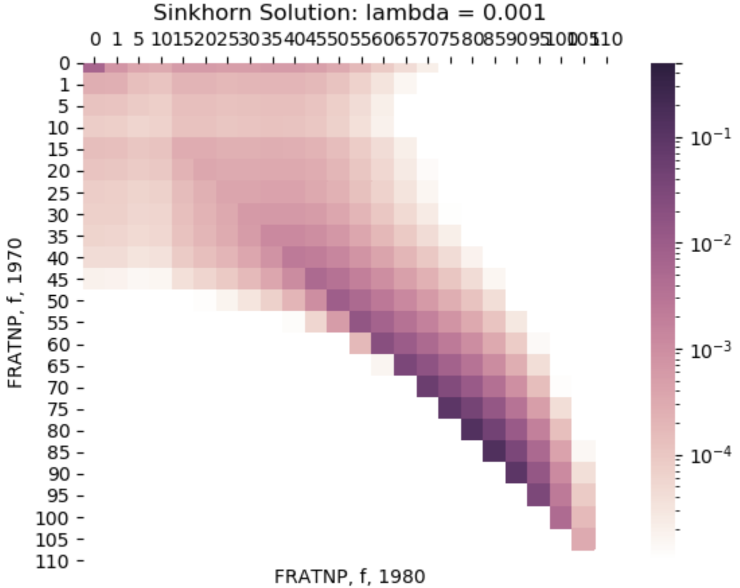
Entropic Regularization

$$\text{Entropy } H(x) = -\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \log x_{ij} \tag{5}$$

$$\text{minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} - \lambda H(x) \tag{6}$$

$$\lambda = \text{Regularization Parameter} \tag{7}$$

Strong concavity of Entropy.
No need for power on transfer costs.
Variability: a.k.a Frailty, Cross–Hauling?


Sinkhorn Solution: lambda = 0.001
0 1 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110
FRATNP, f, 1970
FRATNP, f, 1980
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Outline

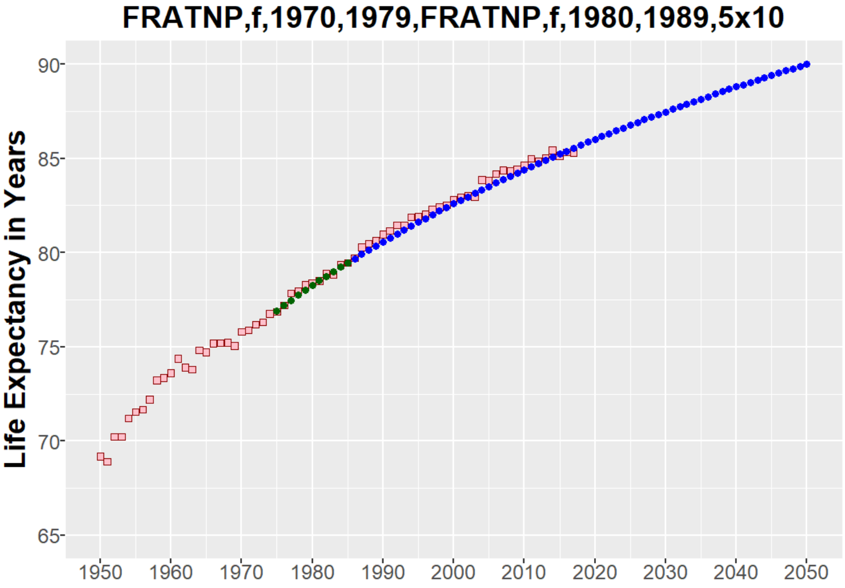
1 The Life–Table as Earth–Moving

2 The Cost of Life–Saving

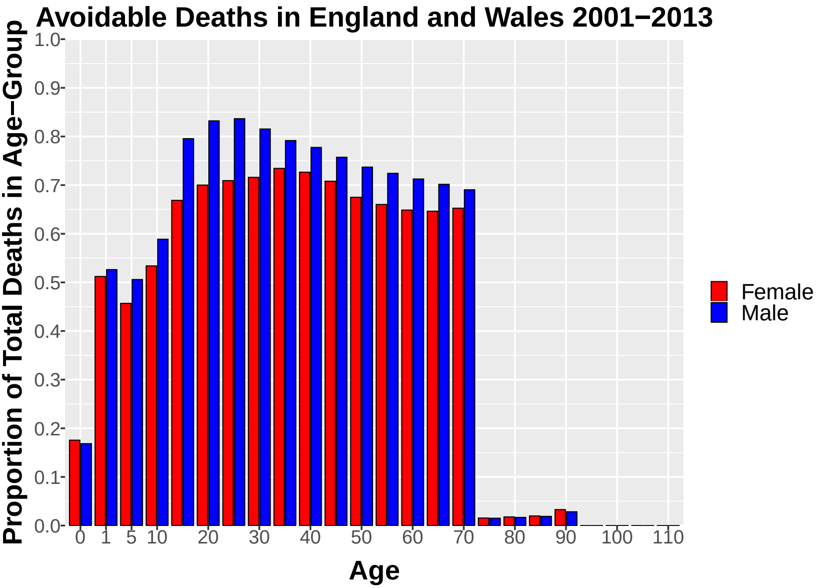
3 Optimisation Algorithm

4 Individual Variability

5 Forecasting


FRATNP,f,1970,1979,FRATNP,f,1980,1989,5x10
Life Expectancy in Years
90
85
80
75
70
65
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050

Thank You


Avoidable Deaths in England and Wales 2001−2013
Proportion of Total Deaths in Age−Group
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 1 5 10 20 30 40 50 60 70 80 90 100 110
Age
Female
Male

Removing All Avoidable Mortality
Effect on Life Expectancy at Birth

Country	Year	Data	Male	Female
Eng&Wales	2010-16	Observed	79.2	82.9
Eng&Wales	2010-16	Hypothetical	84.1	84.2
Japan	2017	Observed	81.1	87.3

Method: Preston *et al.* (2001) Box 4.2, p. 85

The Last Pair of Life Tables with a Feasible Solution