A model for the forecasting of socioeconomic mortality differentials

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September 8th 2012

Agenda

- Motivation
- Literature review
- Modelling mortality differentials
- Case study: Mortality by deprivation in England
- Conclusions

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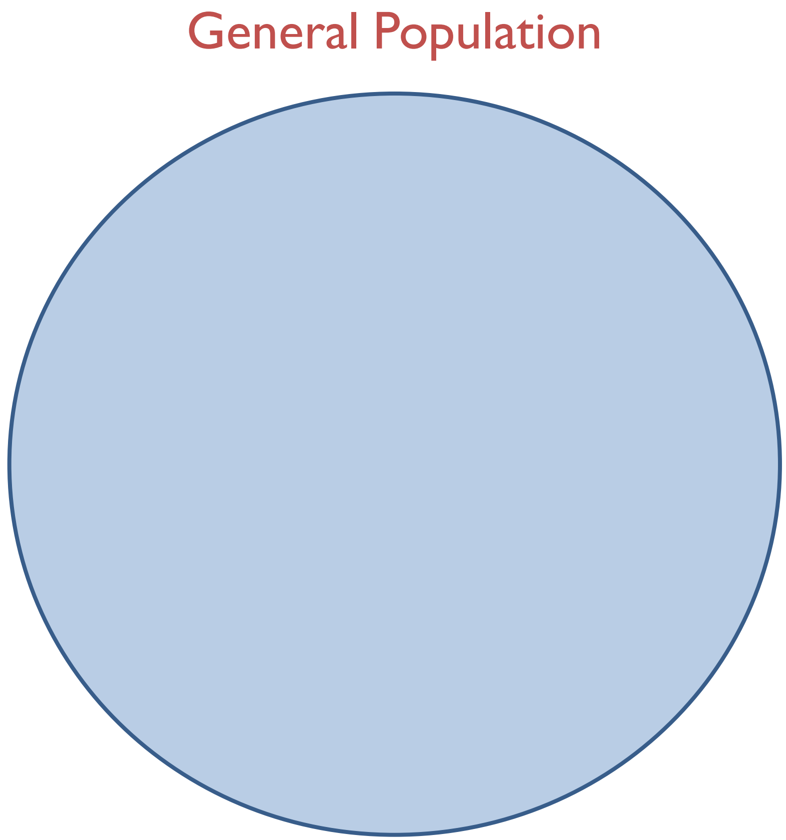
Motivation

Subpopulation mortality

- Mortality and life expectancy differ across subpopulations

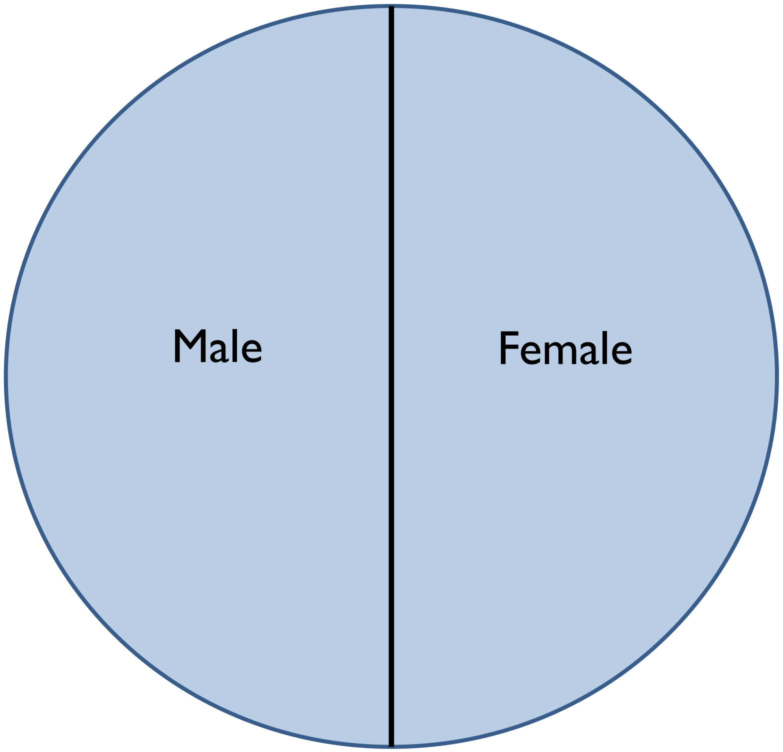


Motivation

Subpopulation mortality

- Mortality and life expectancy differ across subpopulations
  - Gender

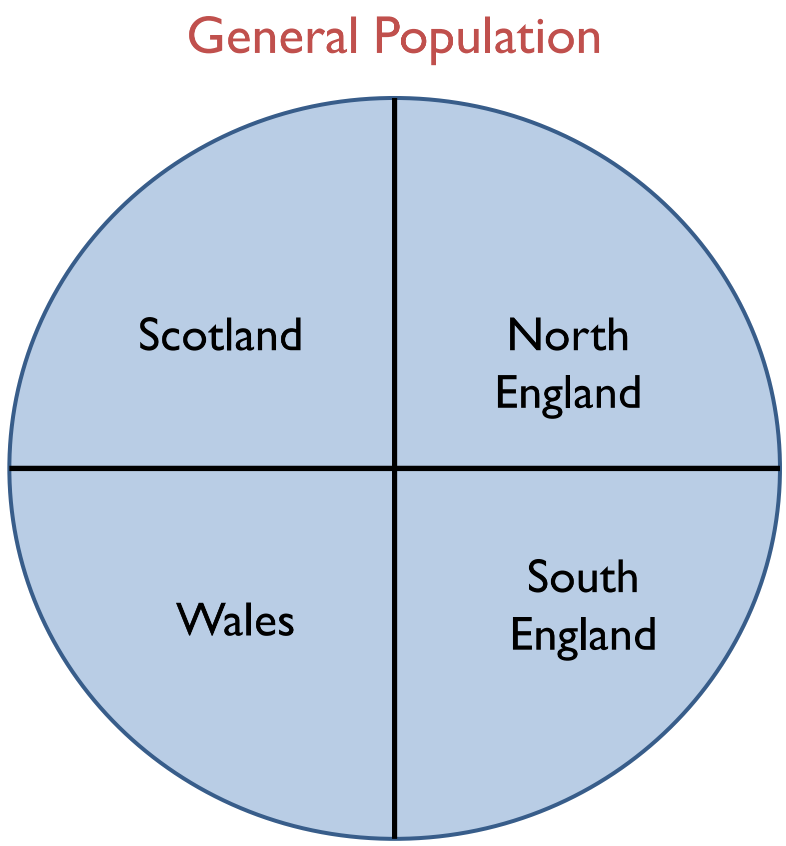
General Population



Motivation

Subpopulation mortality

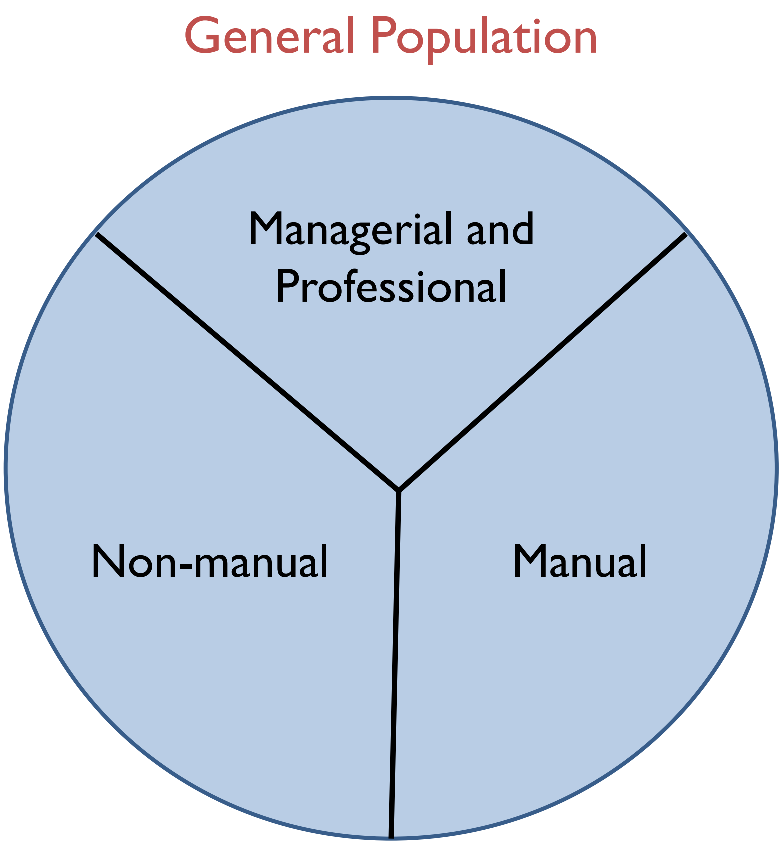
- Mortality and life expectancy differ across subpopulations
  - Gender
  - Geographic area



Motivation

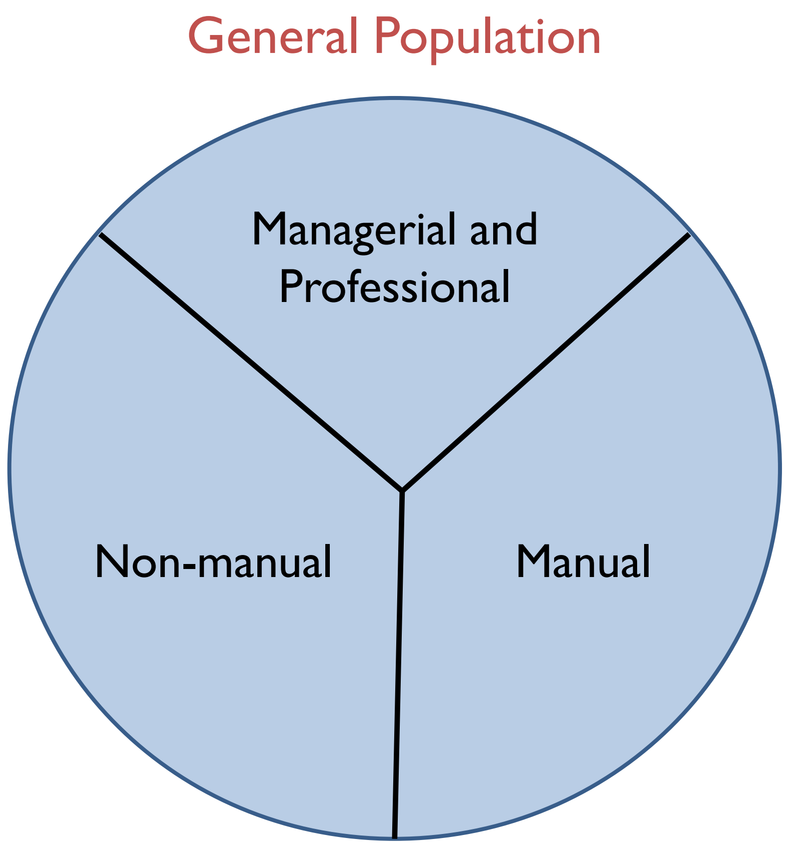
Subpopulation mortality

- Mortality and life expectancy differ across subpopulations
  - Gender
  - Geographic area
  - Socioeconomic variables



Motivation
Subpopulation mortality

- Mortality and life expectancy differ across subpopulations
  - Gender
  - Geographic area
  - Socioeconomic variables
- Mortality forecasts are commonly prepared independently for each subpopulation

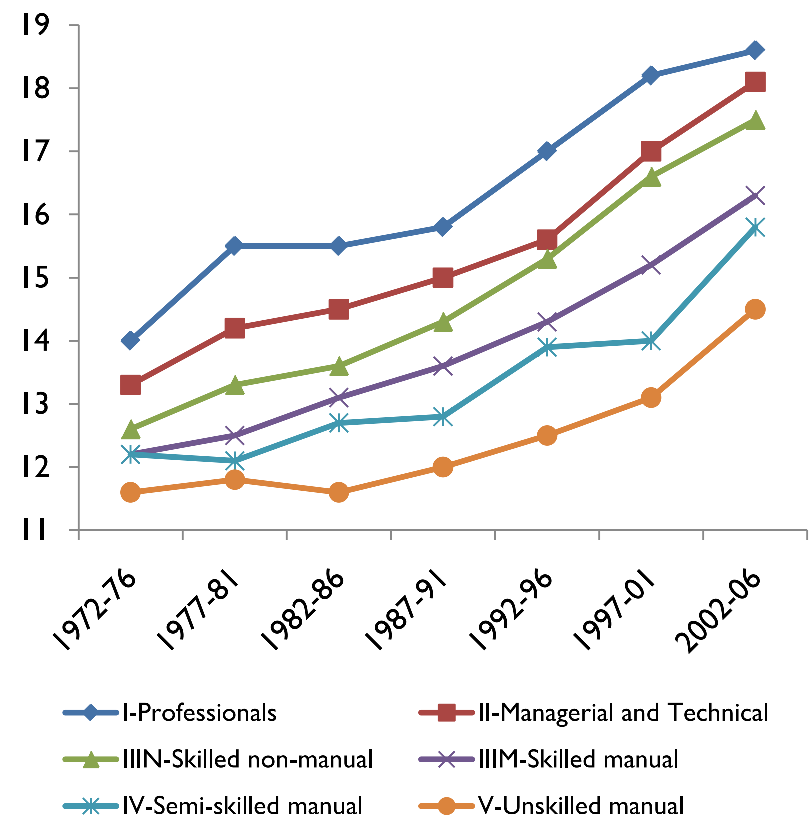


Motivation

SES differences in mortality

- Well-documented relationship between mortality and socioeconomic variables
  - Education
  - Income
  - Occupation
- Important implications on social and financial planning
  - Public policy for tackling inequalities
  - Social security design
  - Annuity reserving and pricing
  - Longevity risk management

Male life expectancy at age 65 by social class -England and Wales



Source: ONS Longitudinal Study

Objectives

- Develop a model for the simultaneous modelling of mortality in a set of socioeconomic subgroups of a population that allows:
  - The quantification of socioeconomic mortality differentials
  - The projection of their possible future evolution
- Evaluate the impact of SES differentials on life expectancy and annuity rates
- Understand the recent trends in socio-economic mortality inequalities in the English population

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Literature review

- Modelling mortality in a single population
  - Lee-Carter model (Lee and Carter, 1992)
  - Cohort effects (Renshaw and Haberman, 2006)
- Modelling mortality in multiple populations
  - Lee-Carter extension for the coherent forecasting of mortality in multiple populations (Li and Lee, 2005)
- Modelling mortality in small populations
  - Modelling a small population relative to a larger population (Jarner and Kryger, 2012)

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Modelling mortality differentials

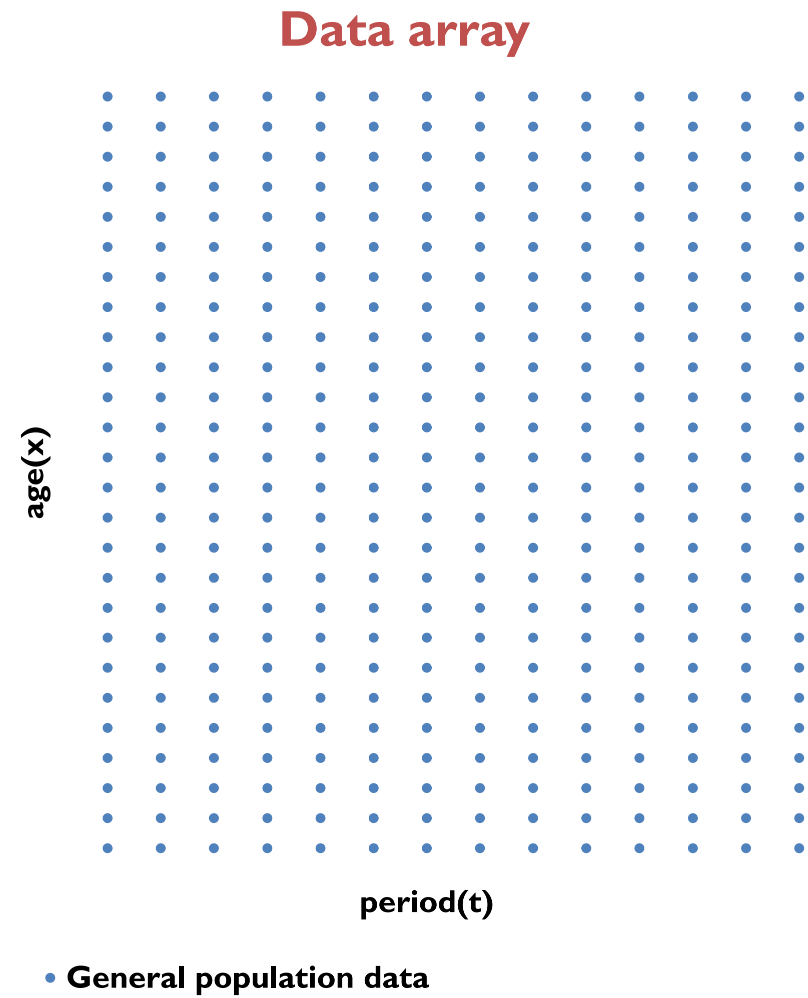
General idea

- Relative modelling approach whereby subpopulation mortality is modelled relative to the mortality of a reference population
- The reference population could be
  - National mortality
  - Sum of the subpopulations
- The rationale of this approach is based on data availability

Modelling mortality differentials

Data

- National mortality data
  - Long periods of time
  - Available at individual ages
  - Full age range

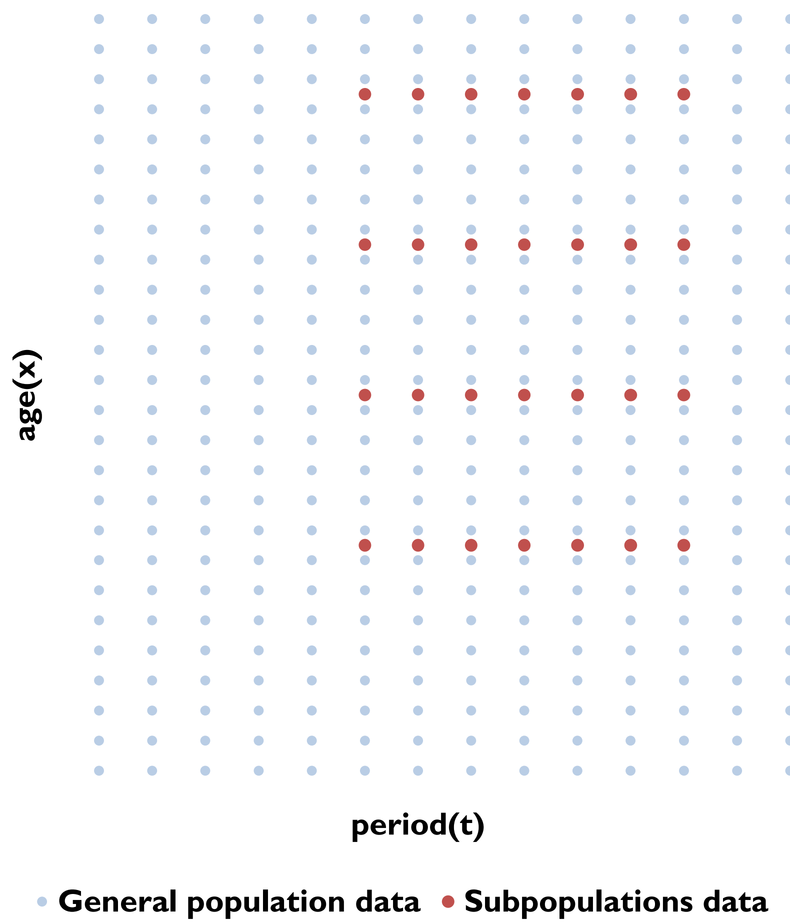


Modelling mortality differentials

Data

- National mortality data
  - Long periods of time
  - Available at individual ages
  - Full age range
- Mortality data disaggregated by SES
  - Short periods of time
  - Available in age-grouped format
  - Limited age range
- Advantages of a relative modelling approach
  - More precise estimation of the long-run mortality trend
  - Coherency with the national mortality trend
  - Enable consideration of cohort effects

Data array



Modelling mortality differentials

Data

- Subpopulation data

$$({}_ne_{xtg}, {}_nd_{xtg}) \qquad \begin{aligned} x \in \mathcal{X} &:= \{x_1, \ldots, x_k\} \\ t \in \mathcal{T} &:= \{t_1, \ldots, t_n\} \\ g \in \mathcal{G} &:= \{g_1, \ldots, g_m\} \end{aligned}$$

- Reference population data

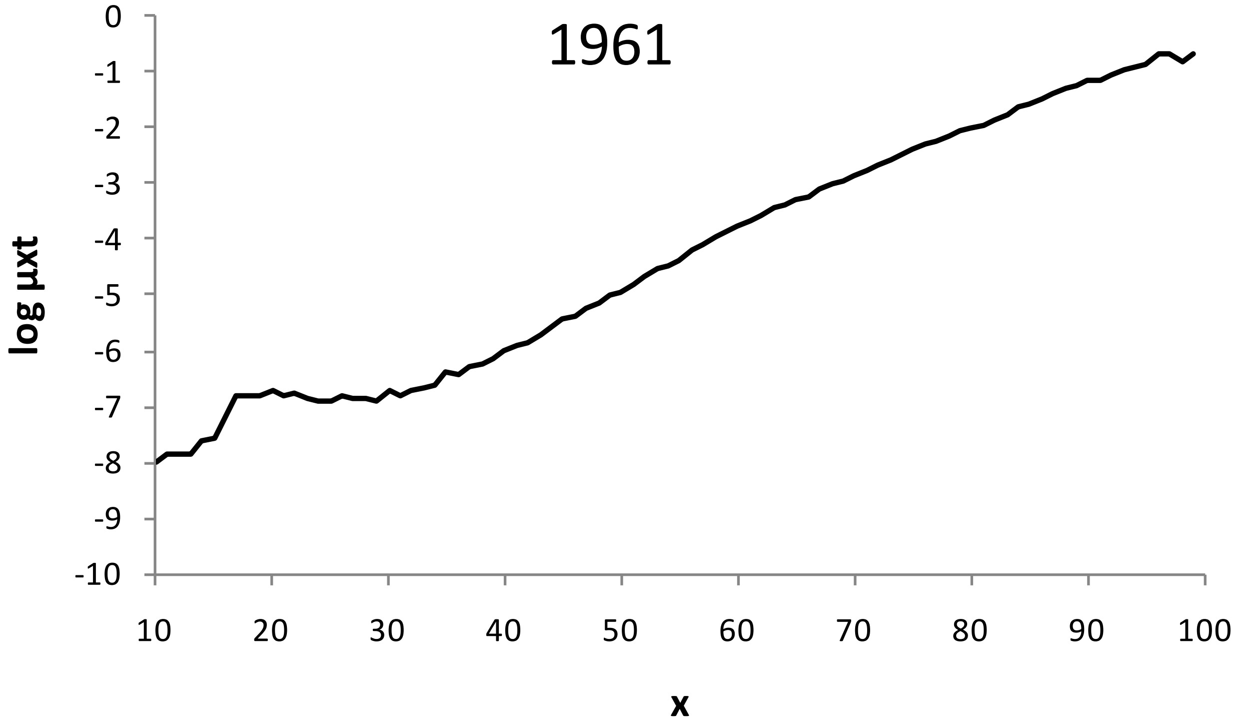
$$(e'_{xt}, d'_{xt}) \qquad \begin{aligned} x \in \mathcal{X}' &:= \{x'_1, \ldots, x'_{k'}\} \\ t \in \mathcal{T}' &:= \{t'_1, \ldots, t'_{n'}\} \end{aligned}$$

- With

$$\begin{aligned} x'_1 &\leq x_1 \\ x'_{k'} &\geq x_k + n \\ t'_1 &\leq t_1 \\ t'_{n'} &\geq t_k \end{aligned}$$

Modelling mortality differentials

Reference population model



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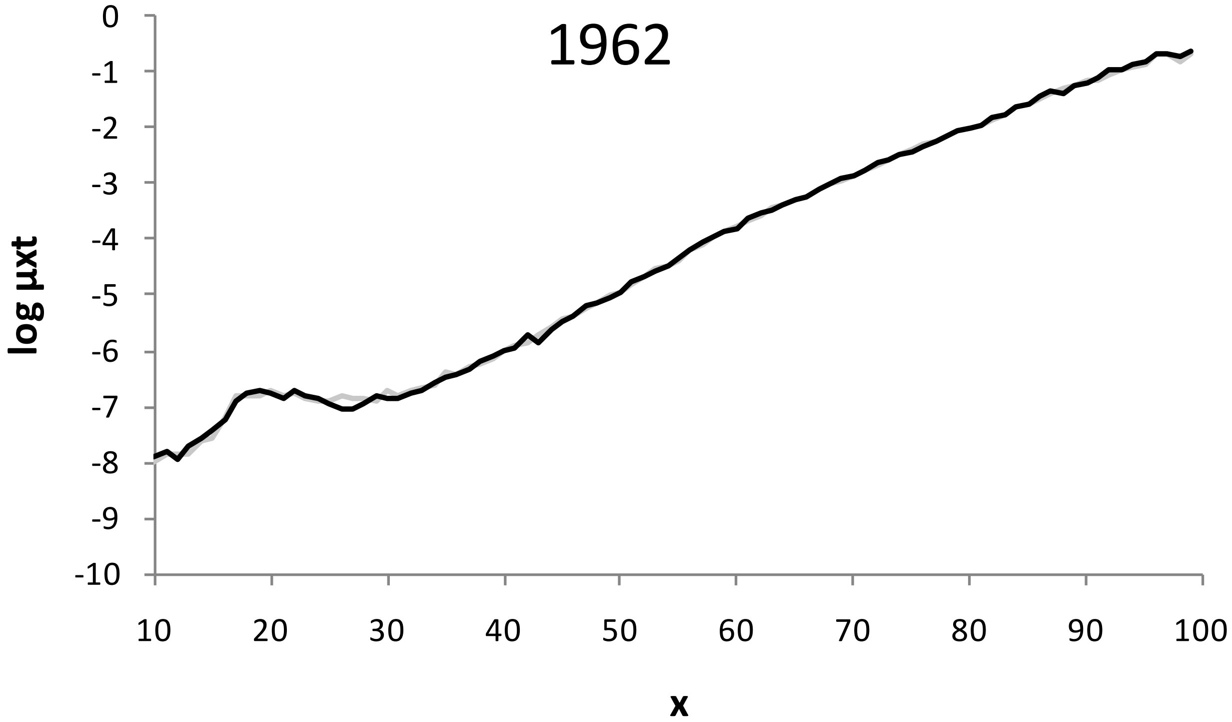

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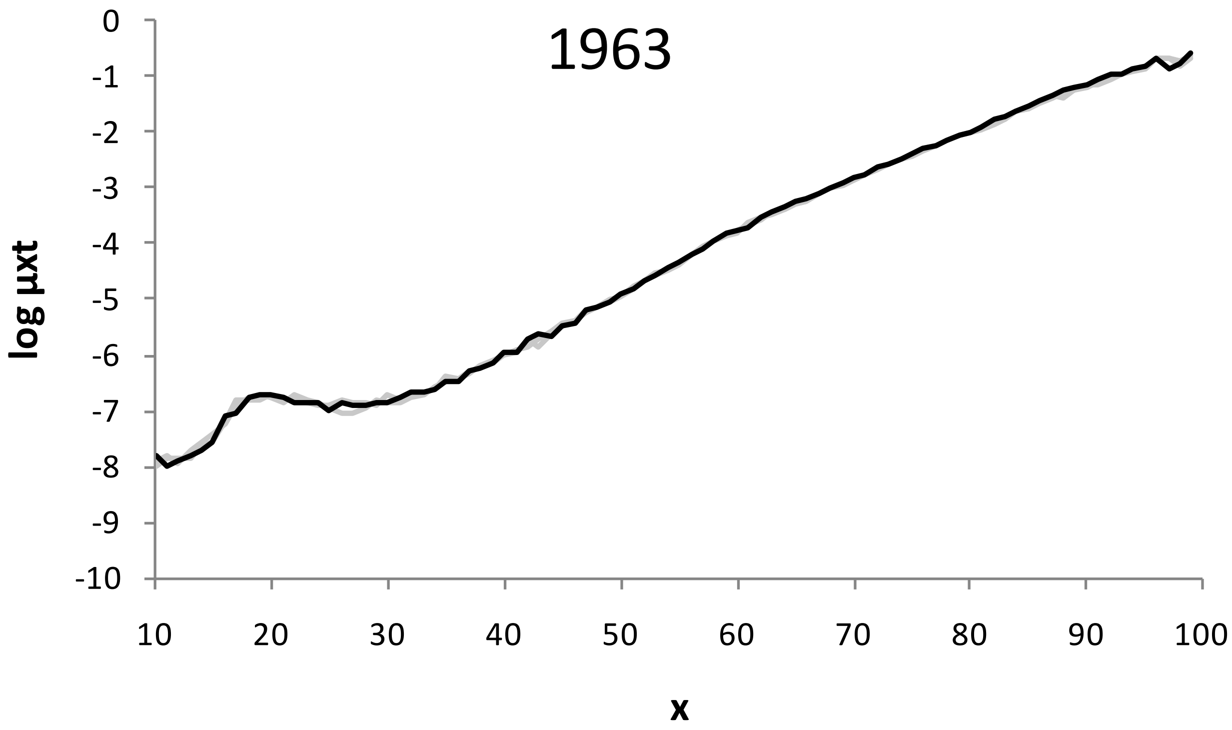

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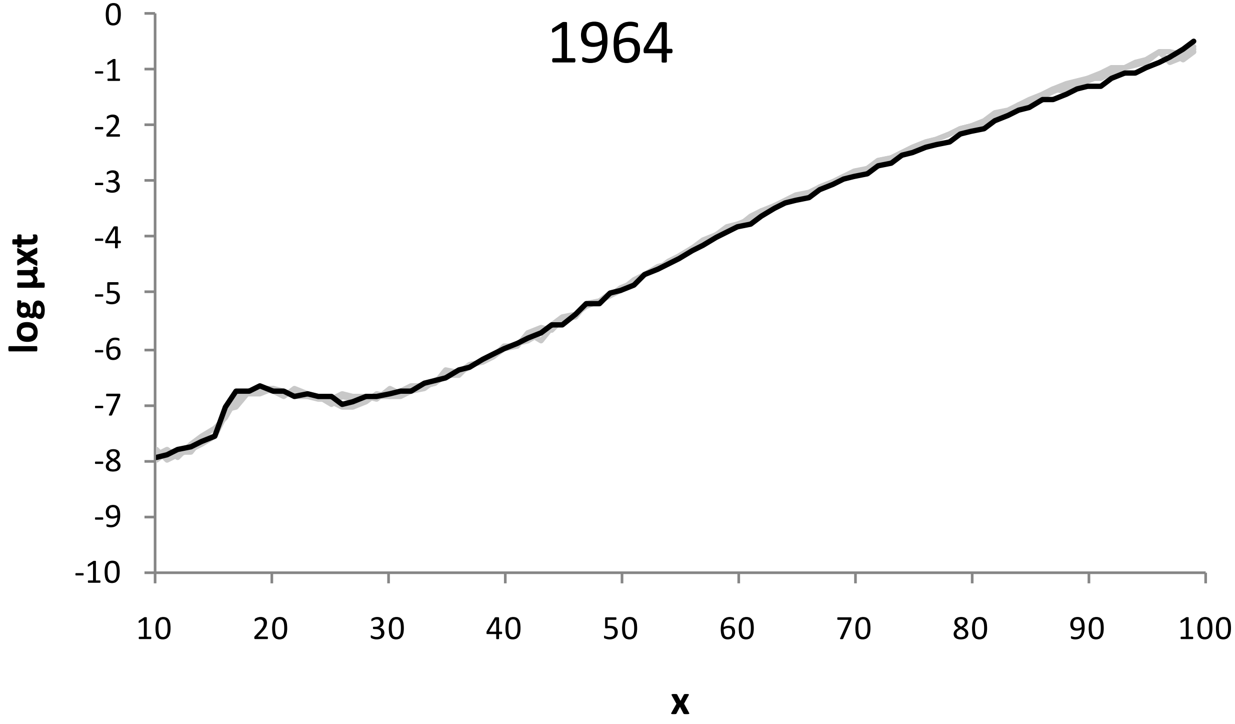

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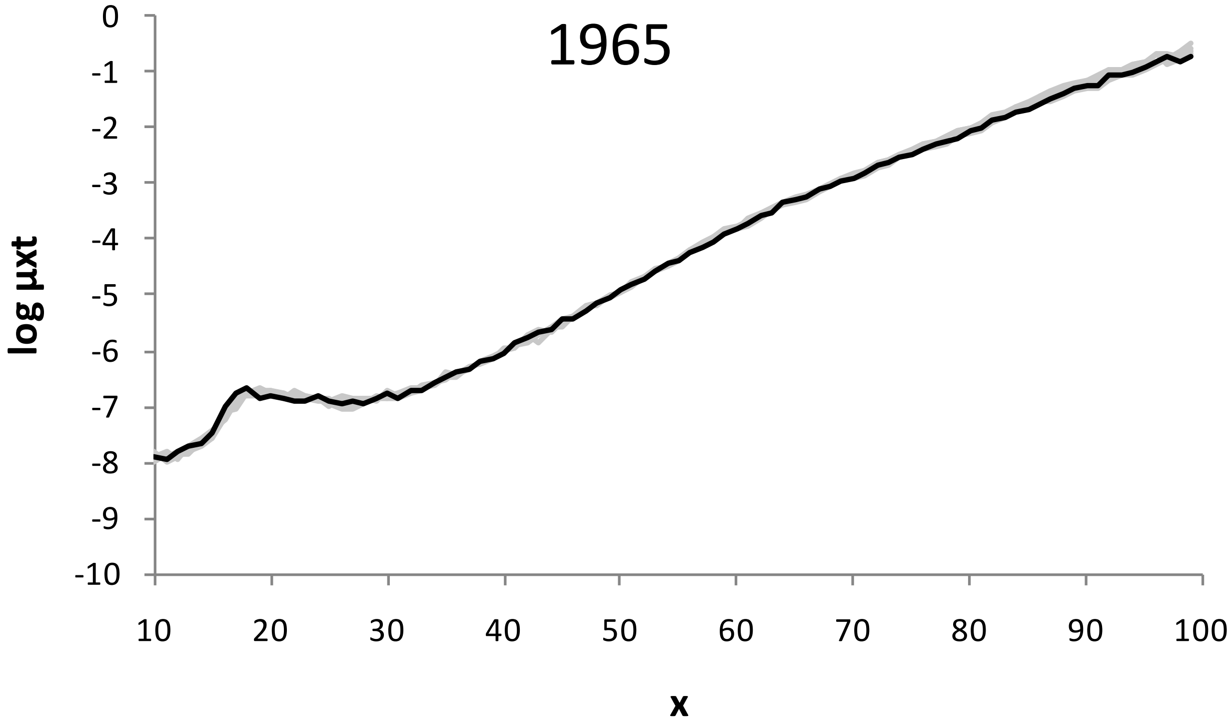

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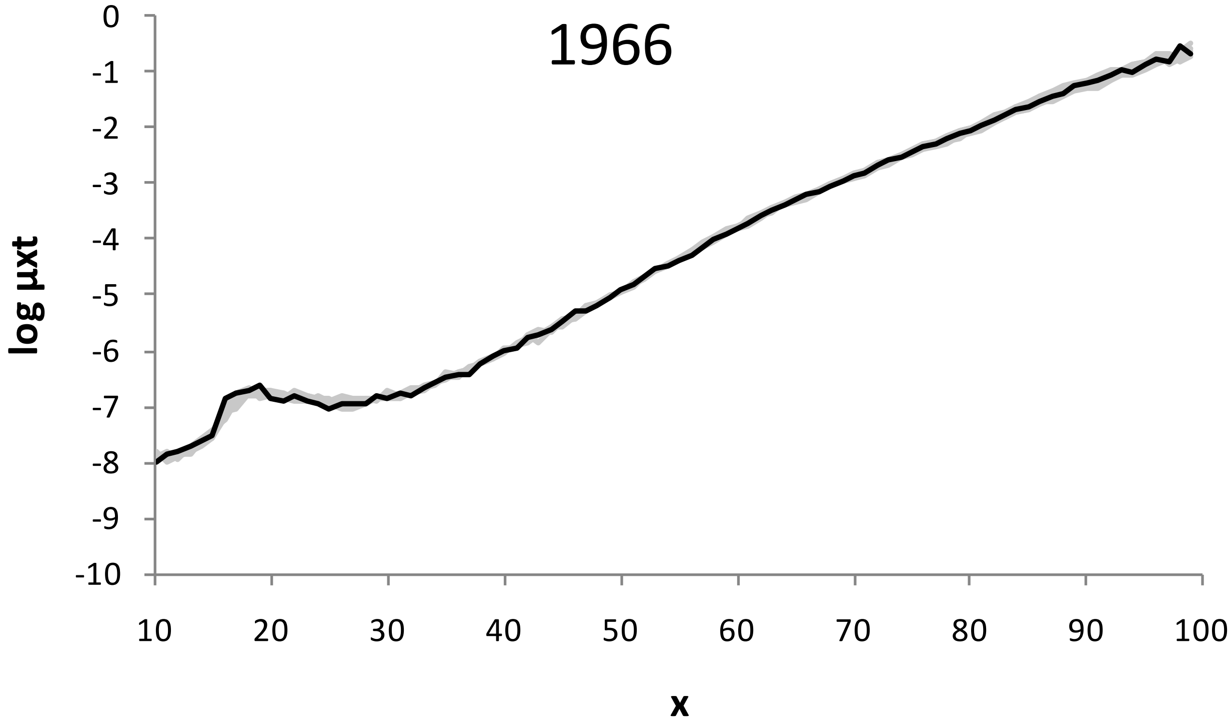

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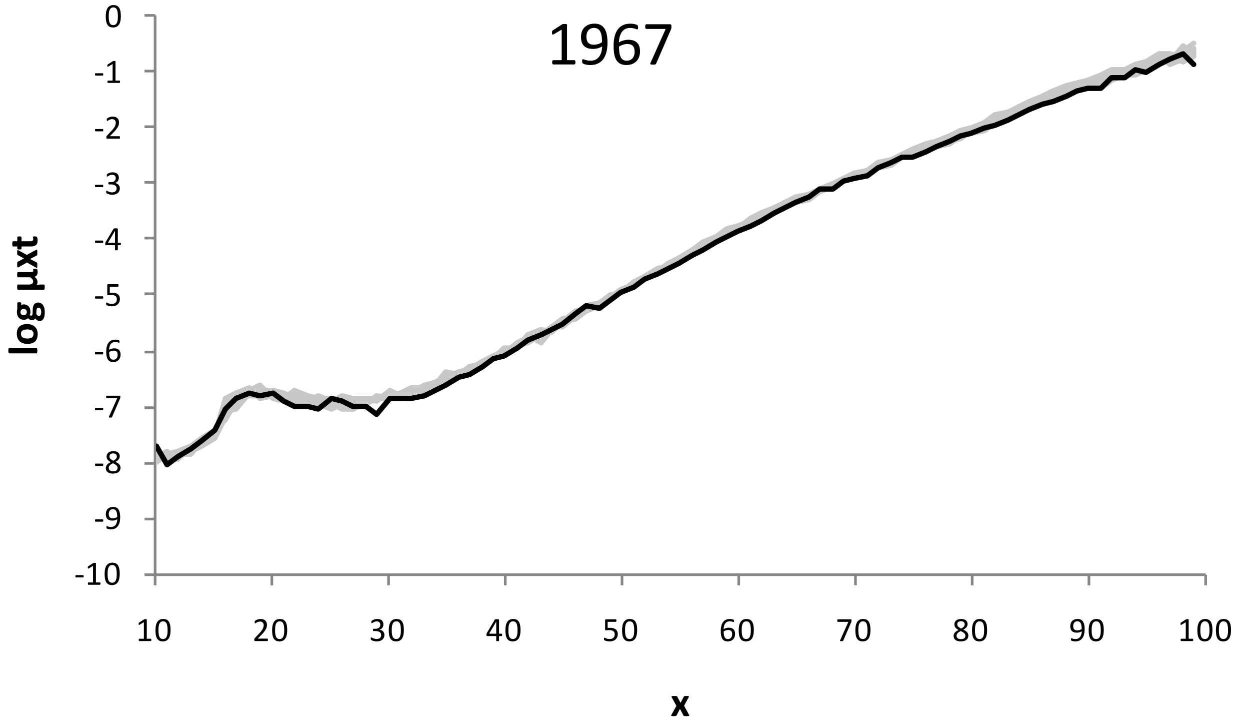

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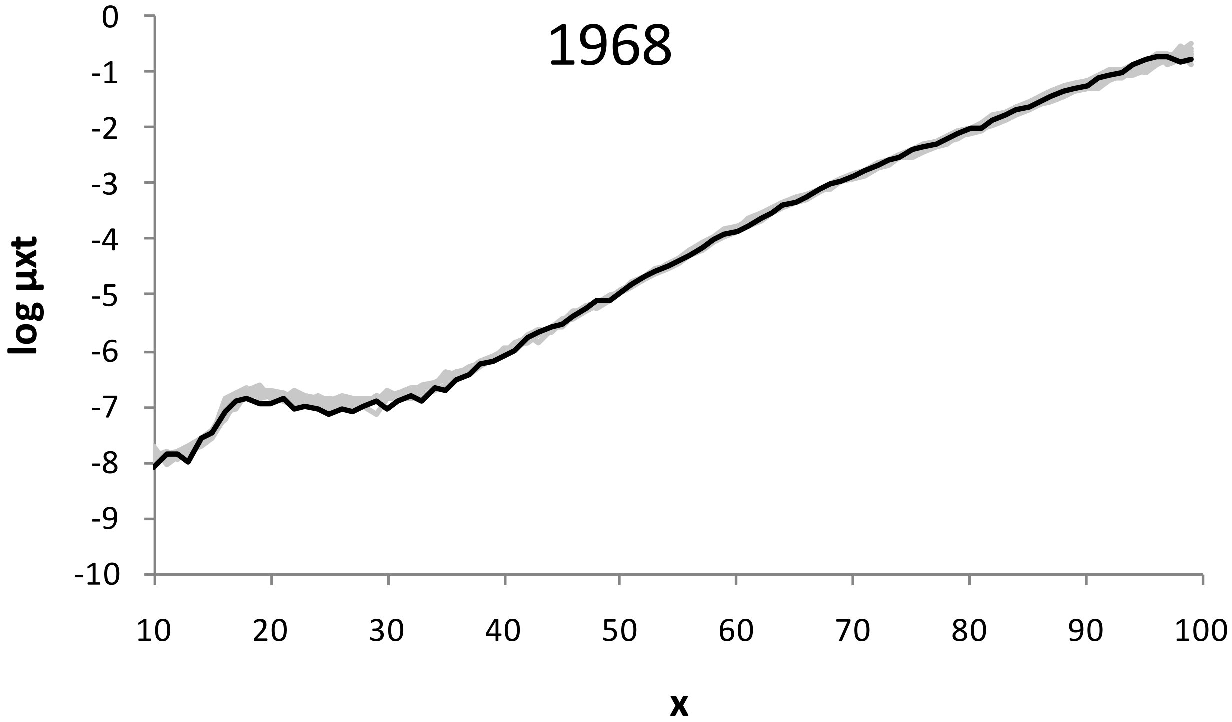

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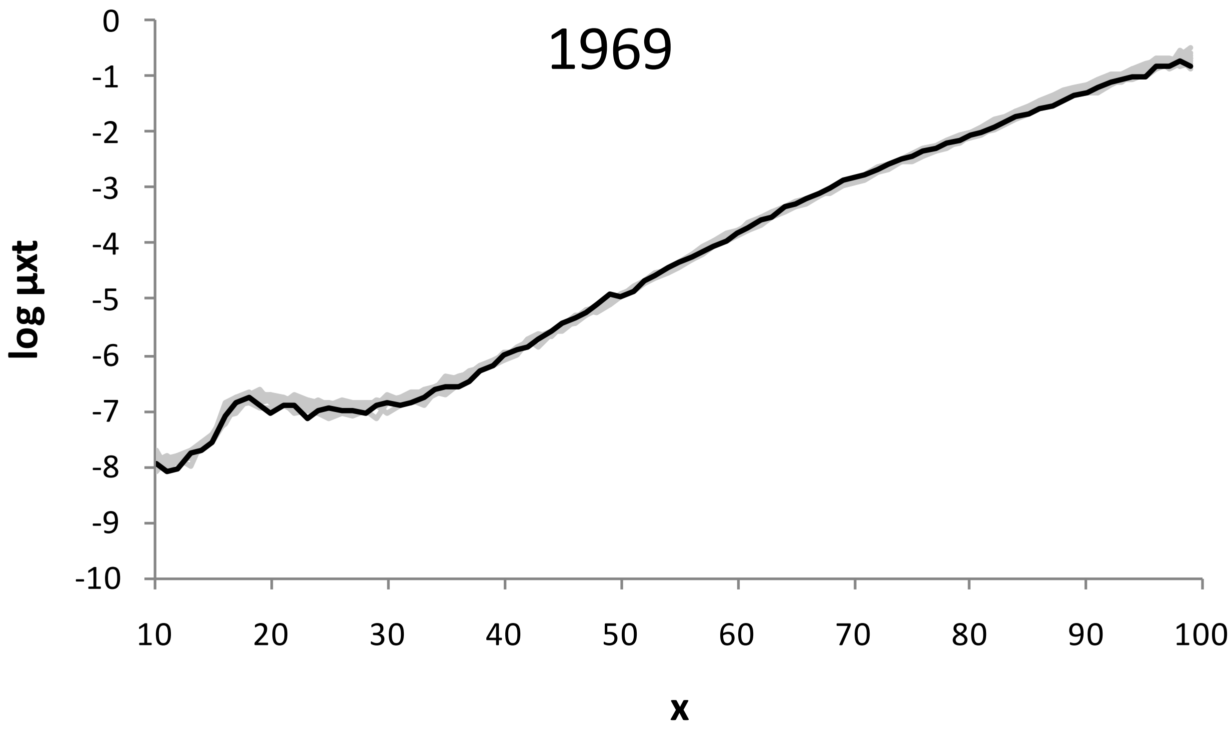

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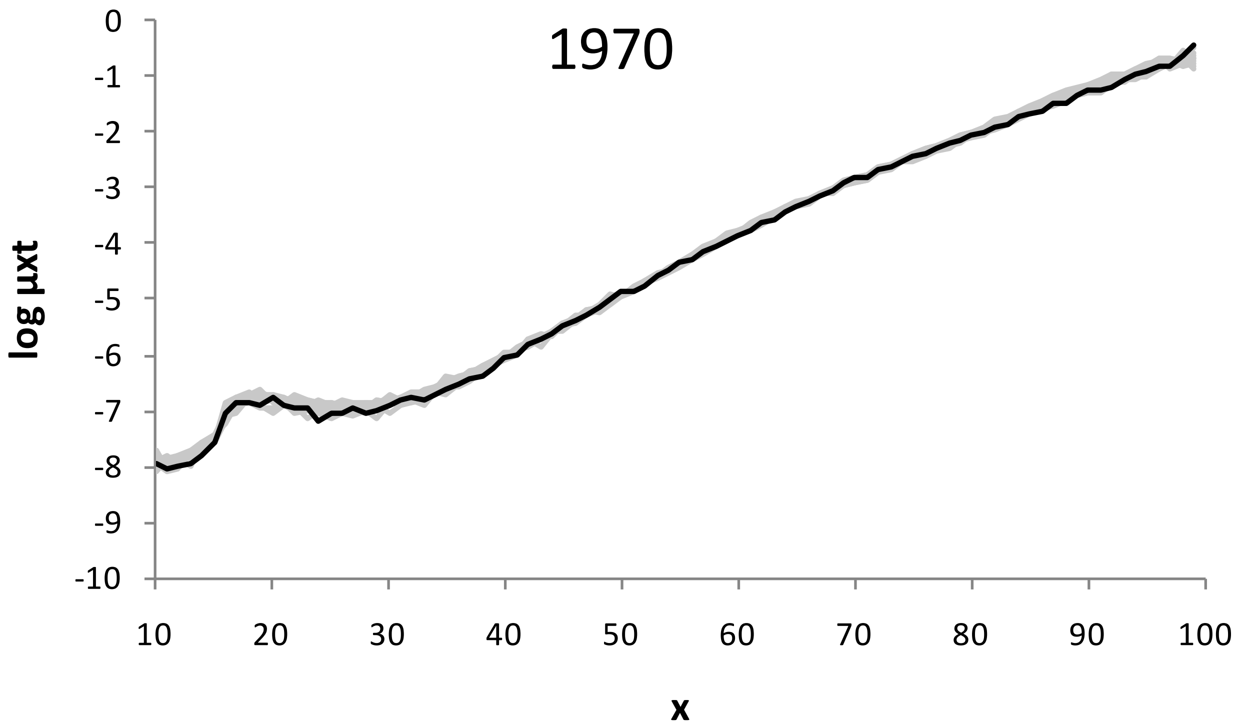

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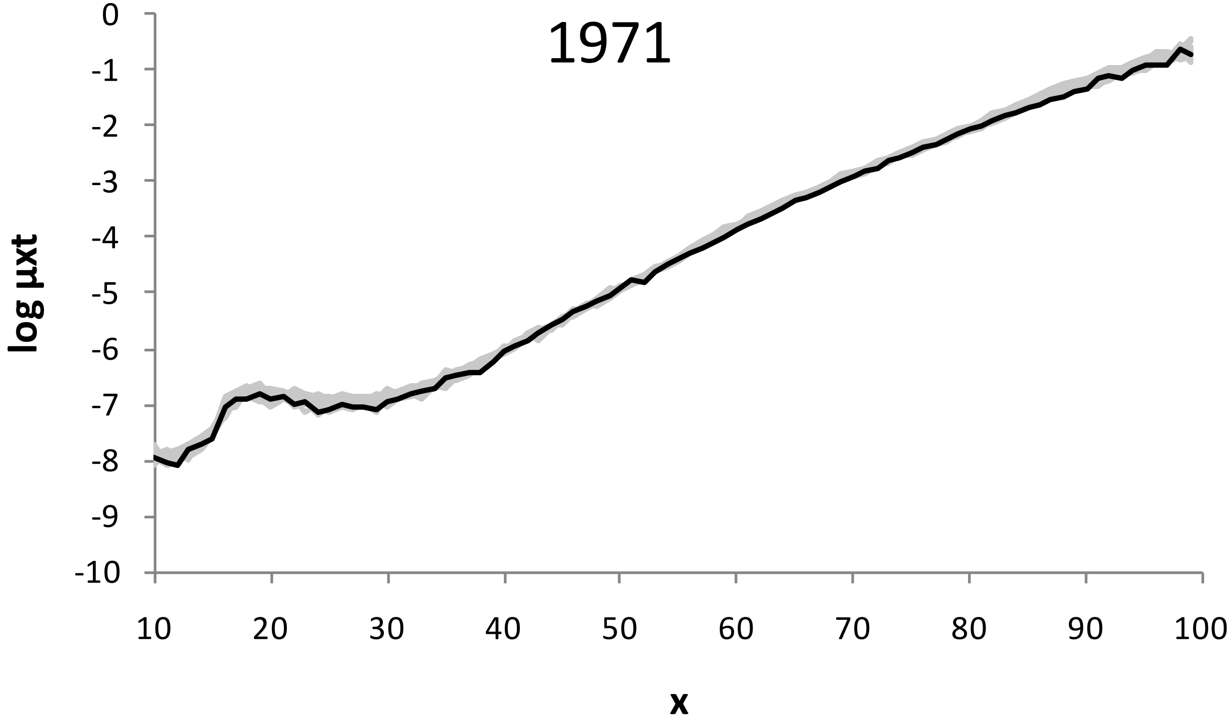

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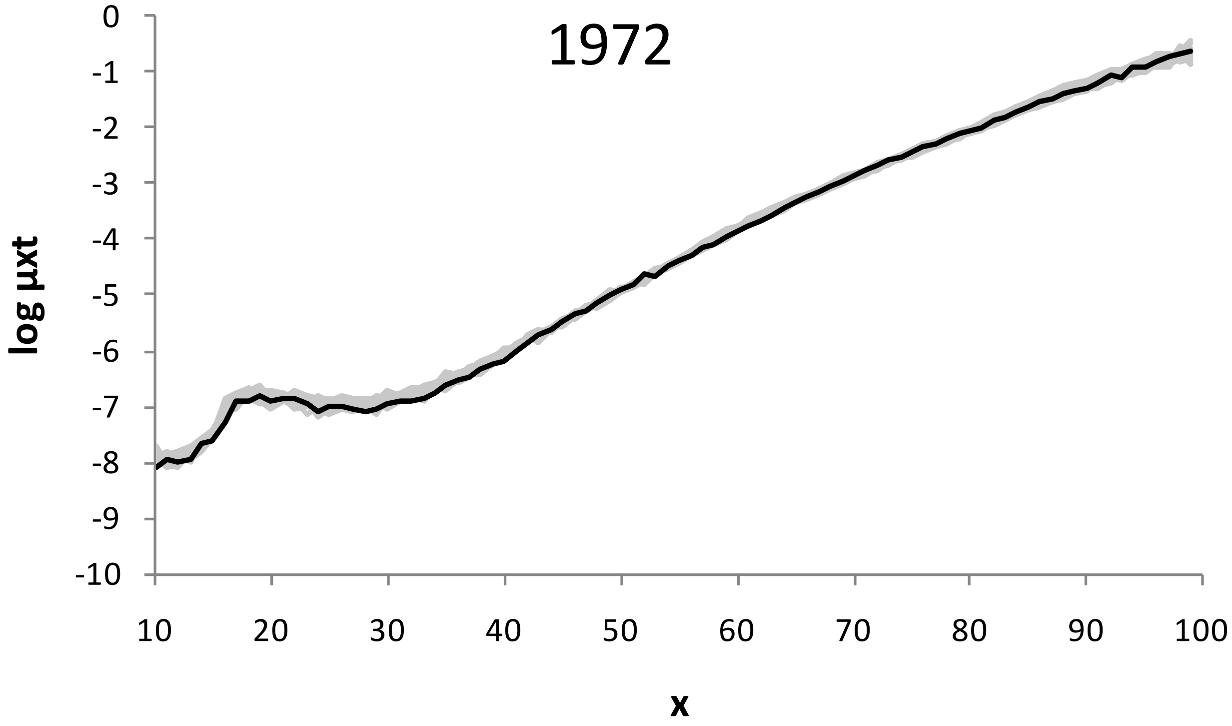

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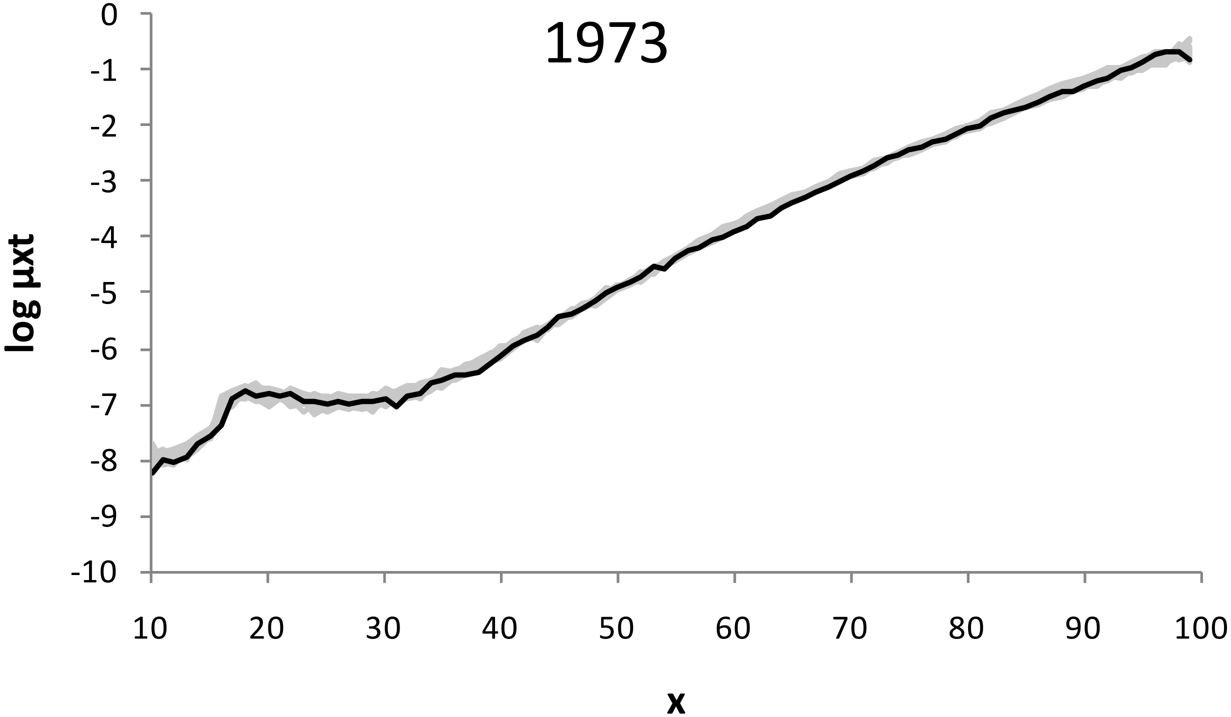

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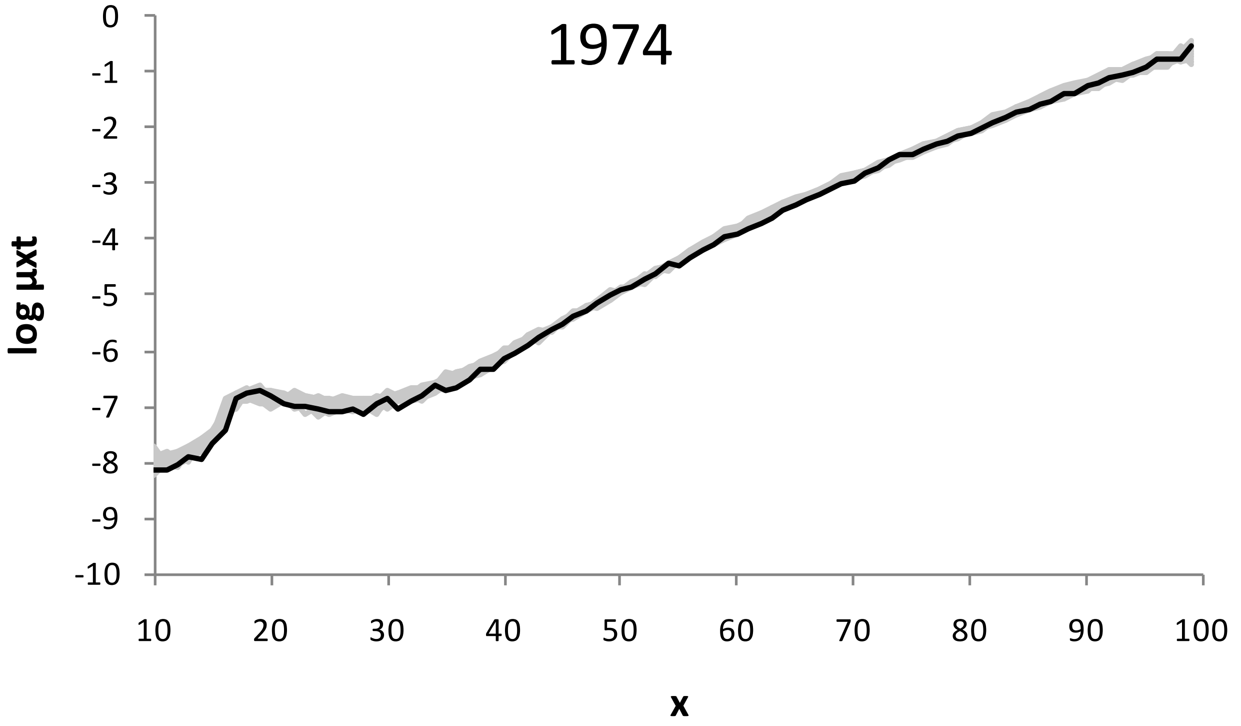

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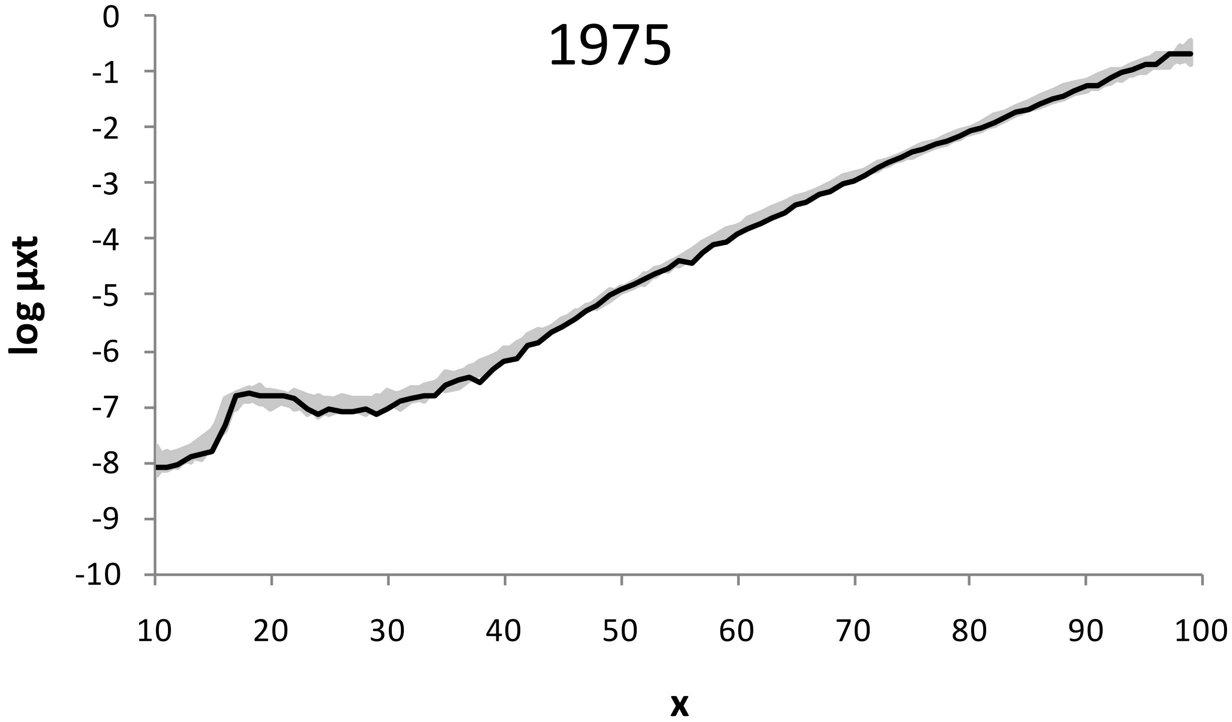

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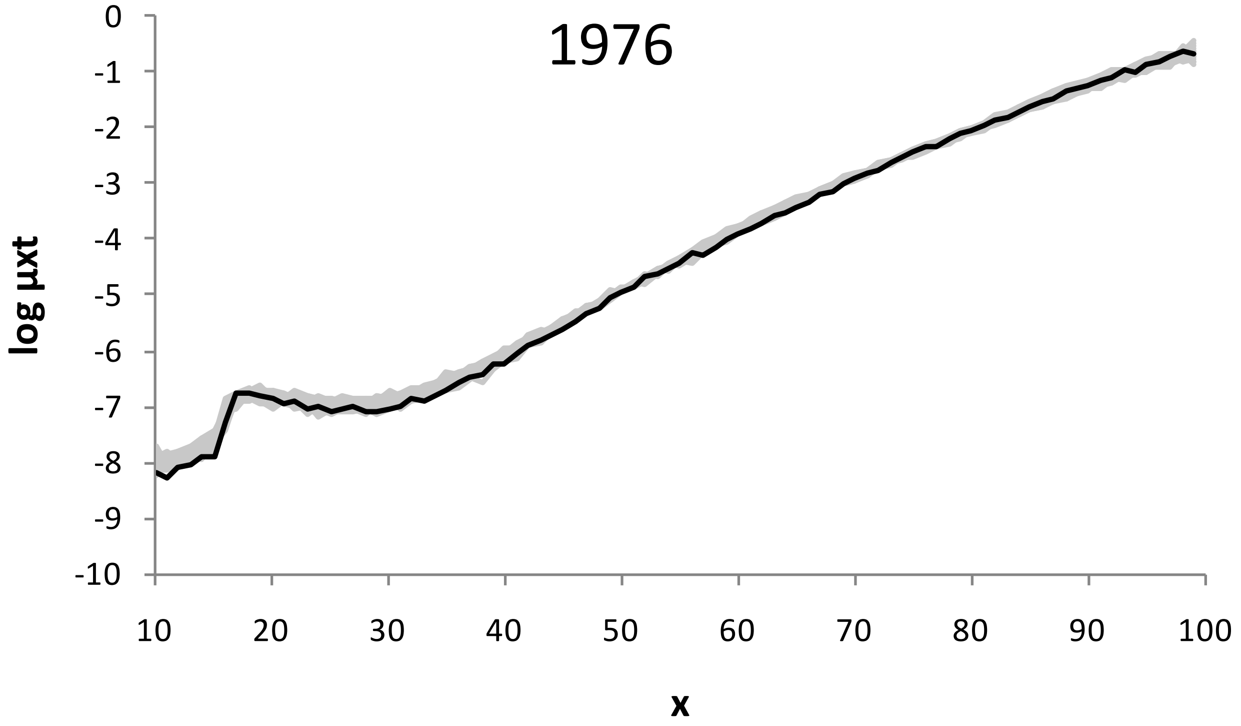

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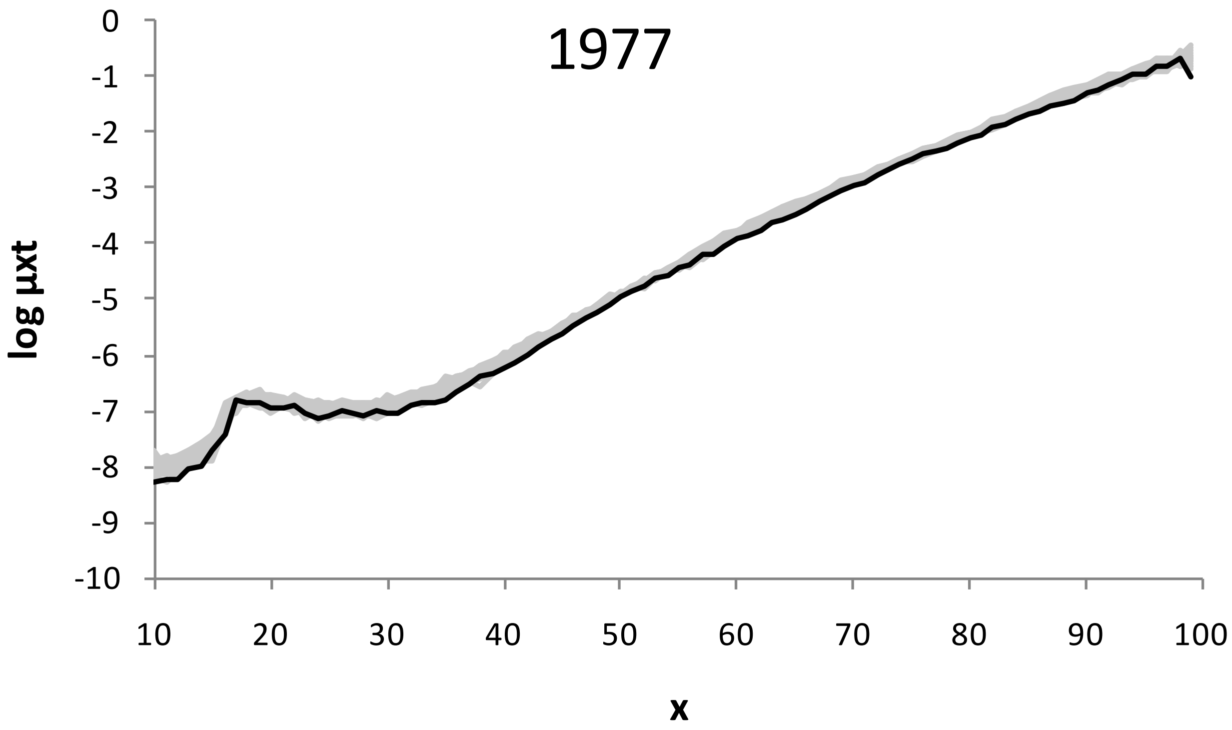

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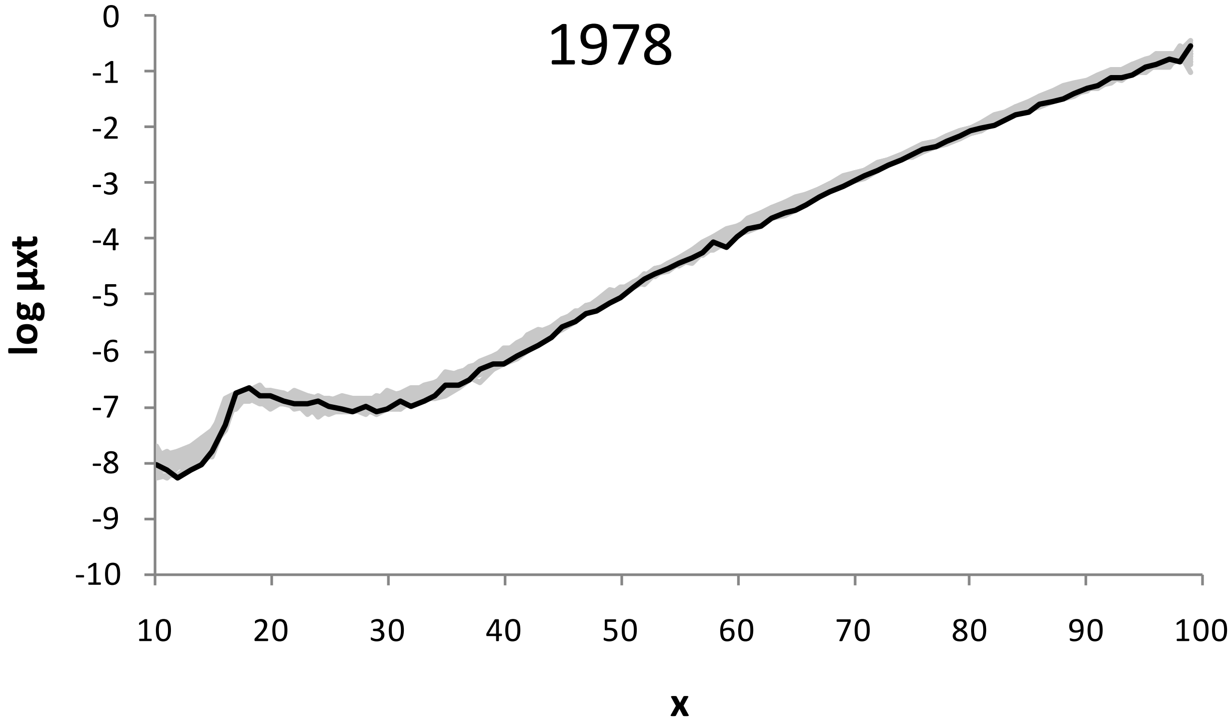

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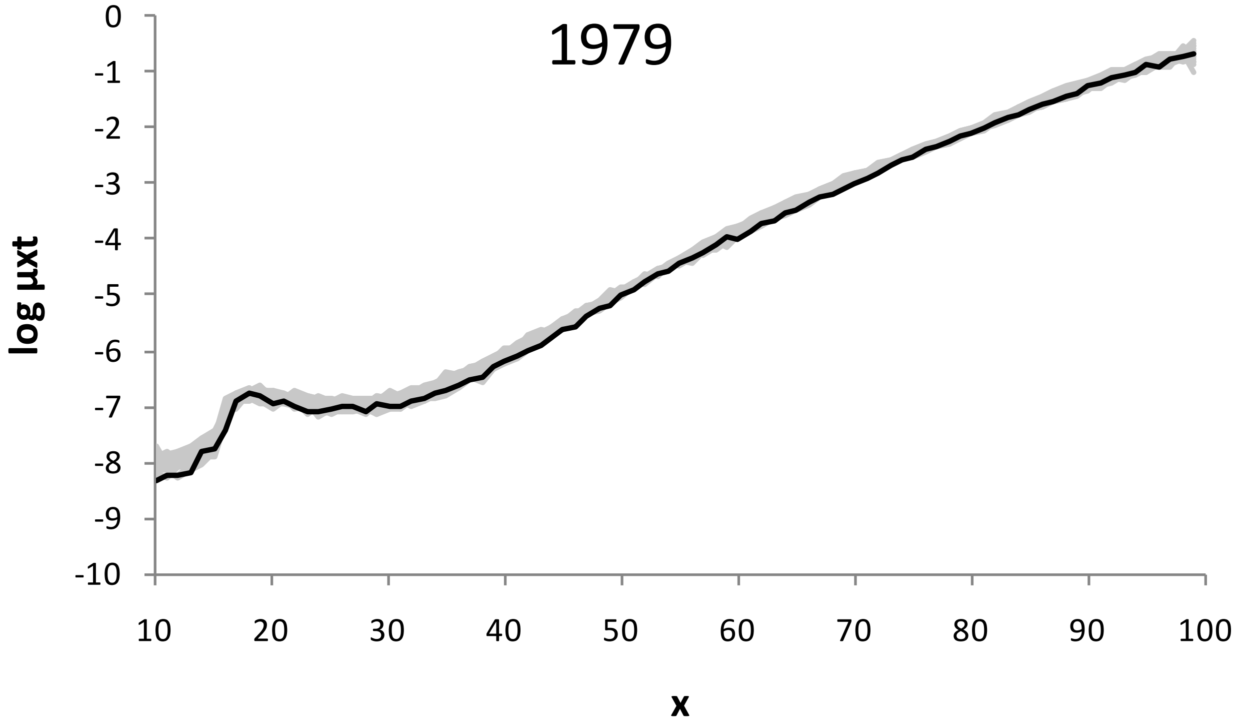

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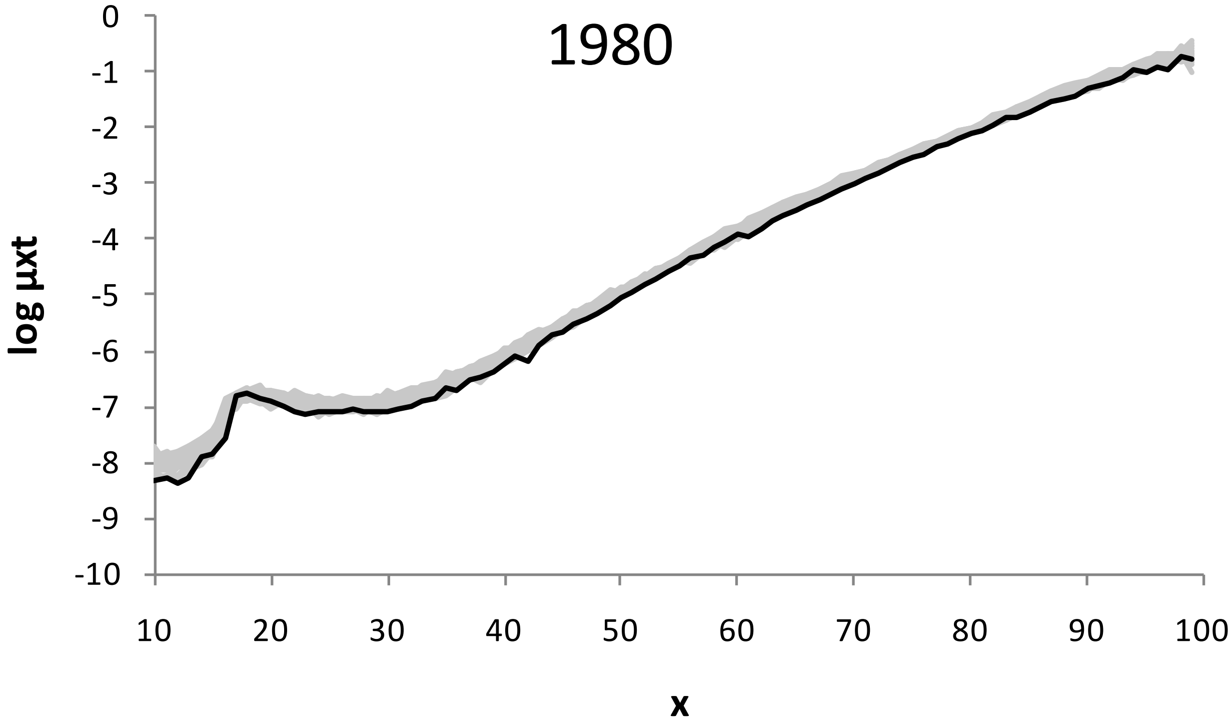

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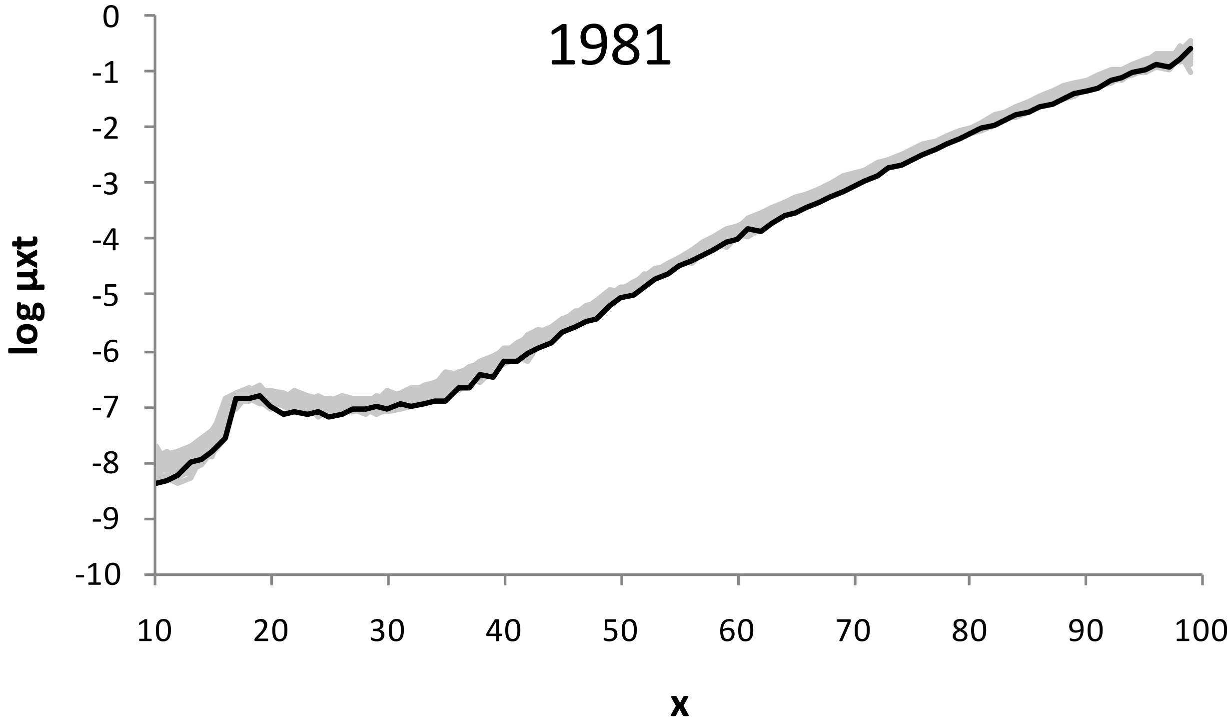

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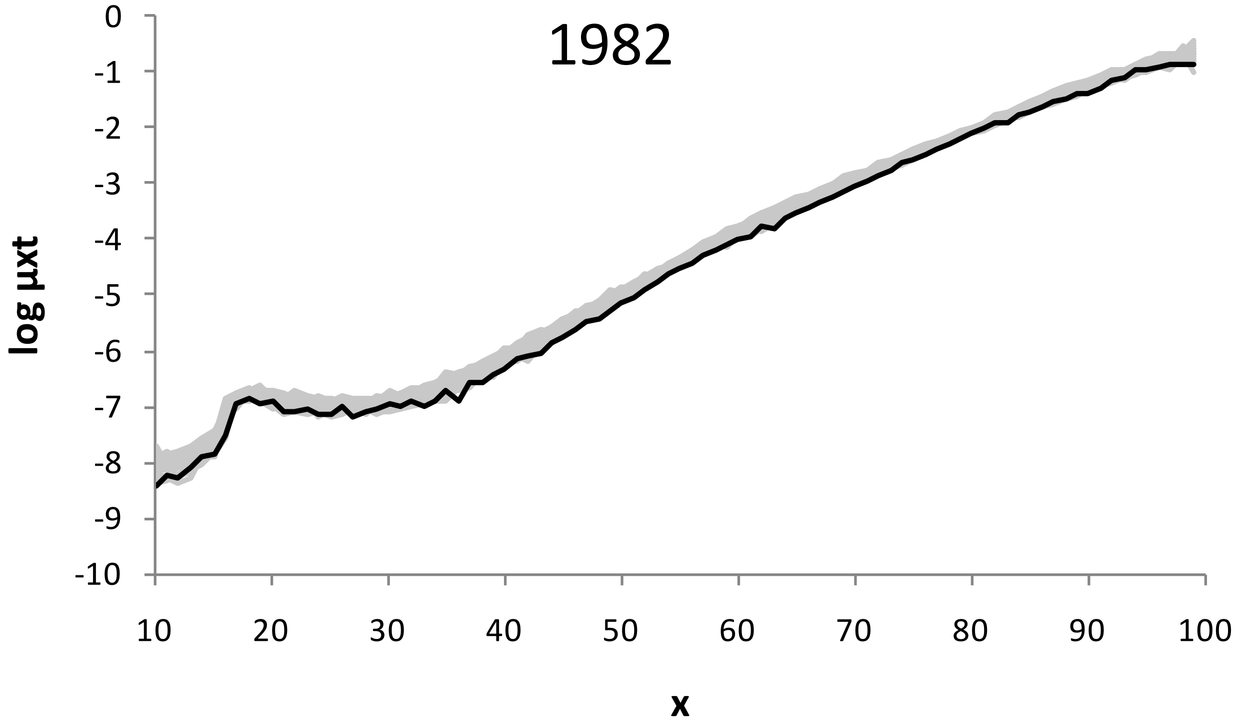

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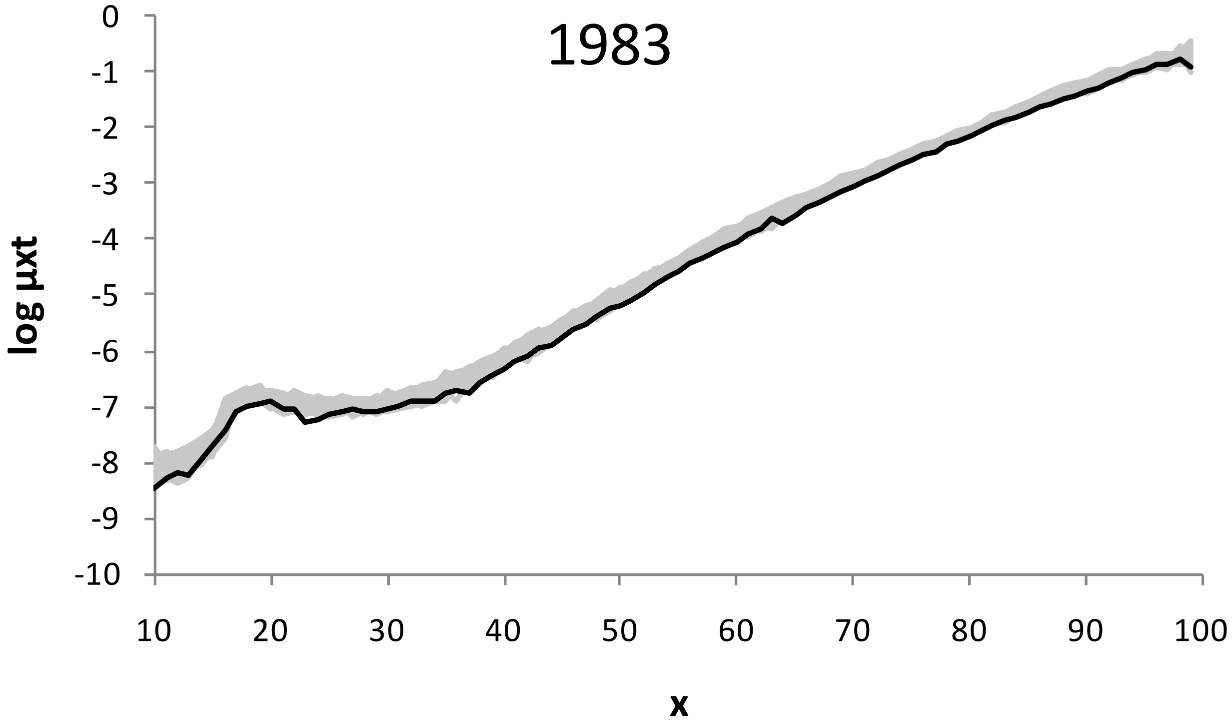

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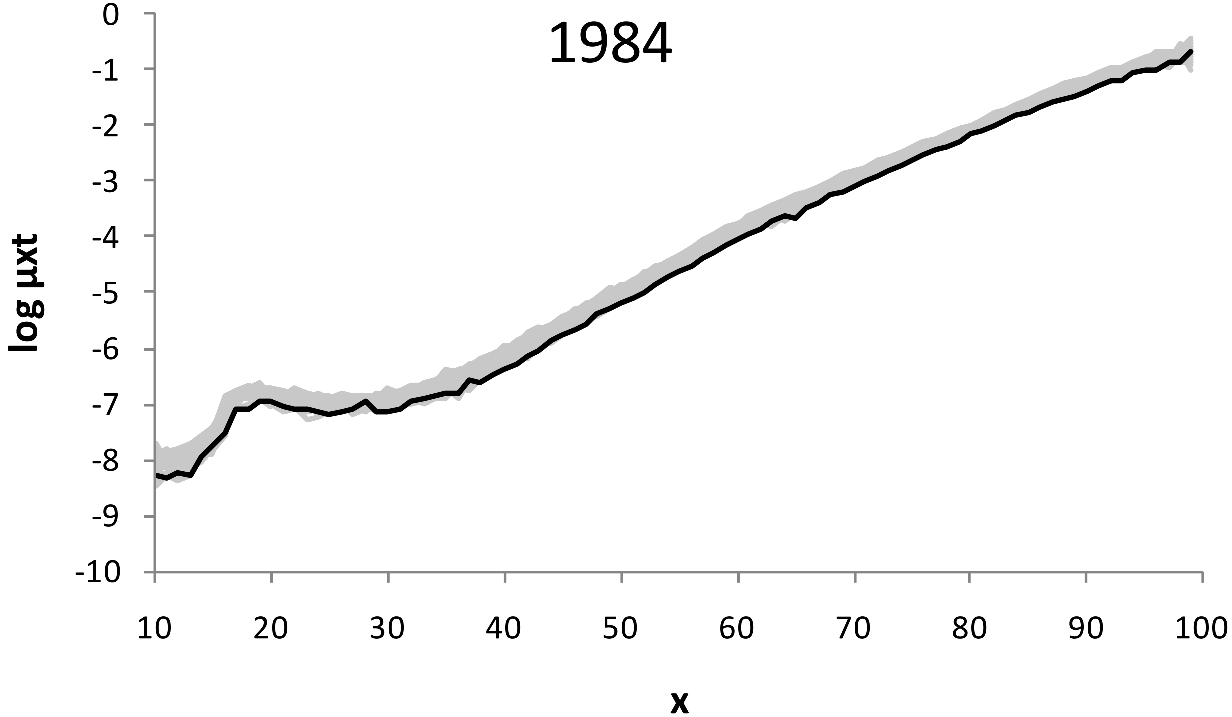

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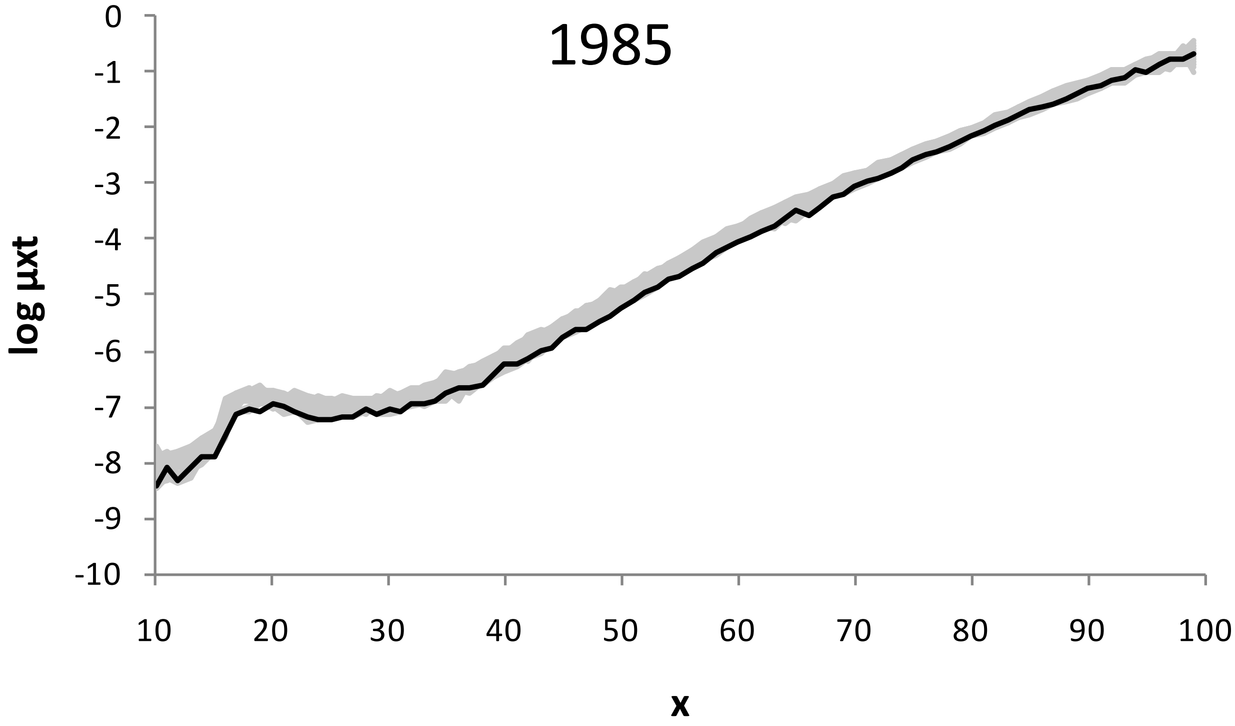

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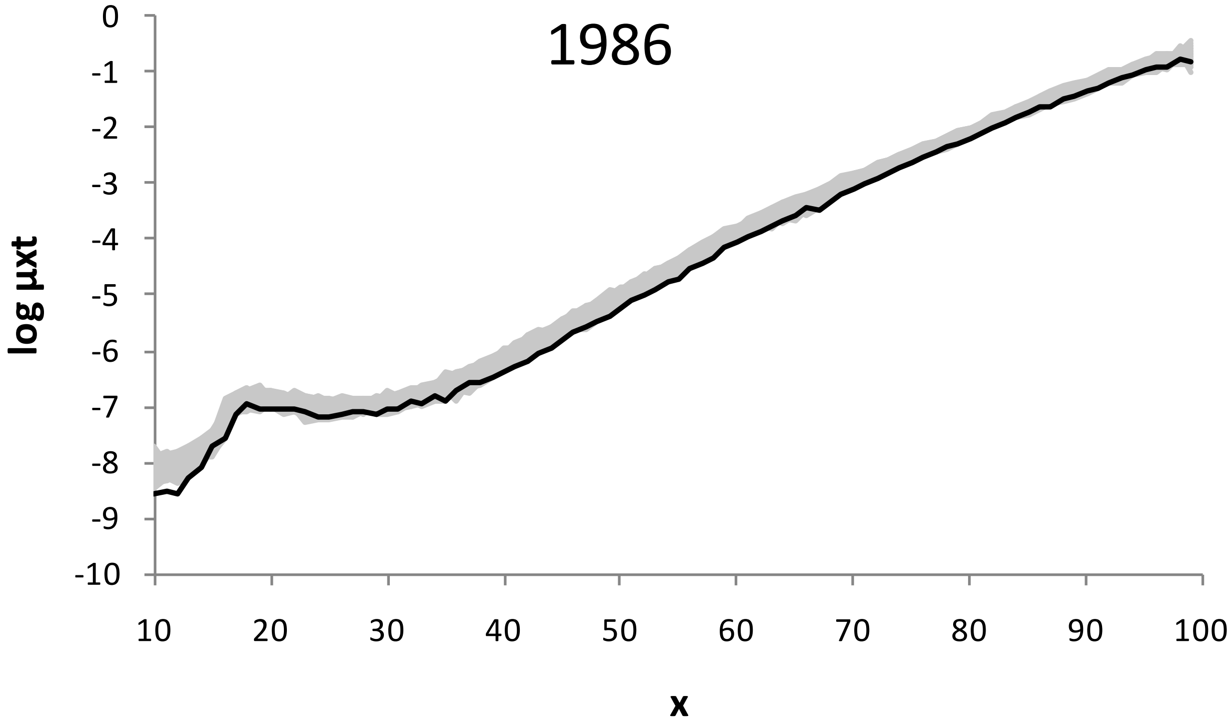

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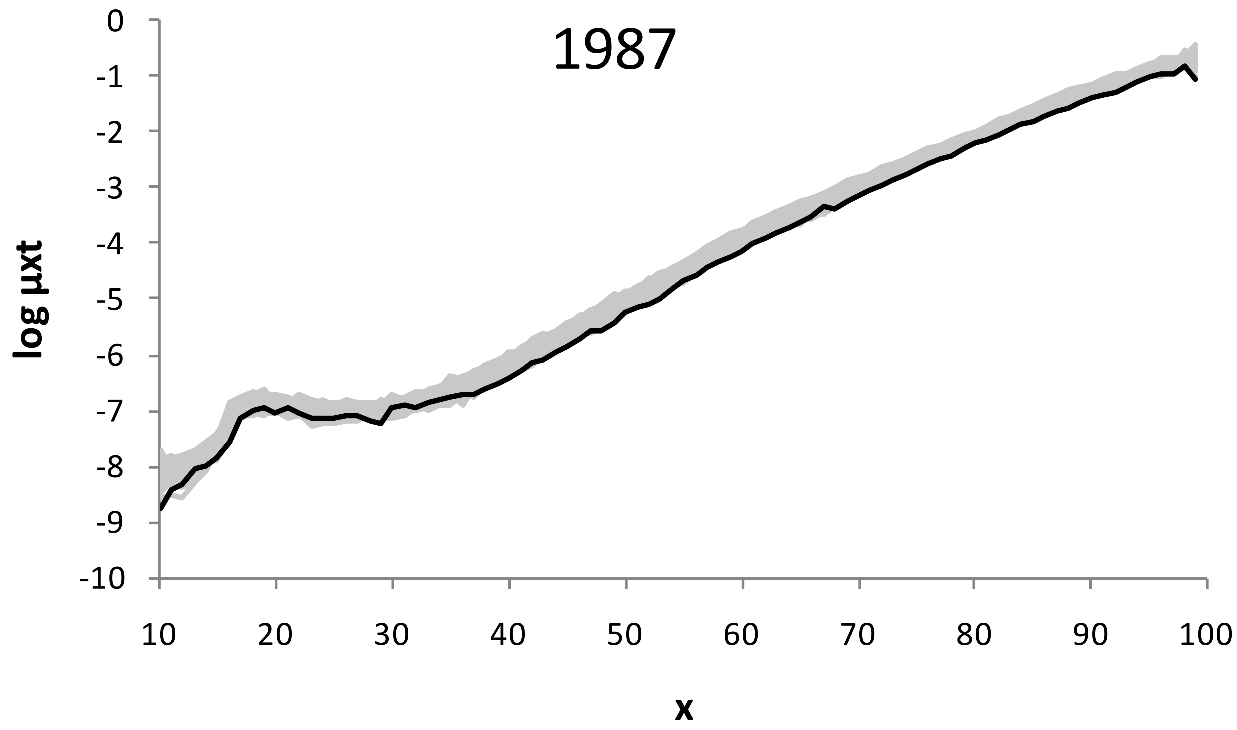

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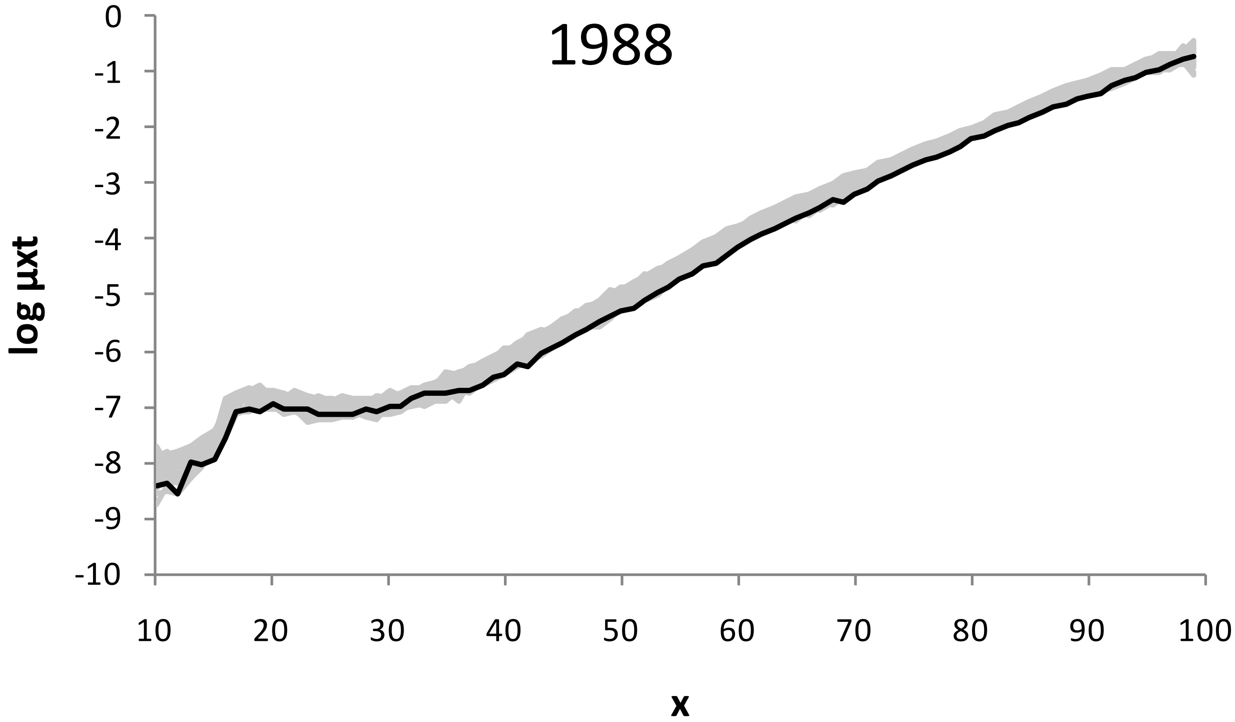

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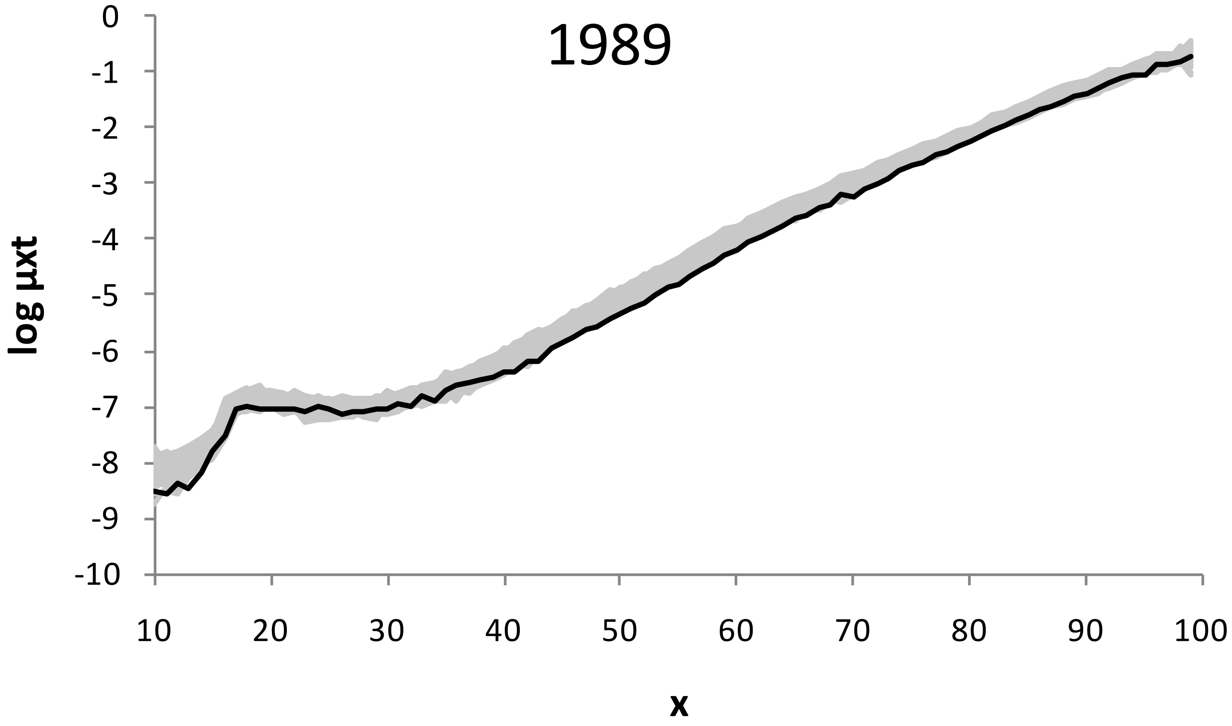

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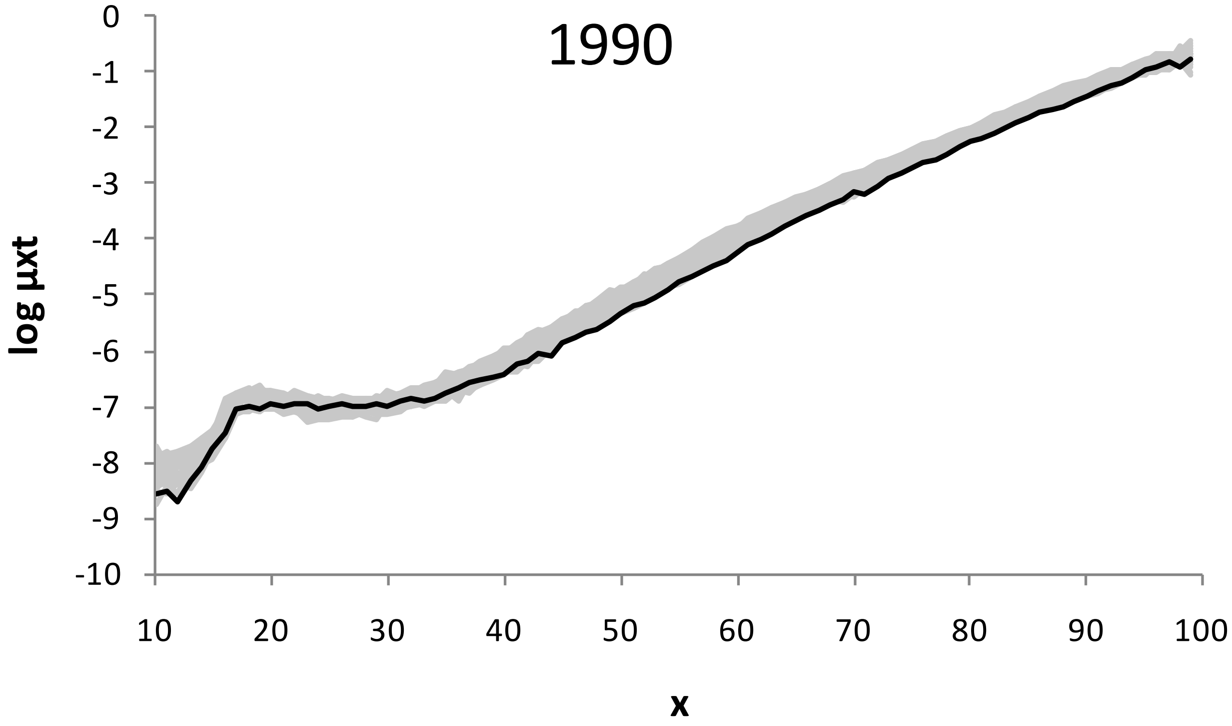

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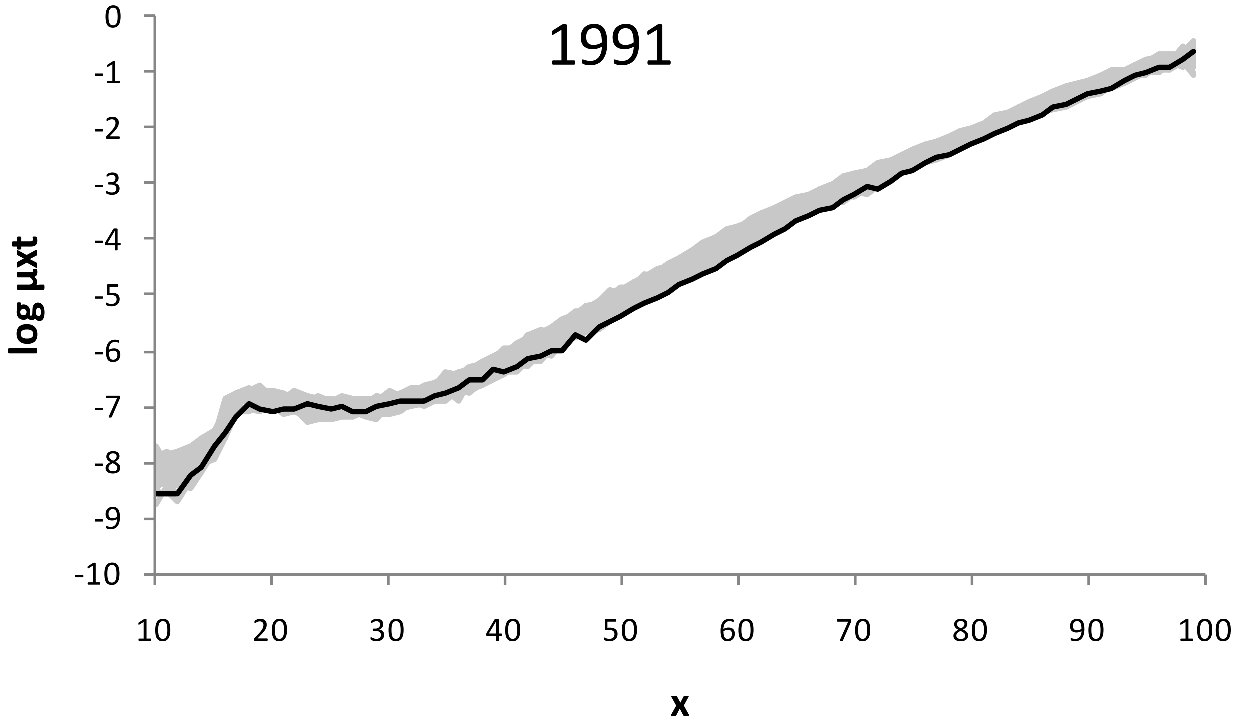

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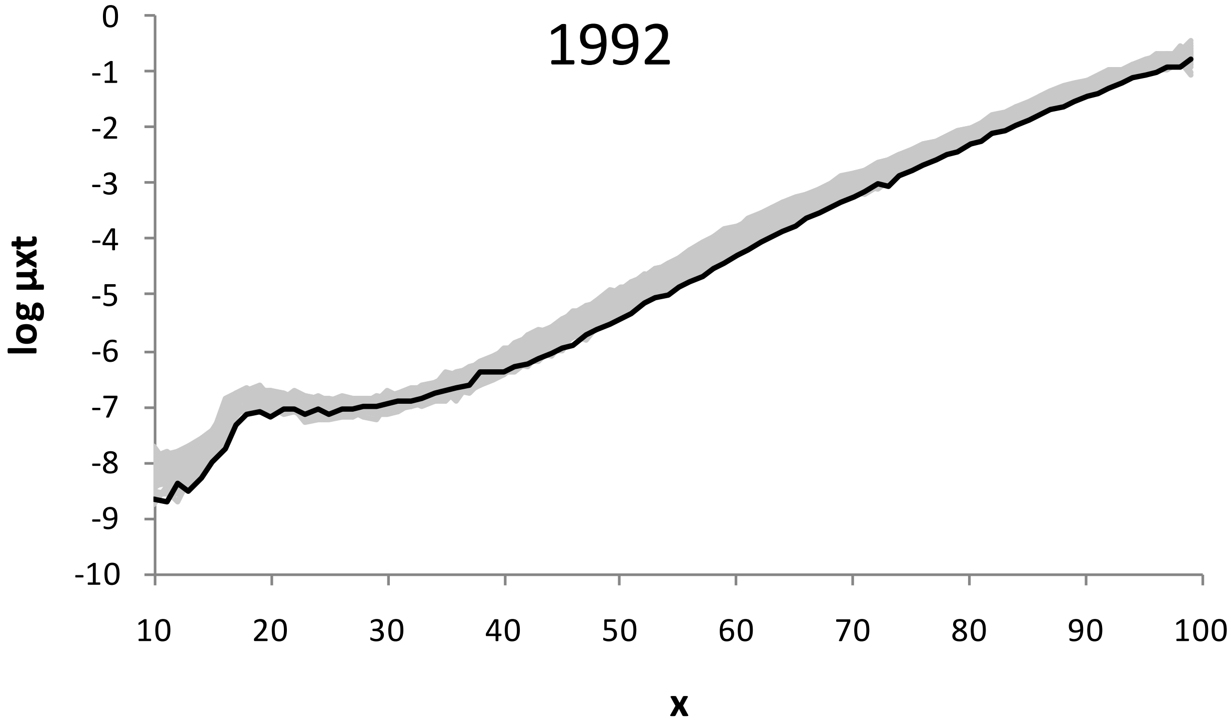

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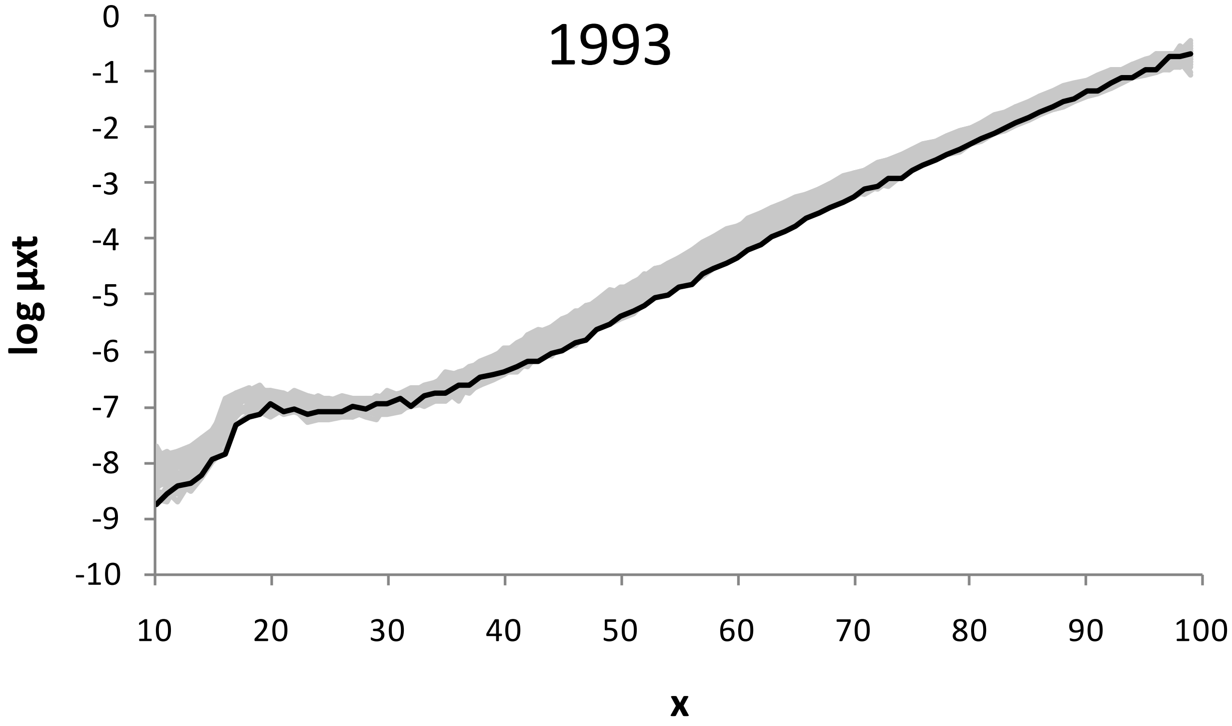

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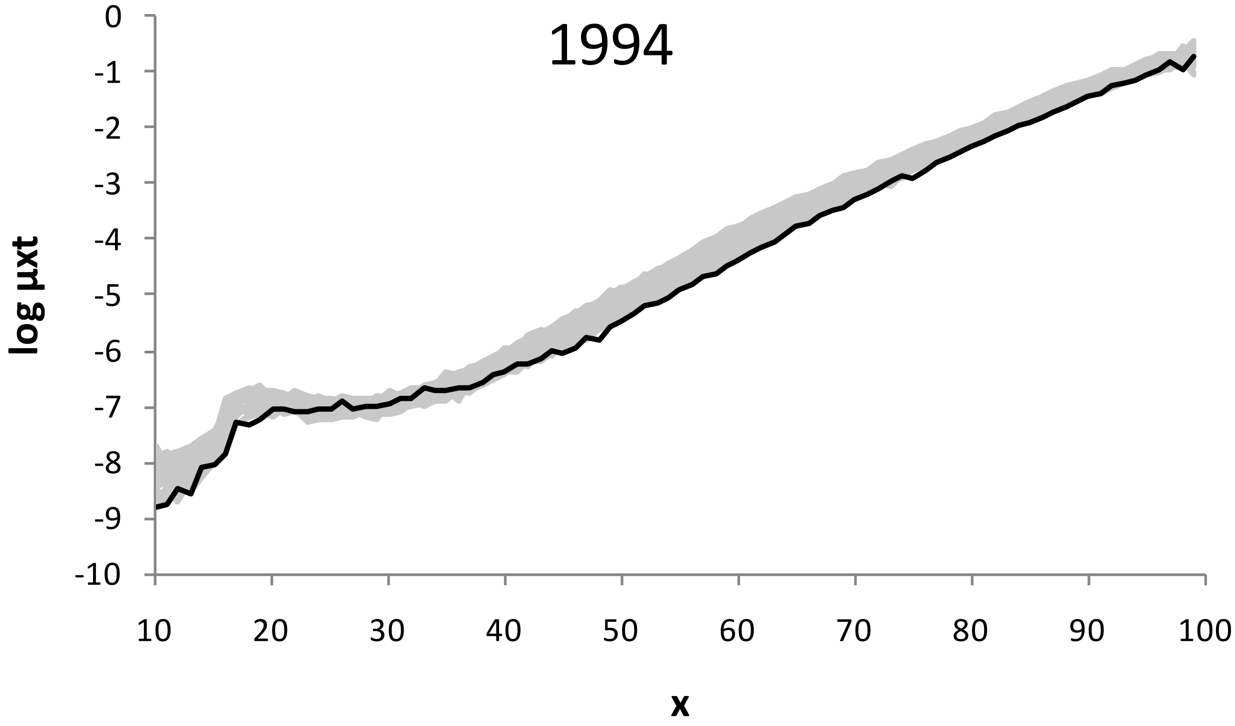

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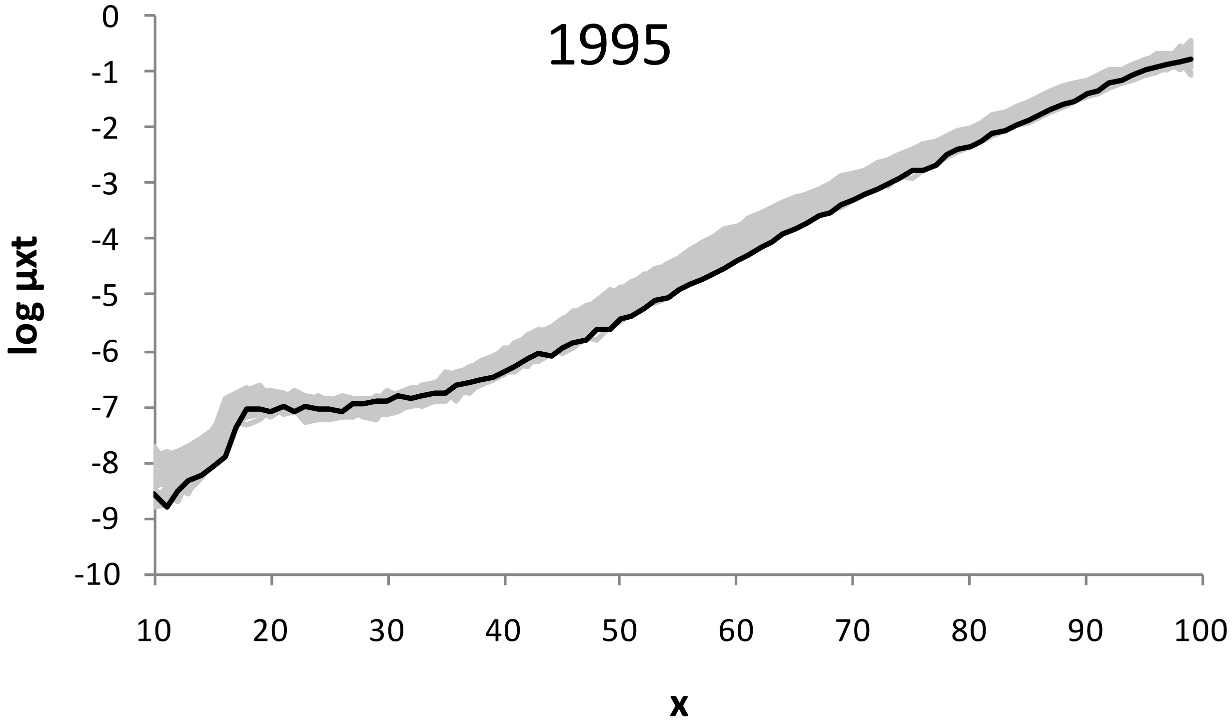

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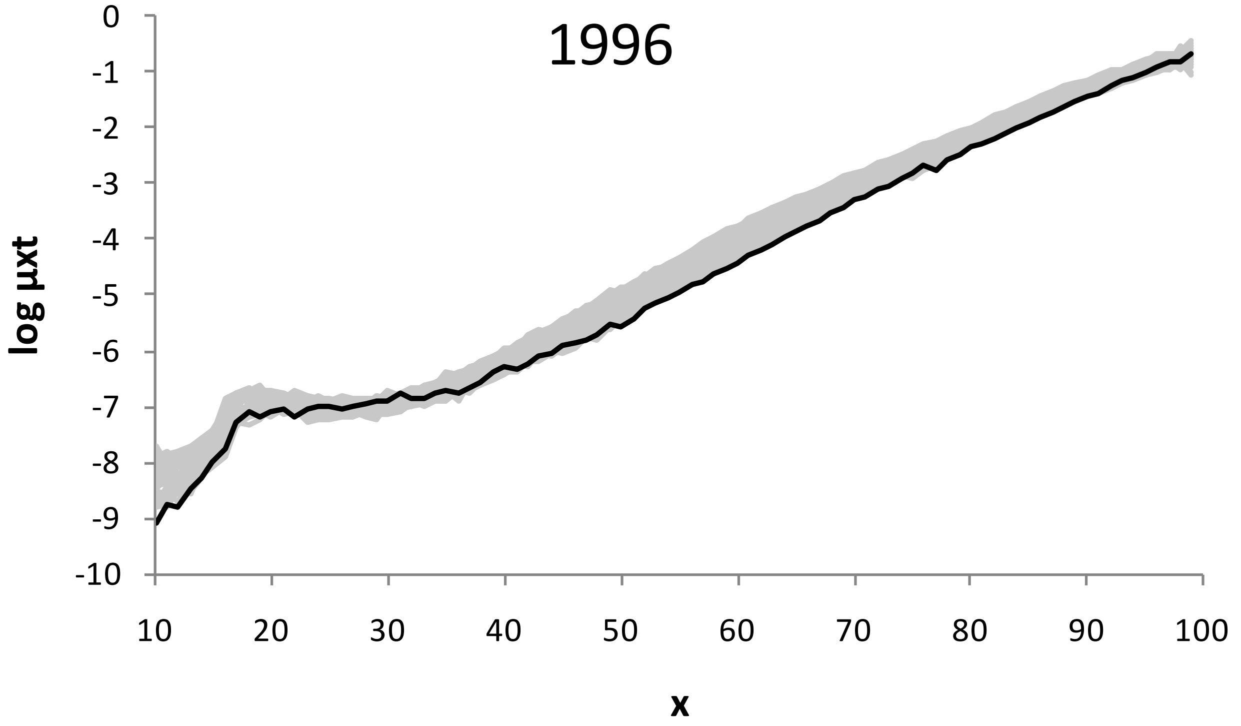

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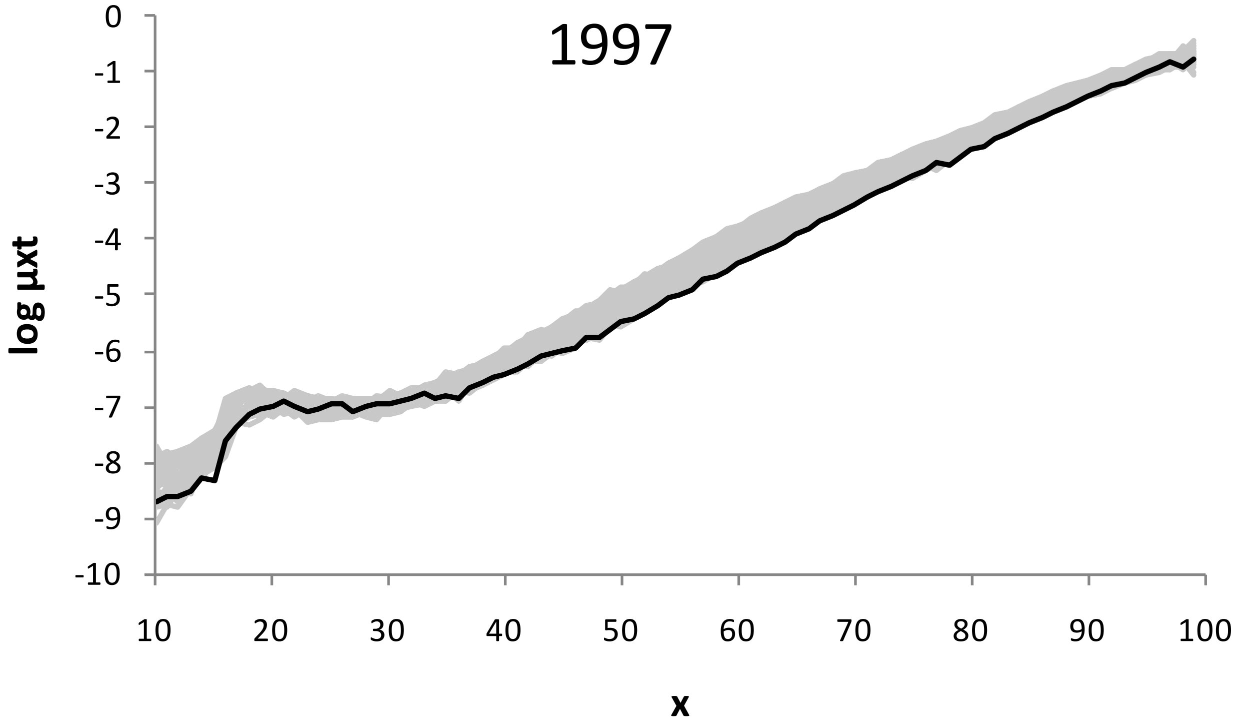

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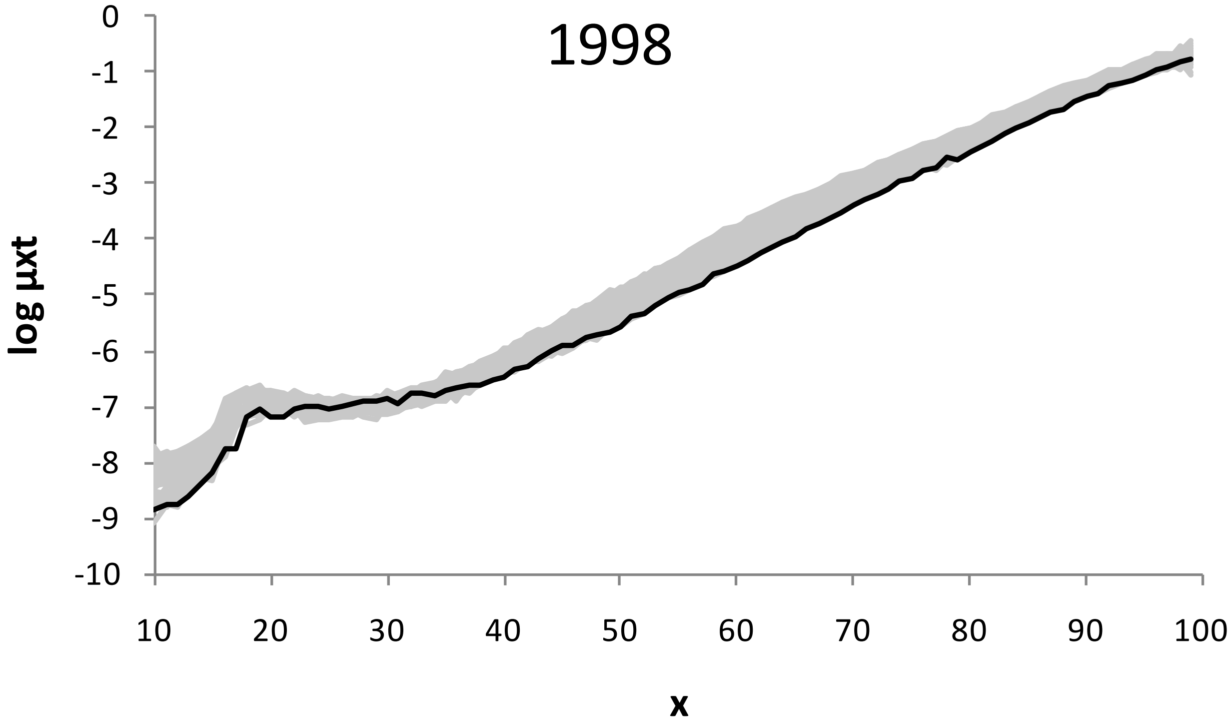

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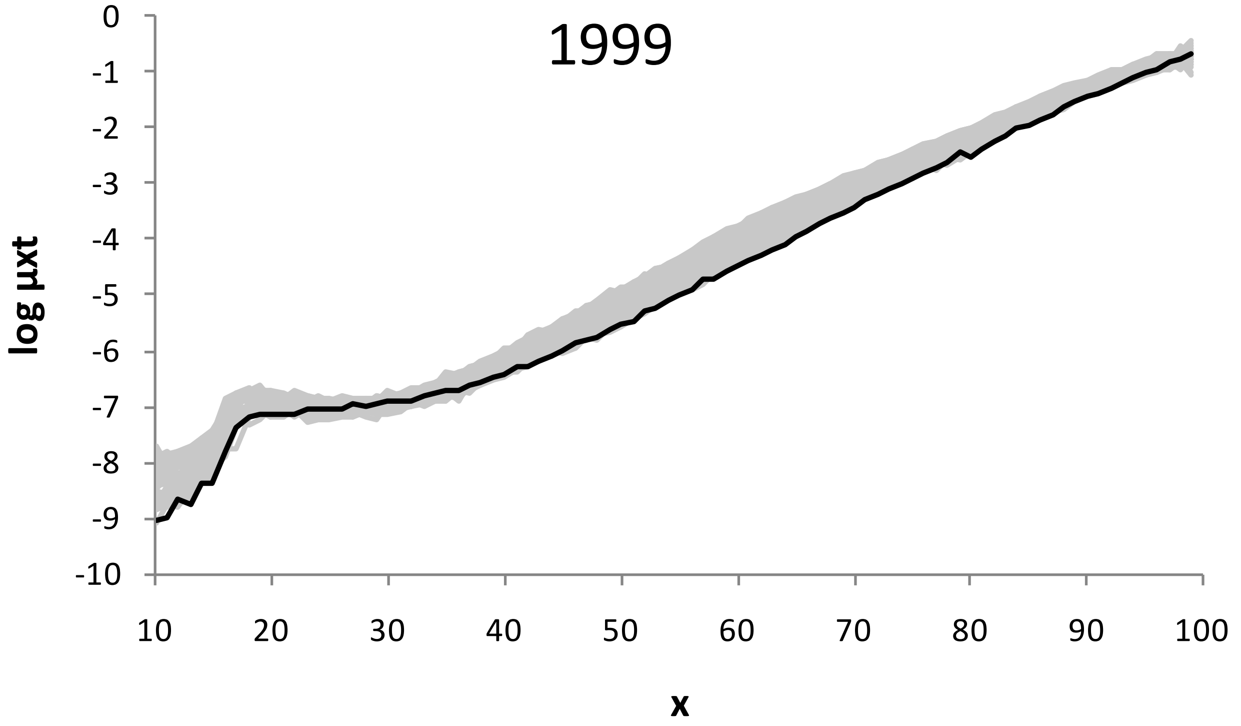

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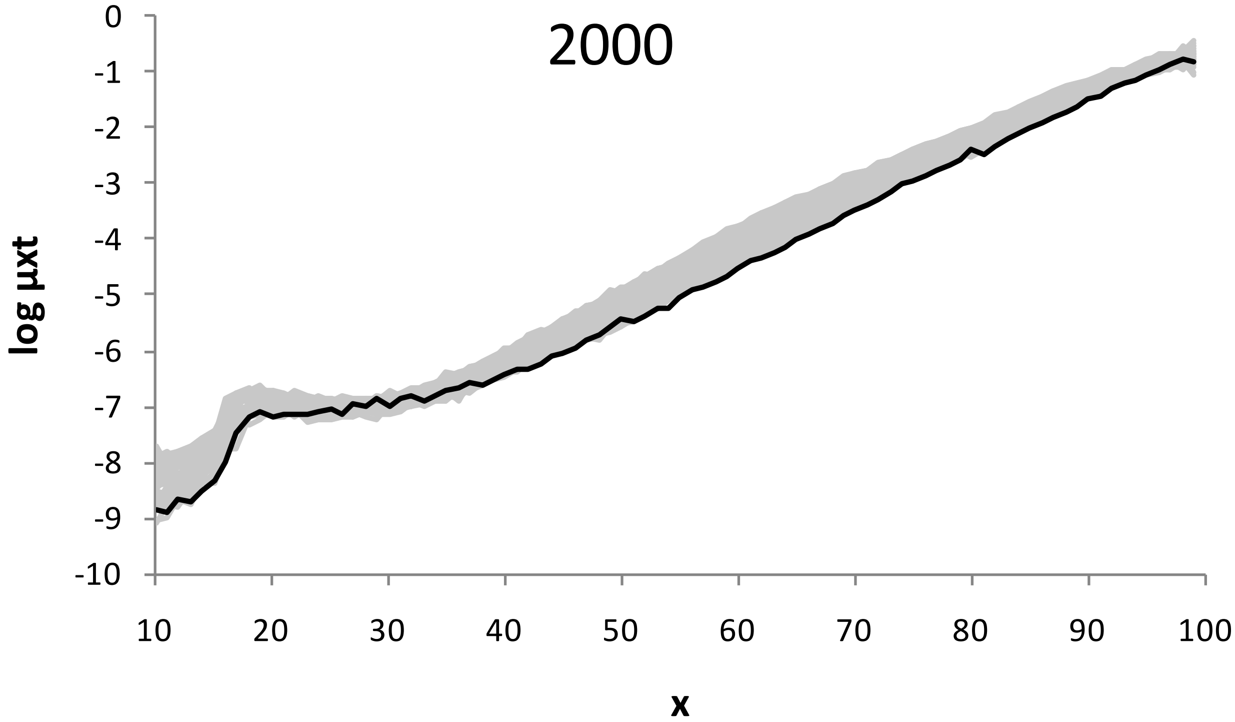

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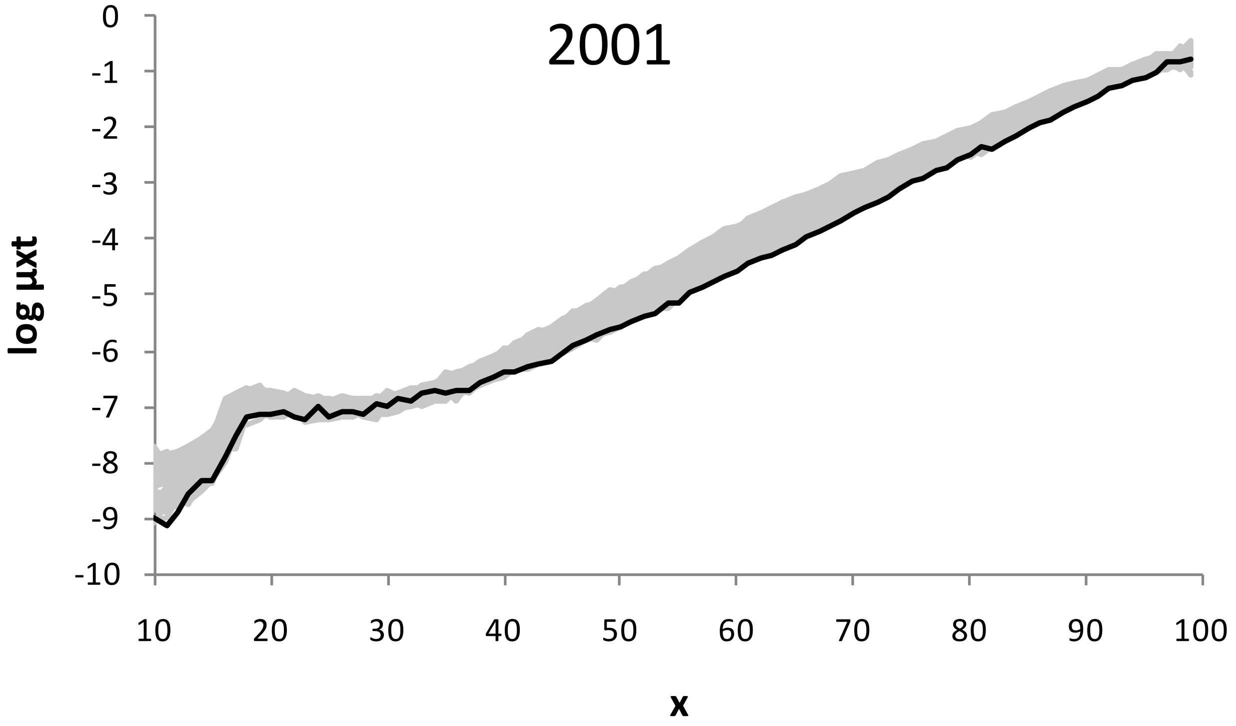

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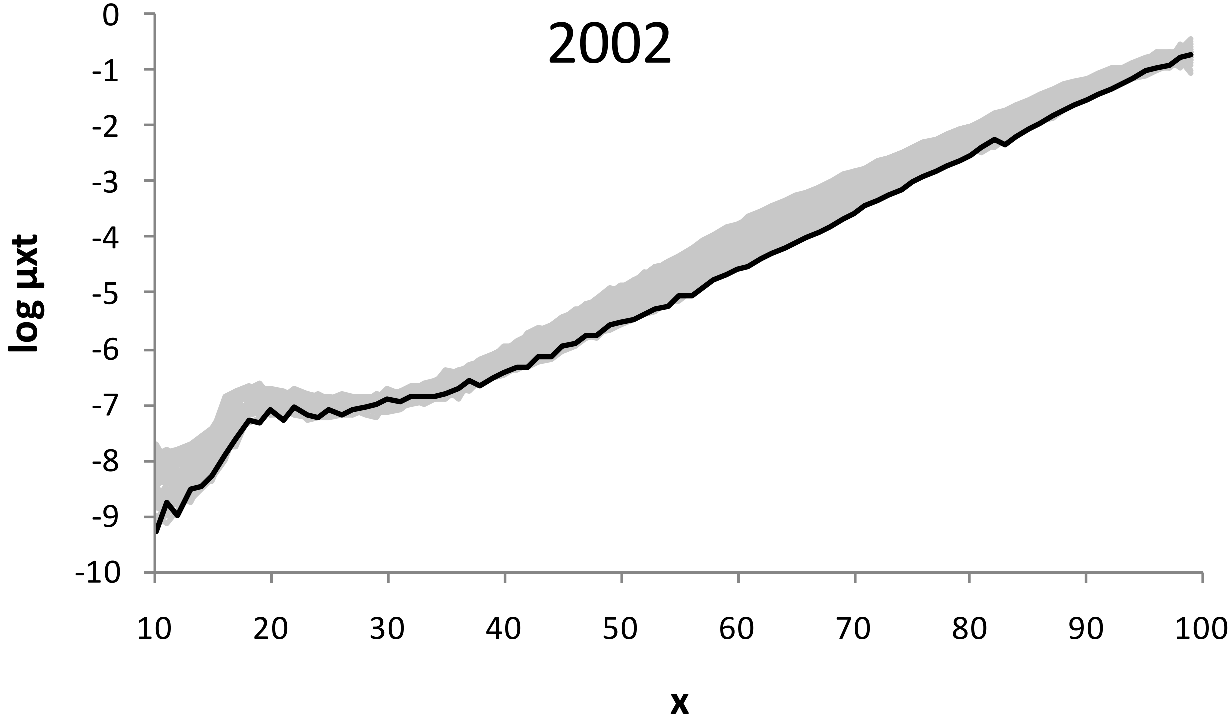

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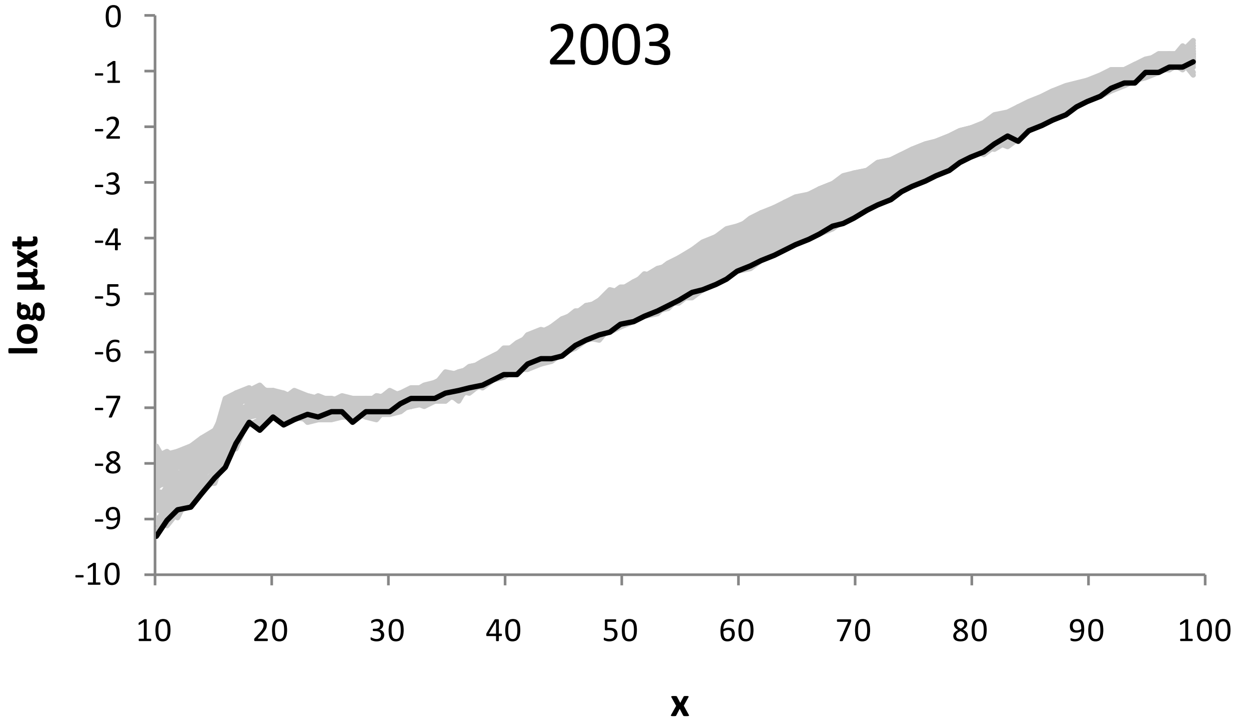

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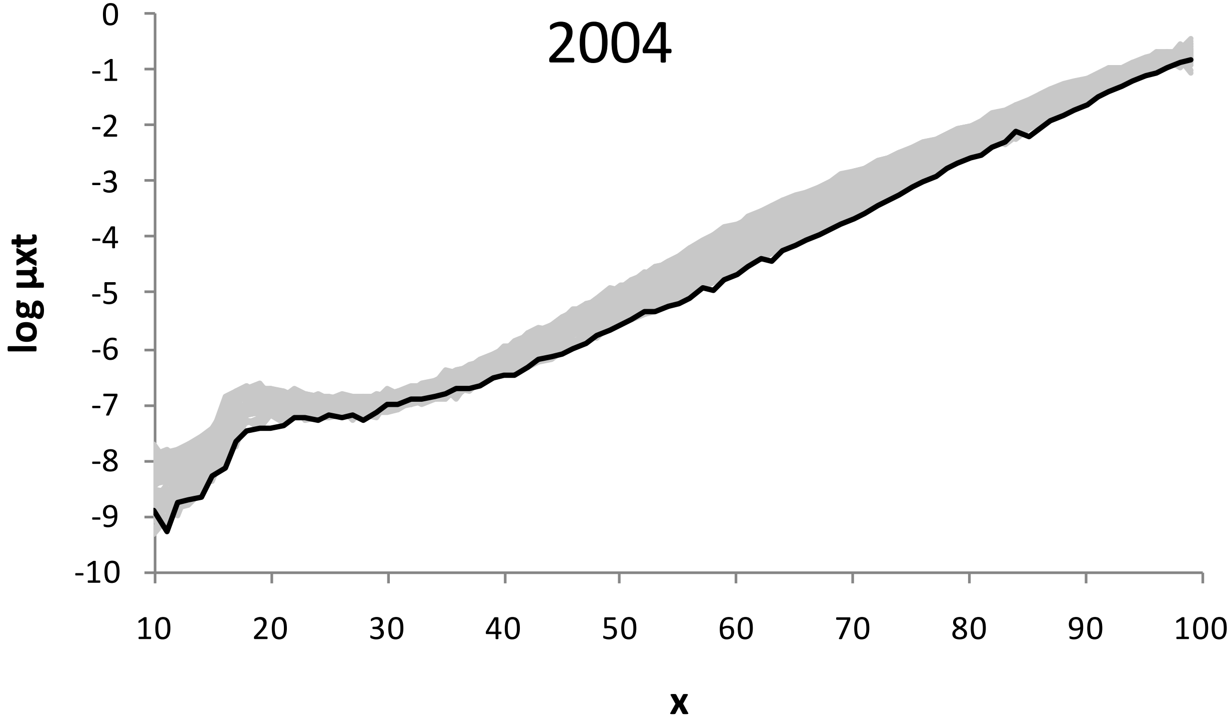

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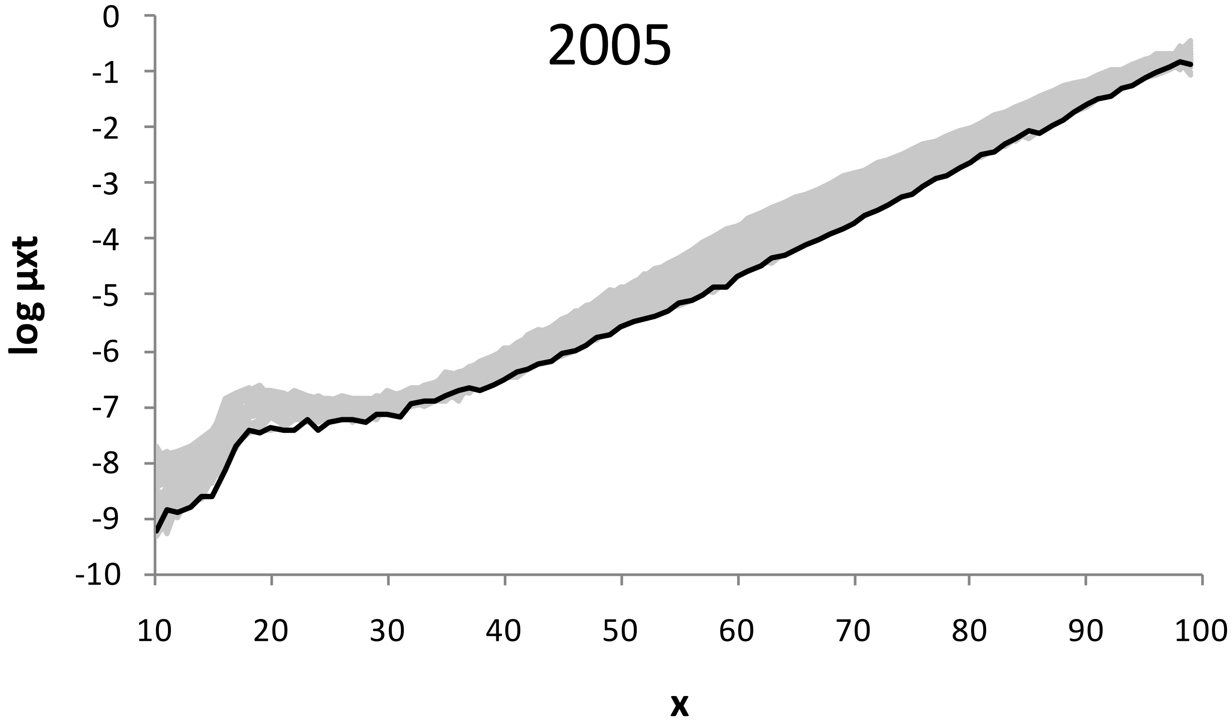

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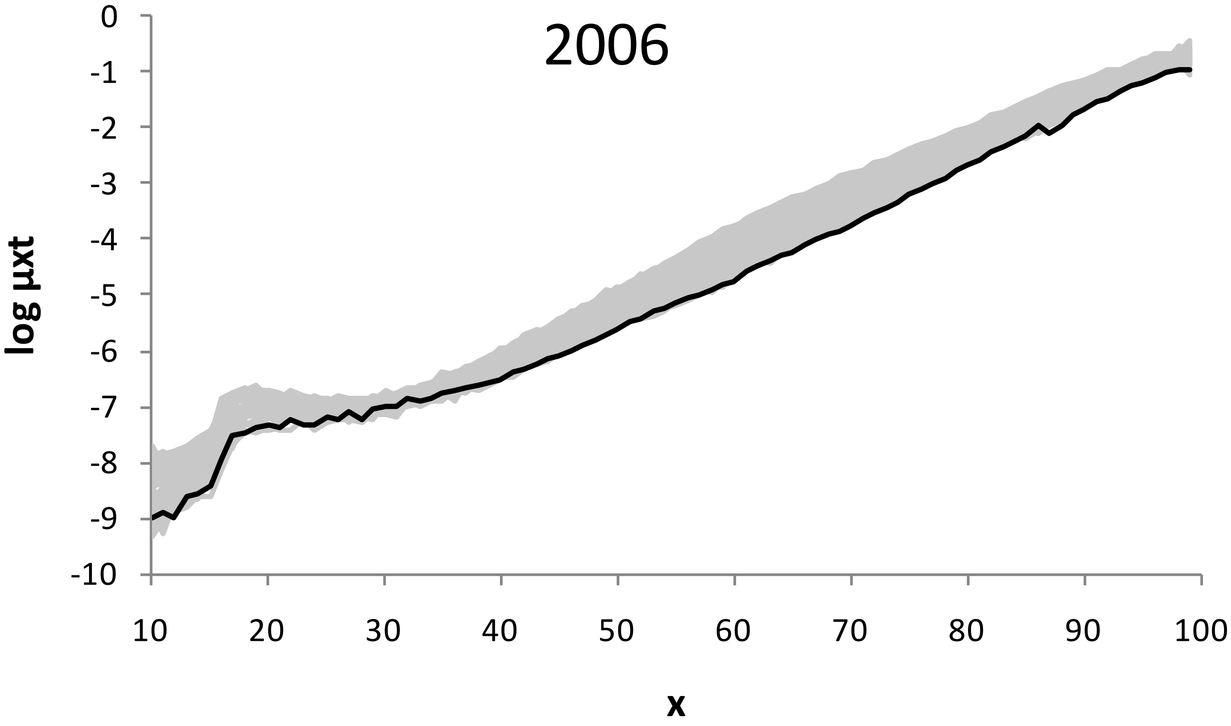

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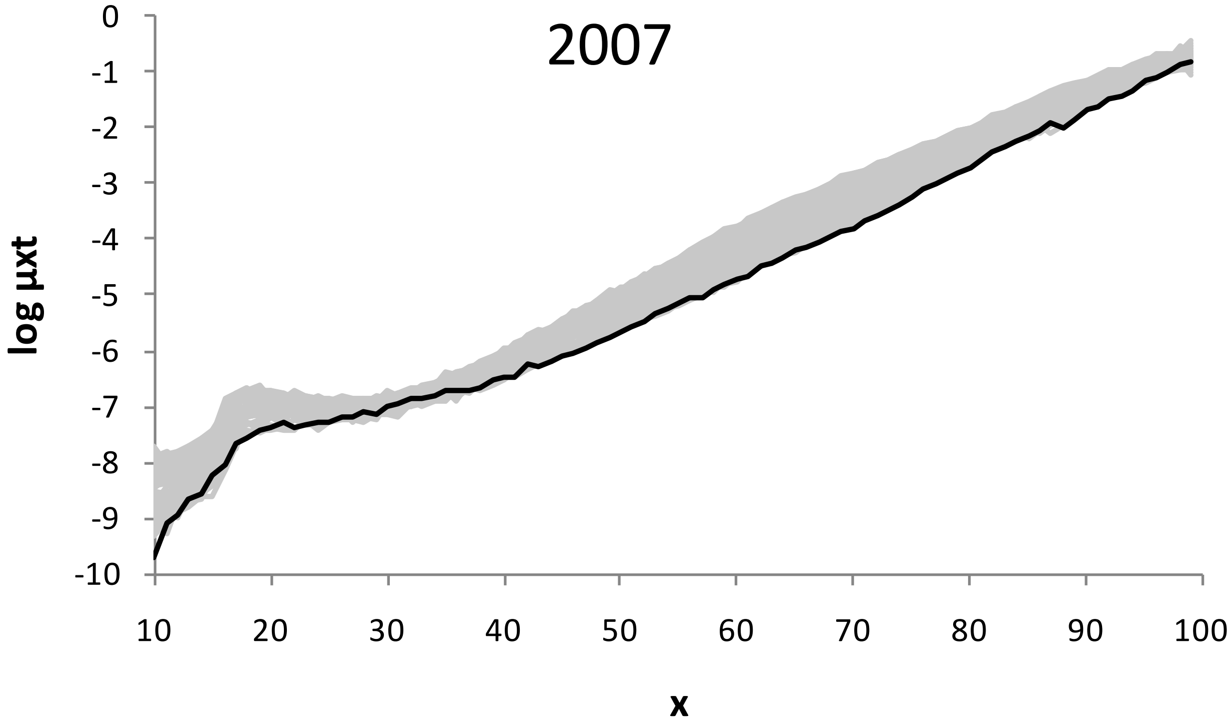

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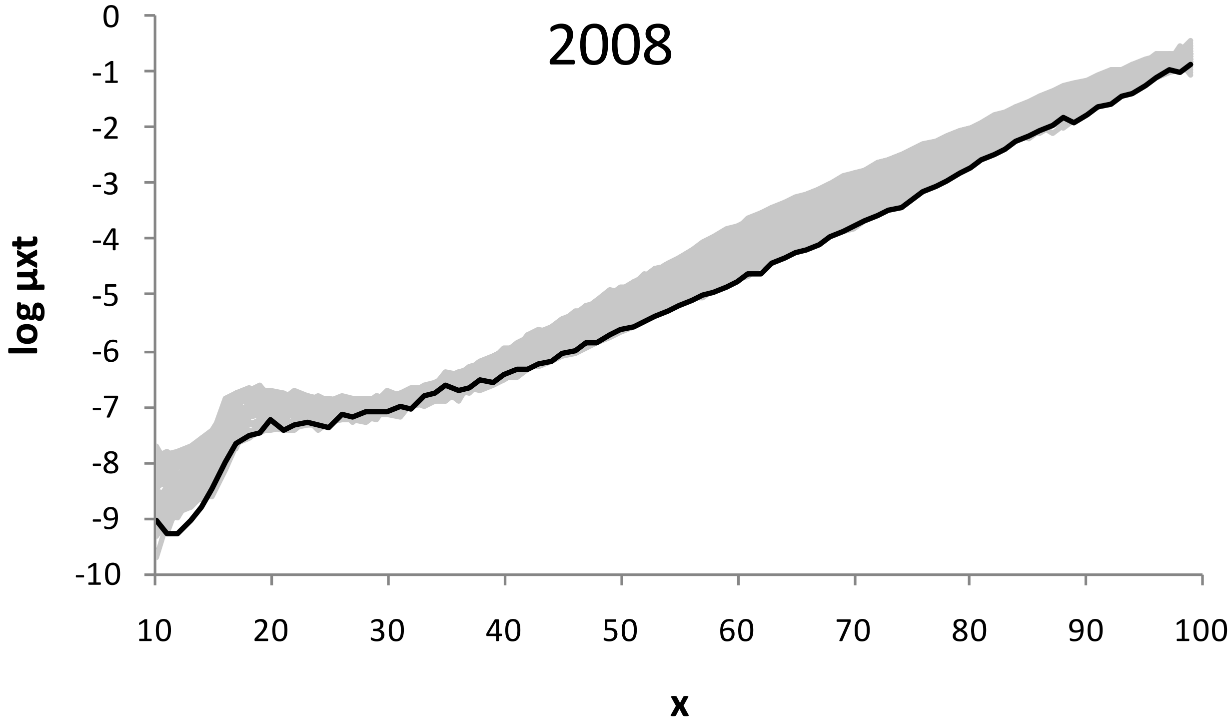

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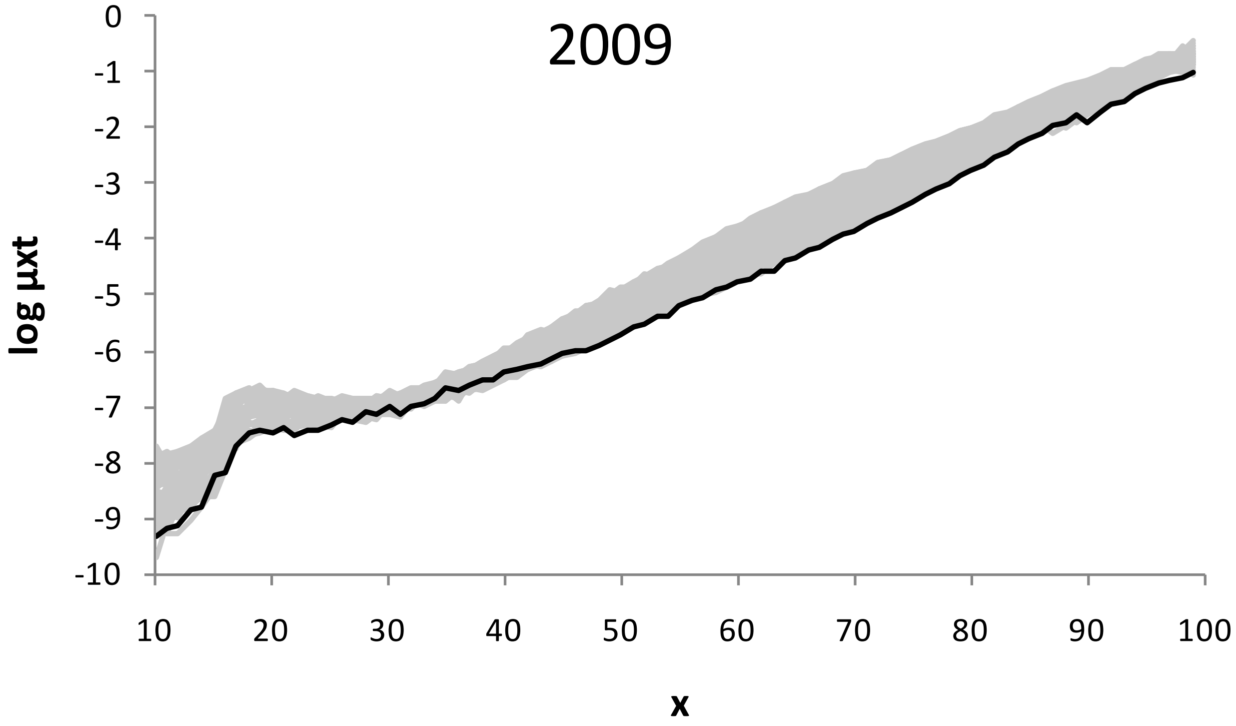

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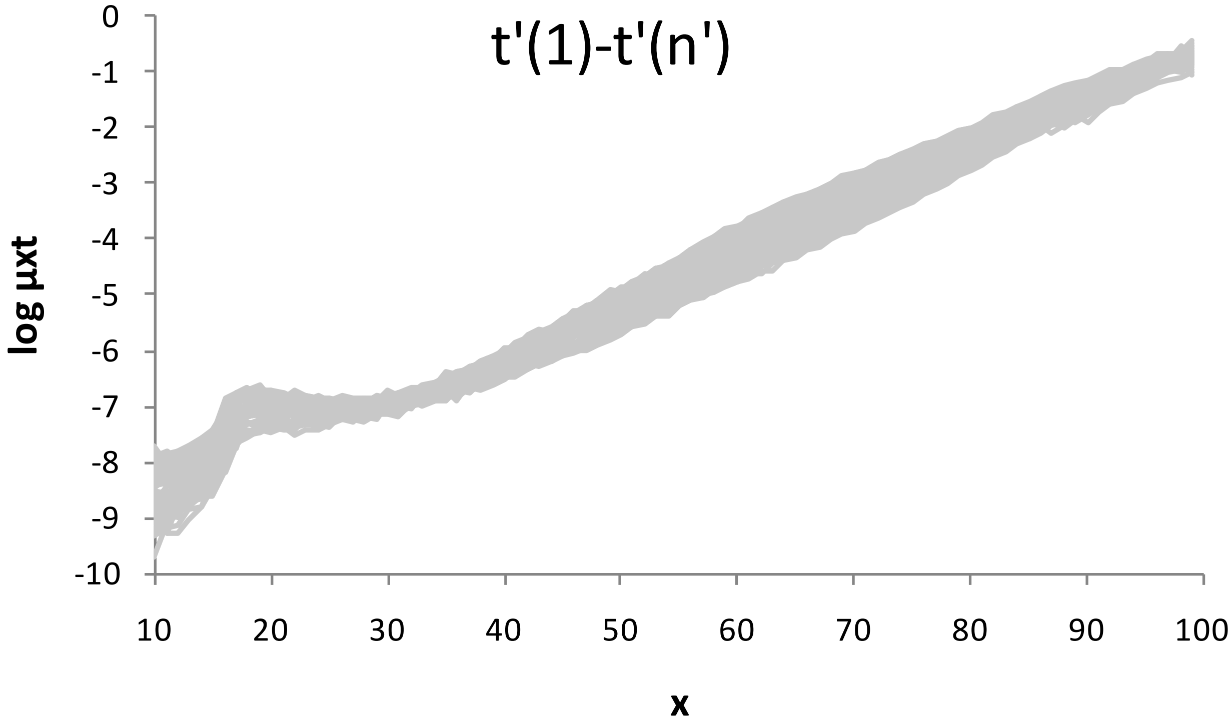

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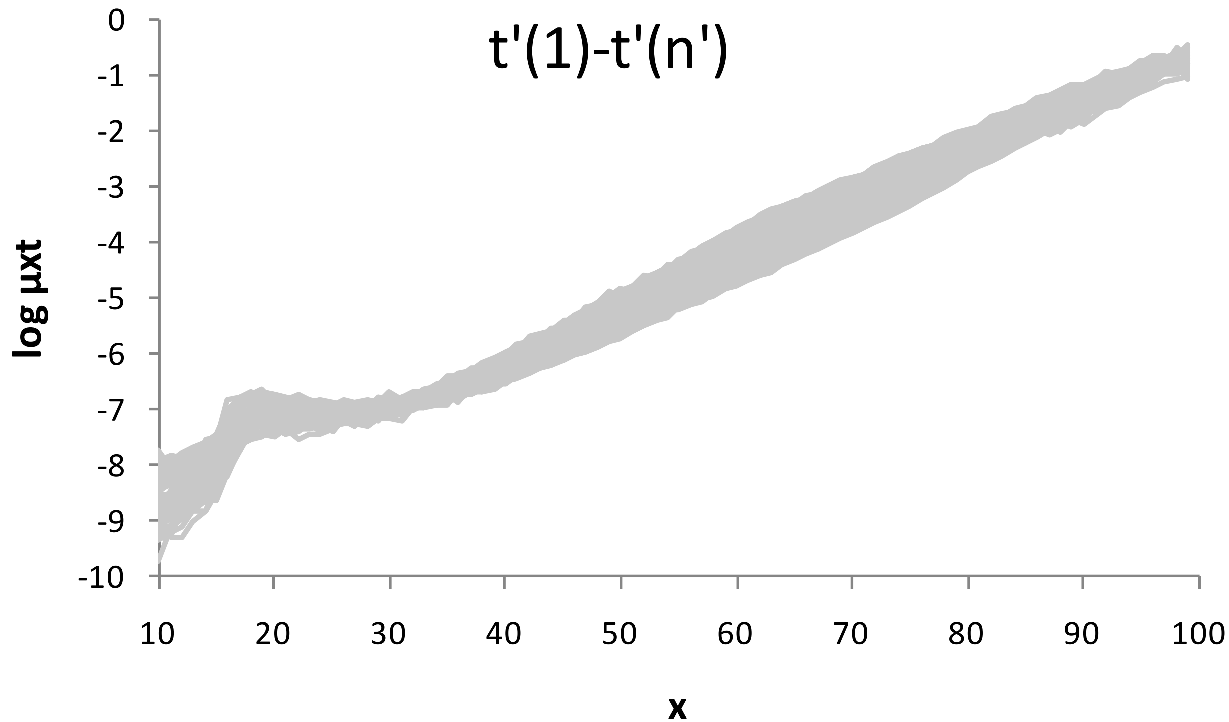

$$D'_{xt} \sim \text{Poisson}(e'_{xt}\mu'_{xt})$$

$$\log \mu_{xt} \quad =$$

Modelling mortality differentials

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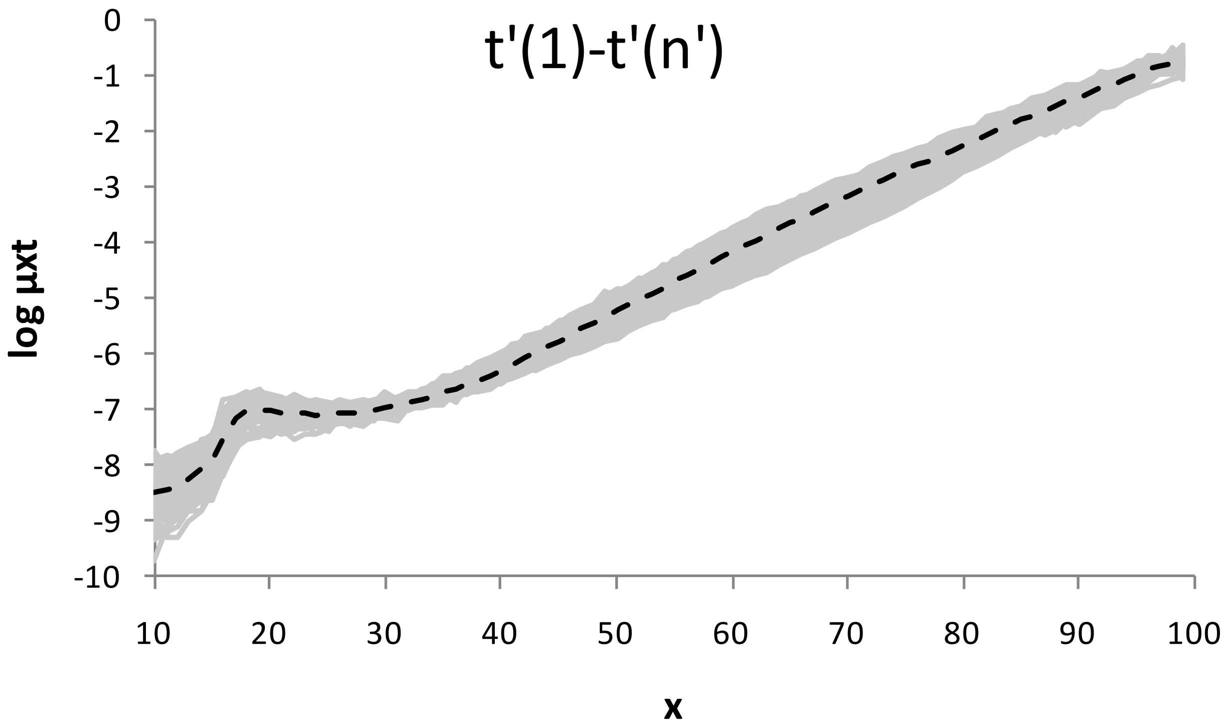
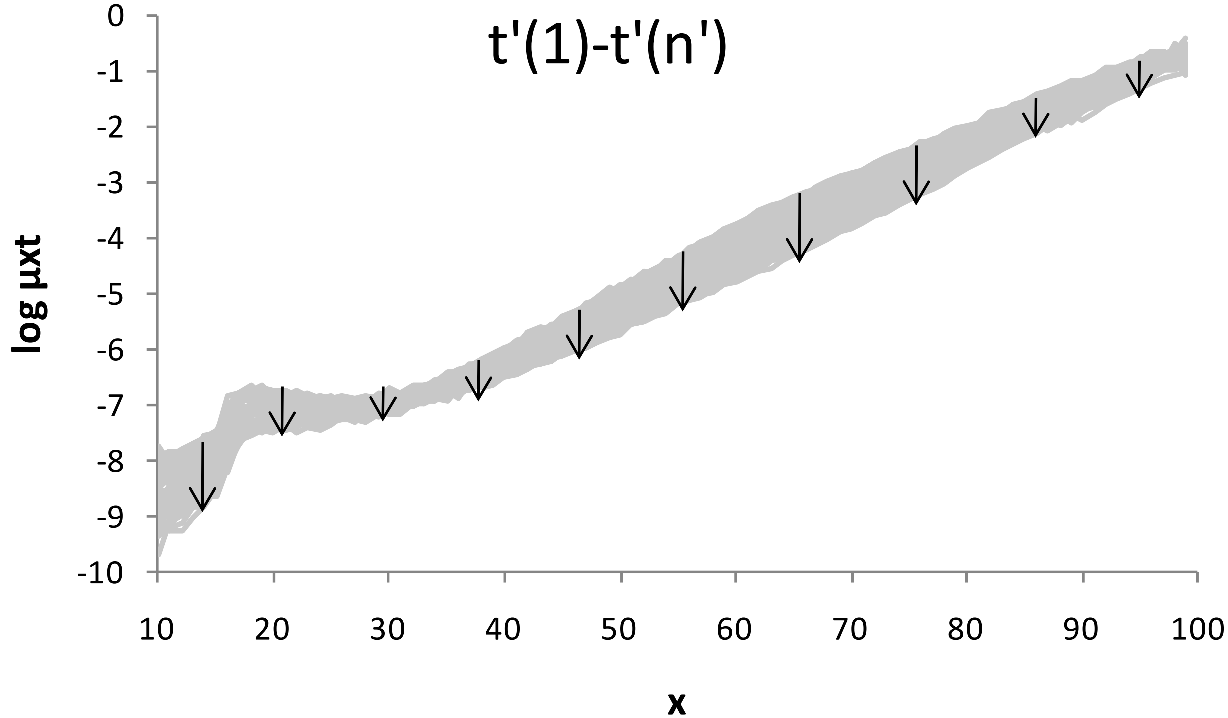


$$D'_{xt} \sim \text{Poisson}(e'_{xt}\mu'_{xt})$$

$$\log \mu_{xt} = \alpha'_x$$

Modelling mortality differentials

Reference population model



$$D'_{xt} \sim \text{Poisson}(e'_{xt}\mu'_{xt})$$

$$\log \mu_{xt} = \alpha'_x + \kappa'_t$$

Modelling mortality differentials

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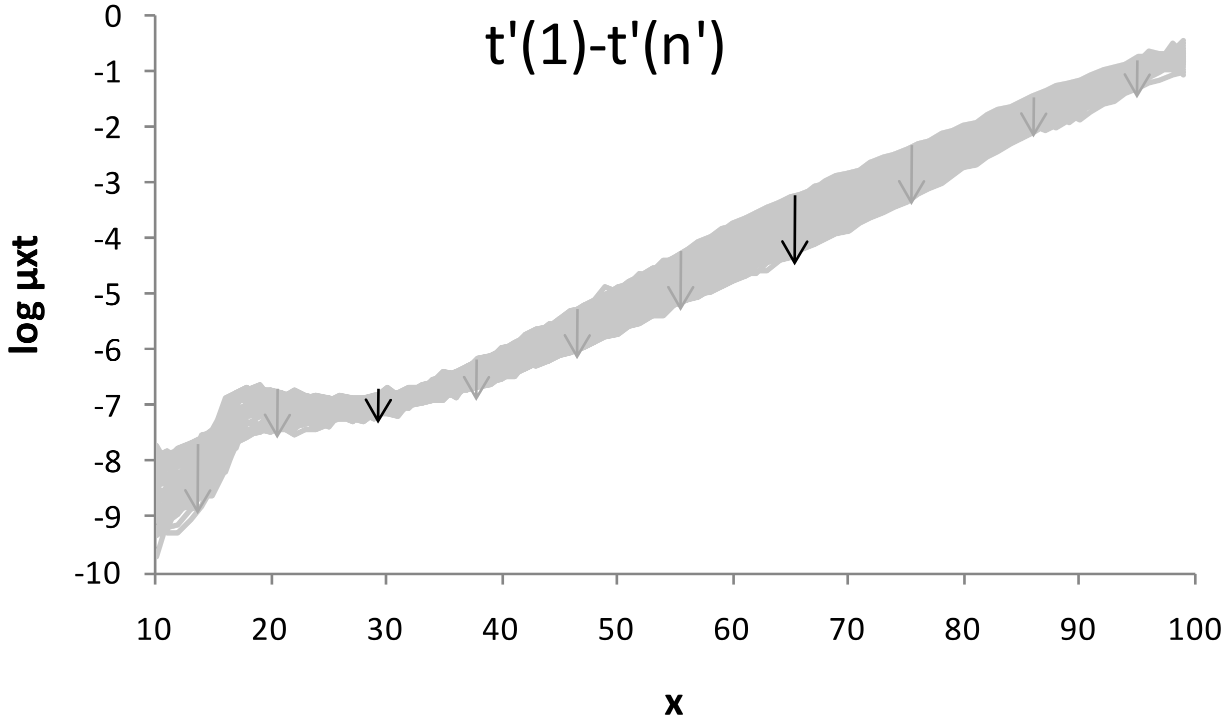

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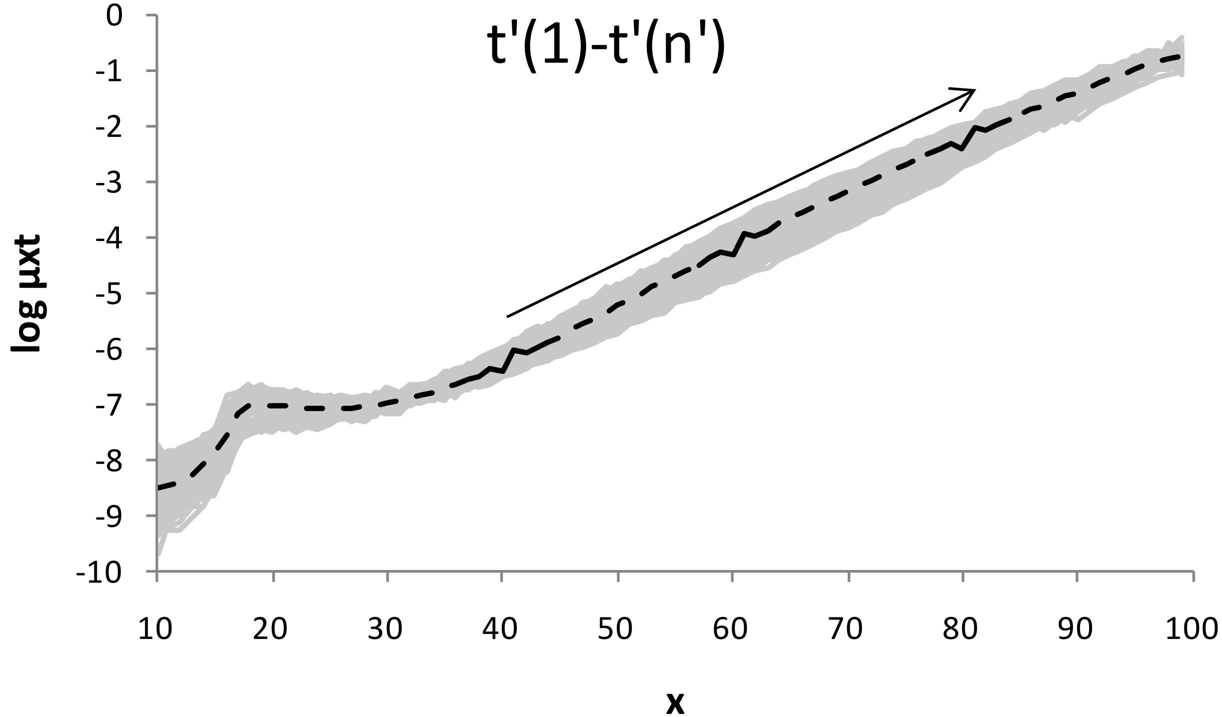
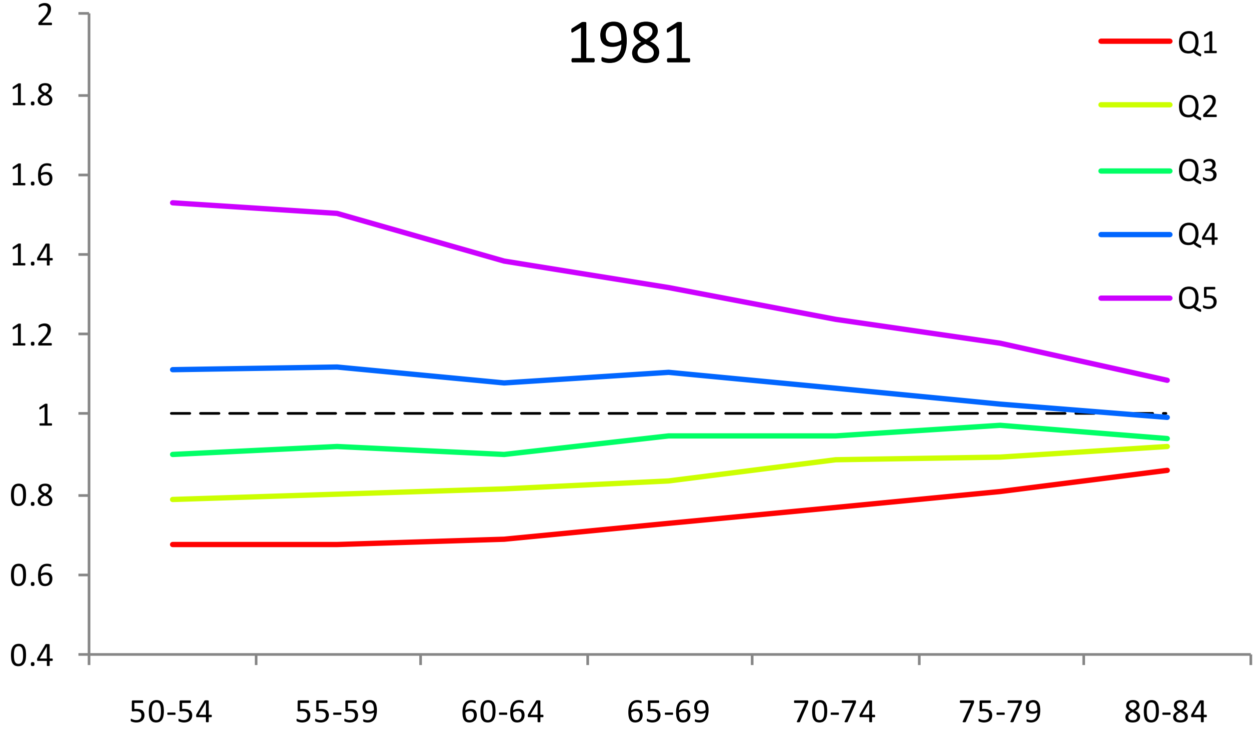


$$D'_{xt} \sim \text{Poisson}(e'_{xt}\mu'_{xt})$$

$$\log \mu_{xt} = \alpha'_x + \beta'_x \kappa'_t + \gamma'_{t-x}$$

Modelling mortality differentials

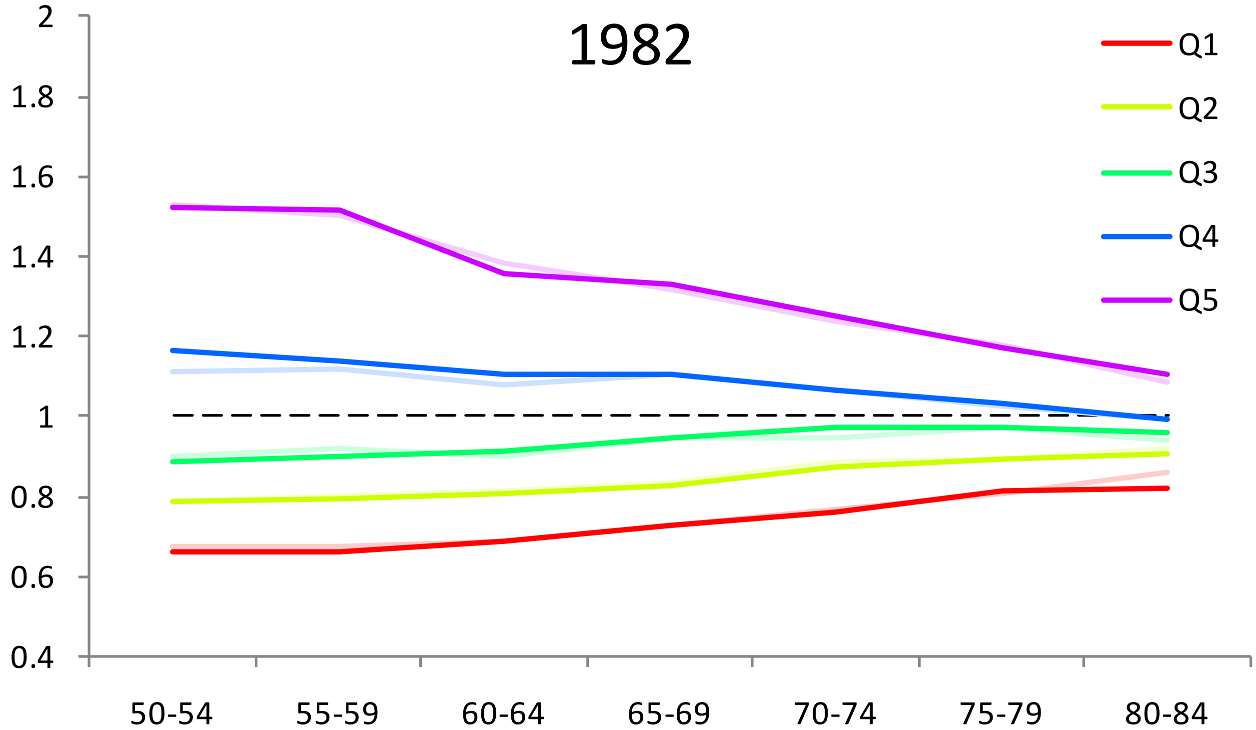
Subpopulations model



$$ {}_n\bar{\mu}'_{xt} = \left(\prod_{i=0}^{n-1} \mu'_{x+i,t} \right)^{\frac{1}{n}} \qquad \frac{{}_n\mu_{xtg}}{{}_n\bar{\mu}'_{xt}} $$

Modelling mortality differentials

Subpopulations model



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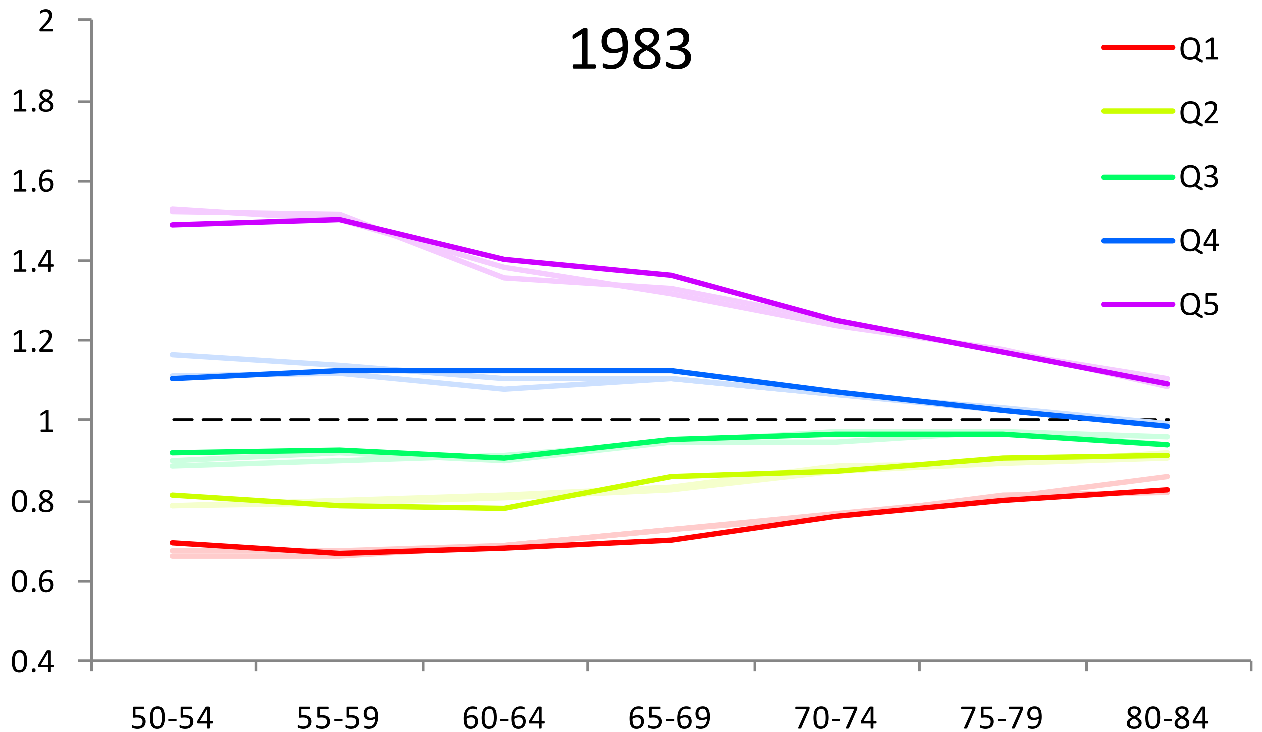

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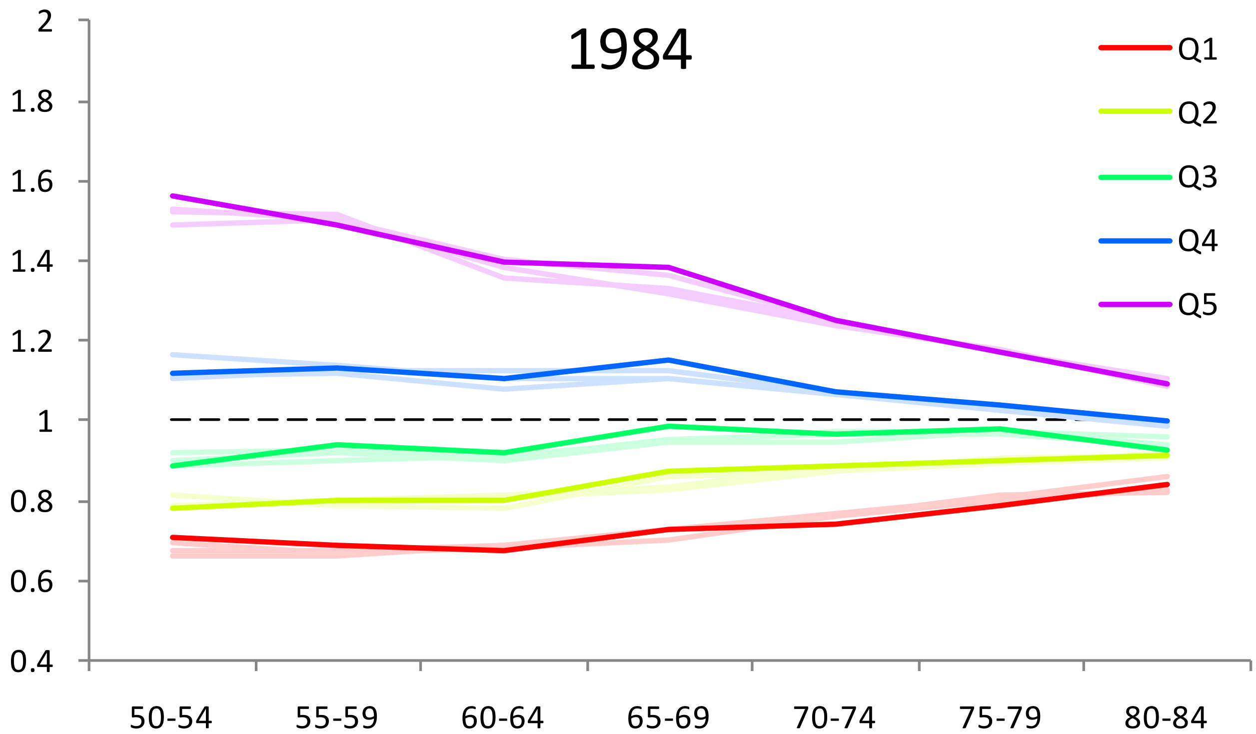
Modelling mortality differentials

Subpopulations model



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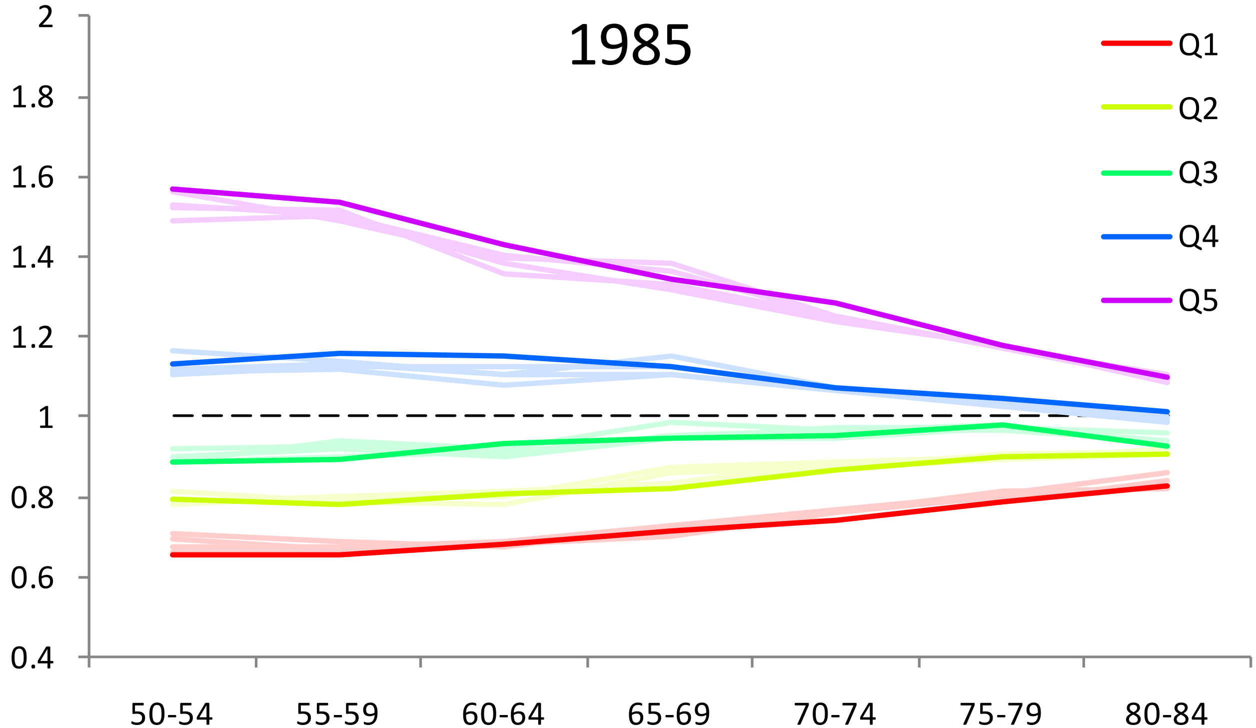
Modelling mortality differentials

Subpopulations model



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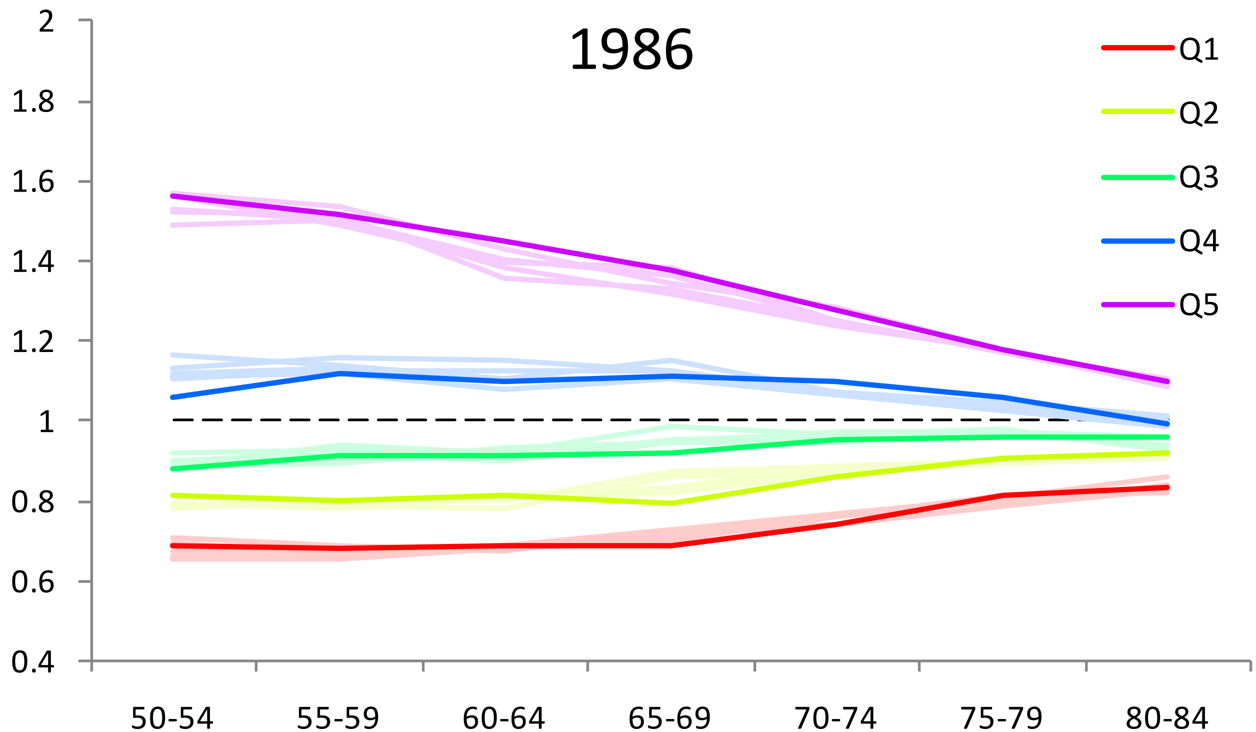

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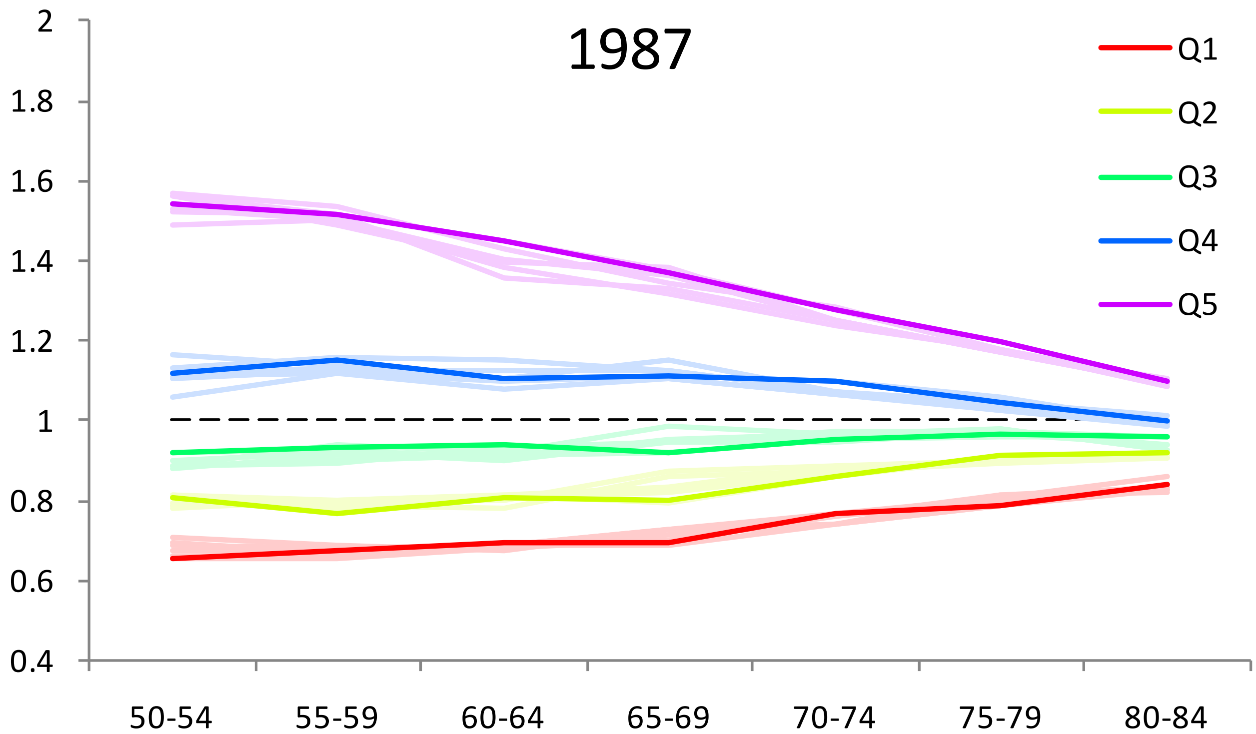

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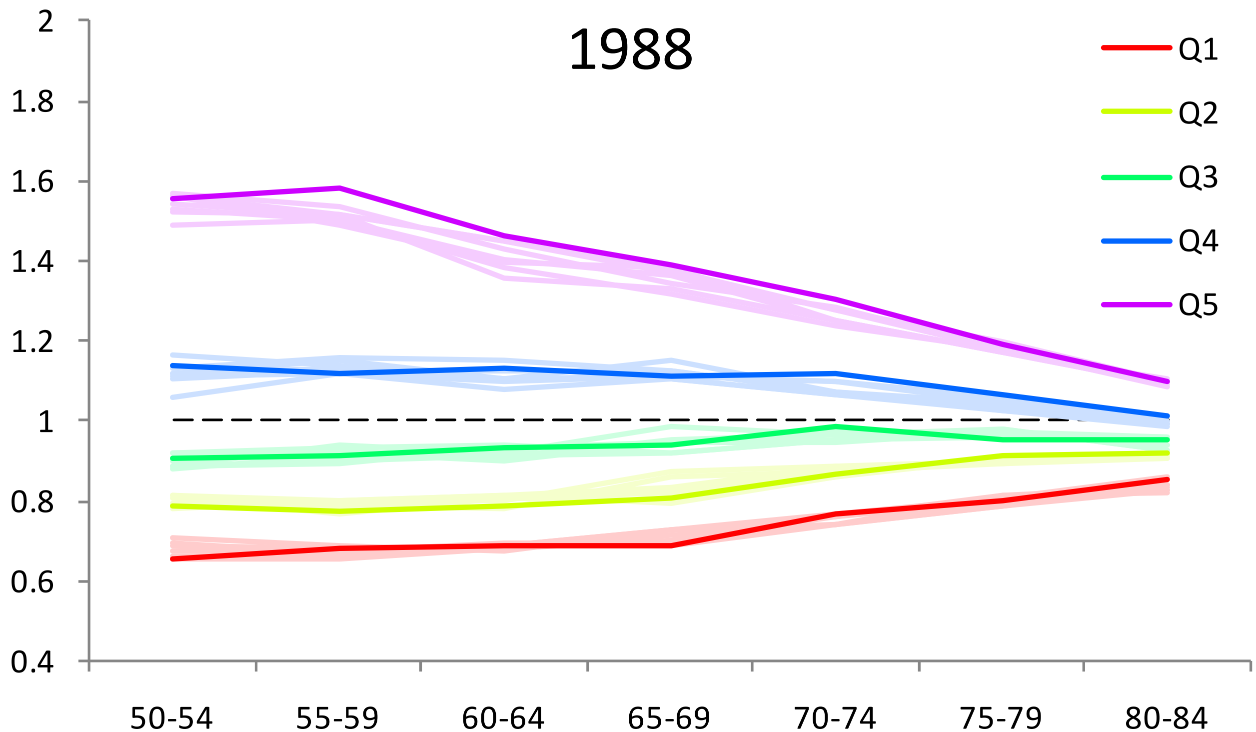

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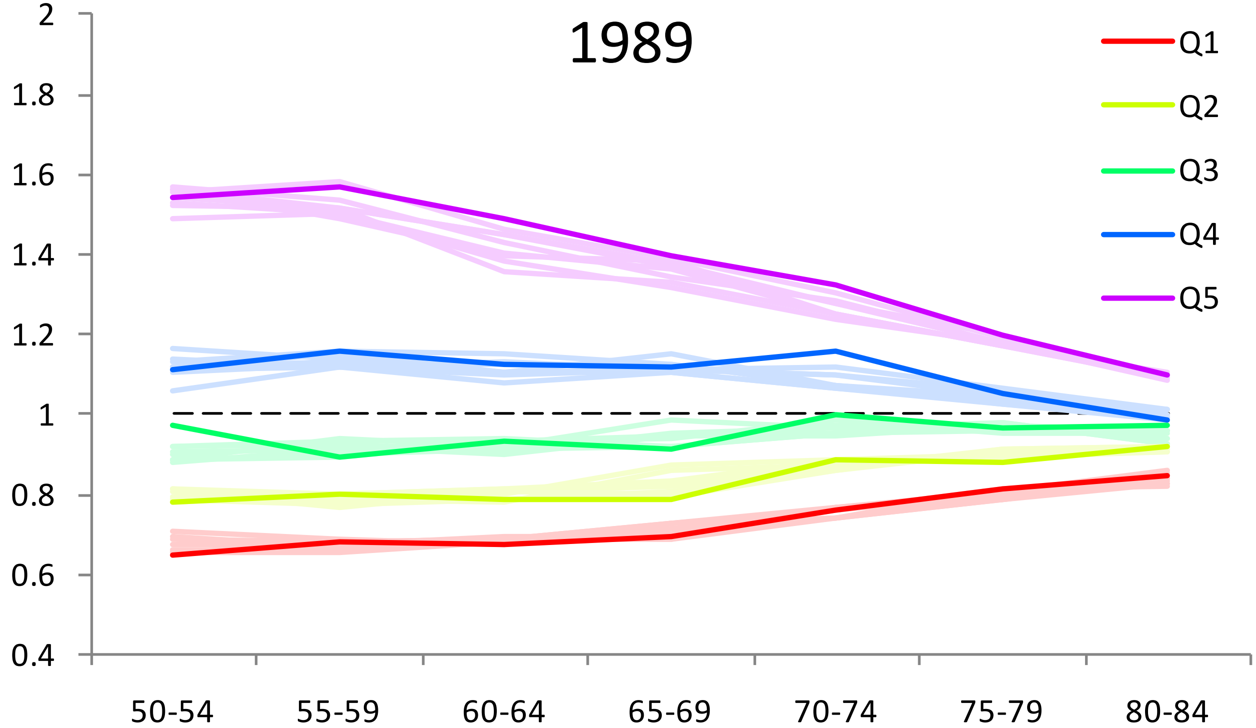

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Modelling mortality differentials

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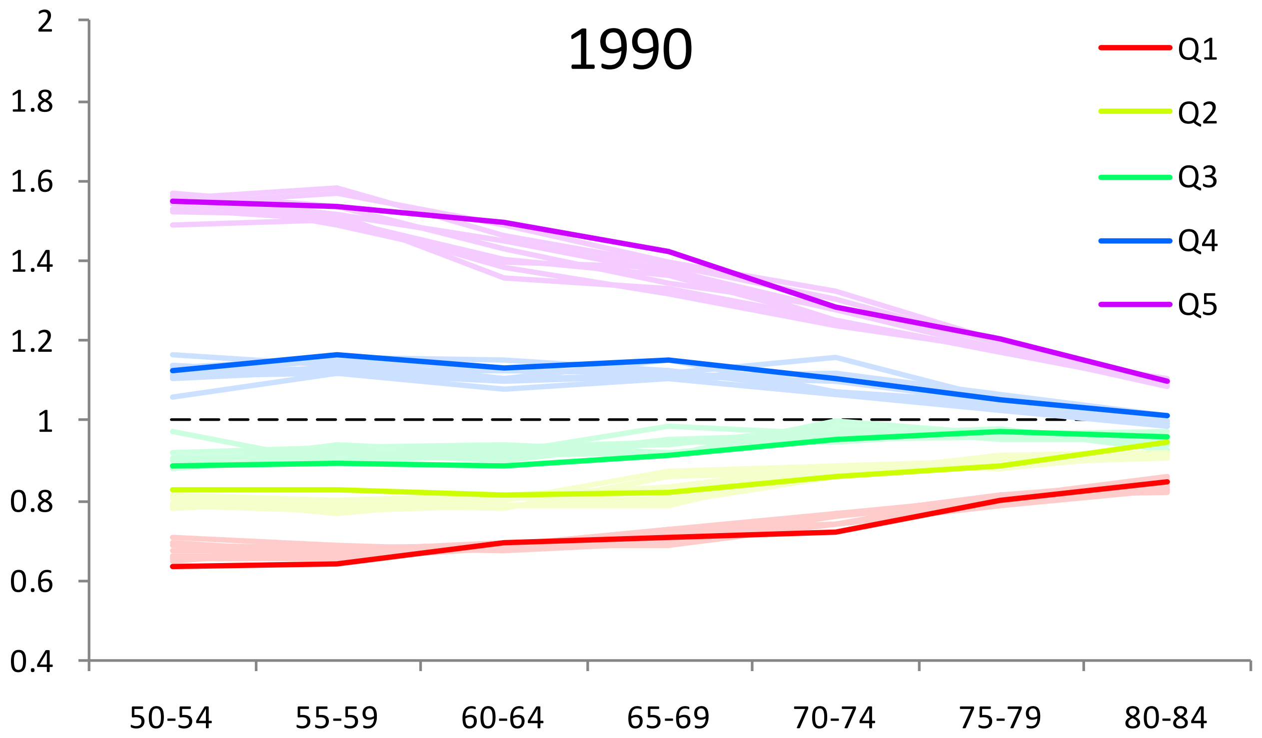

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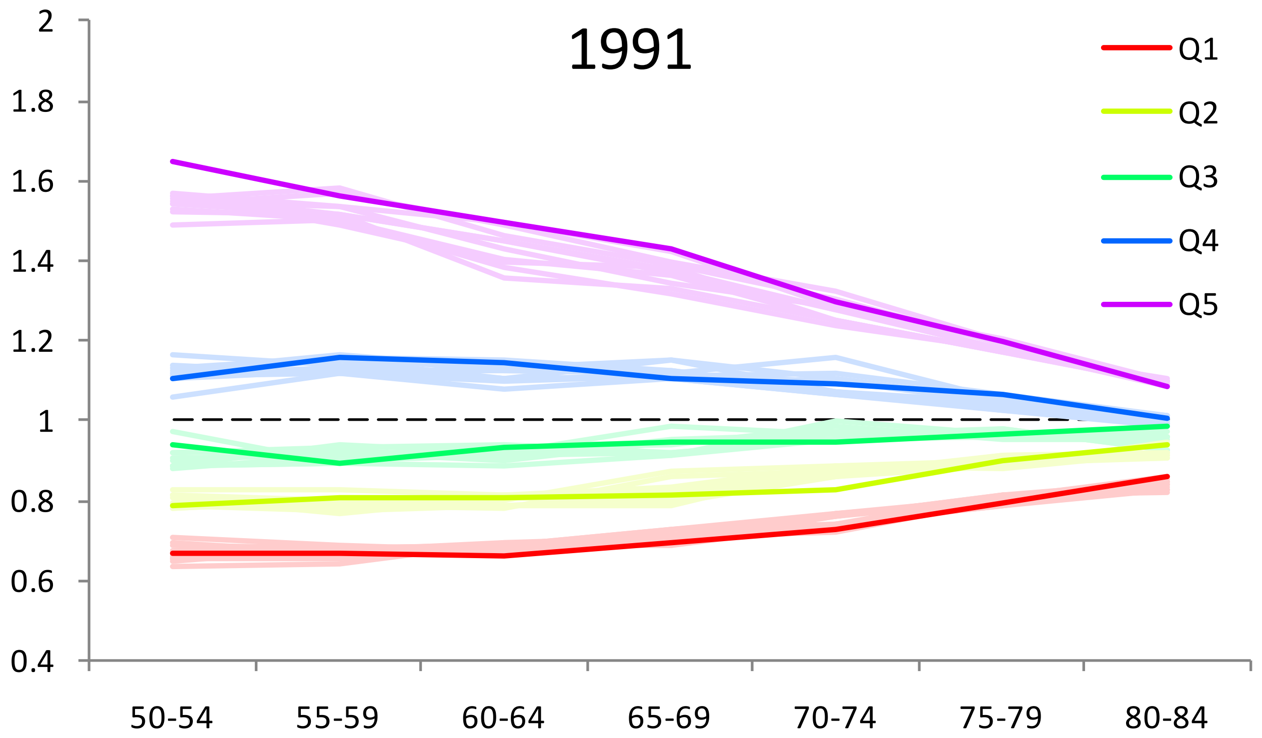
Modelling mortality differentials

Subpopulations model



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Modelling mortality differentials
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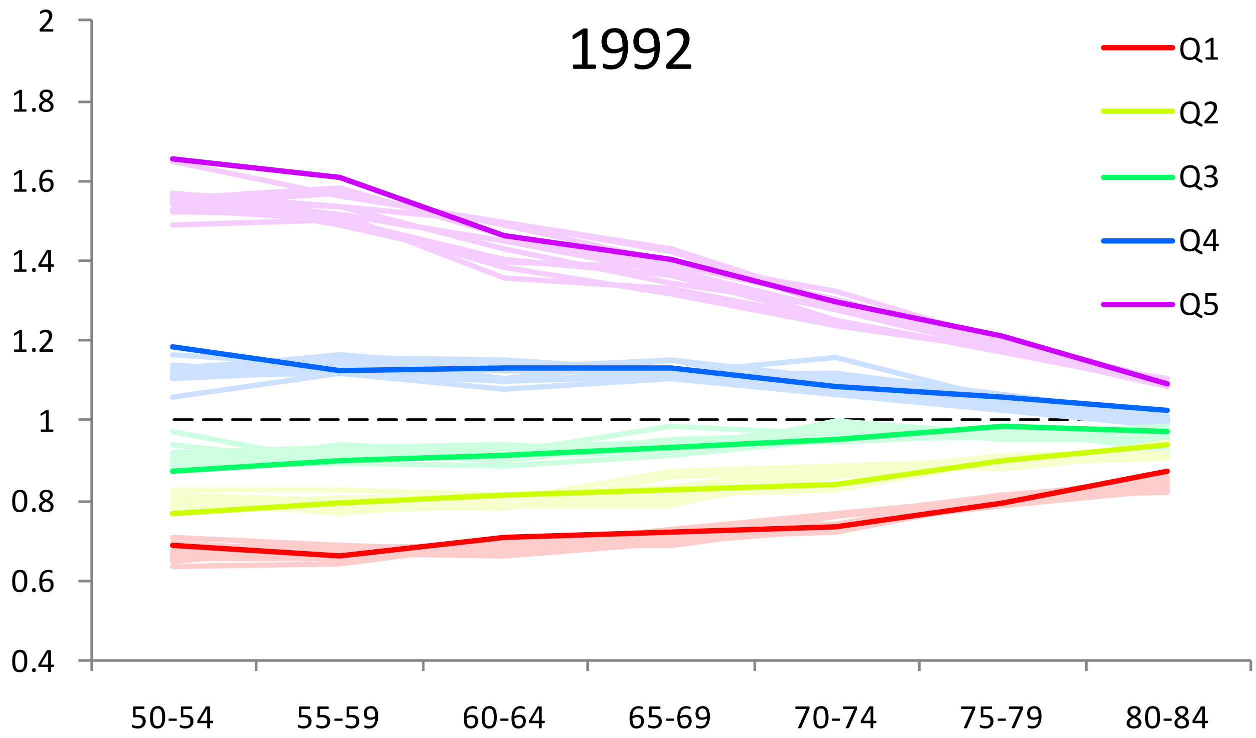

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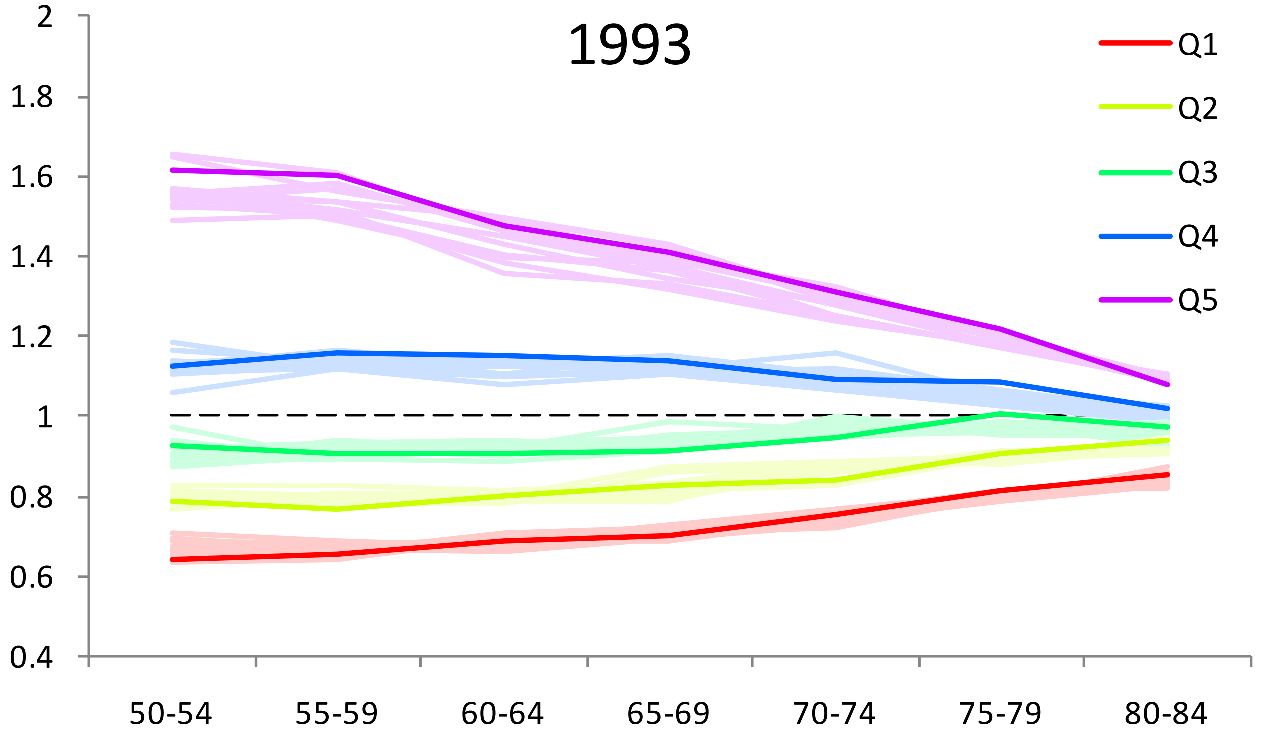
Modelling mortality differentials

Subpopulations model



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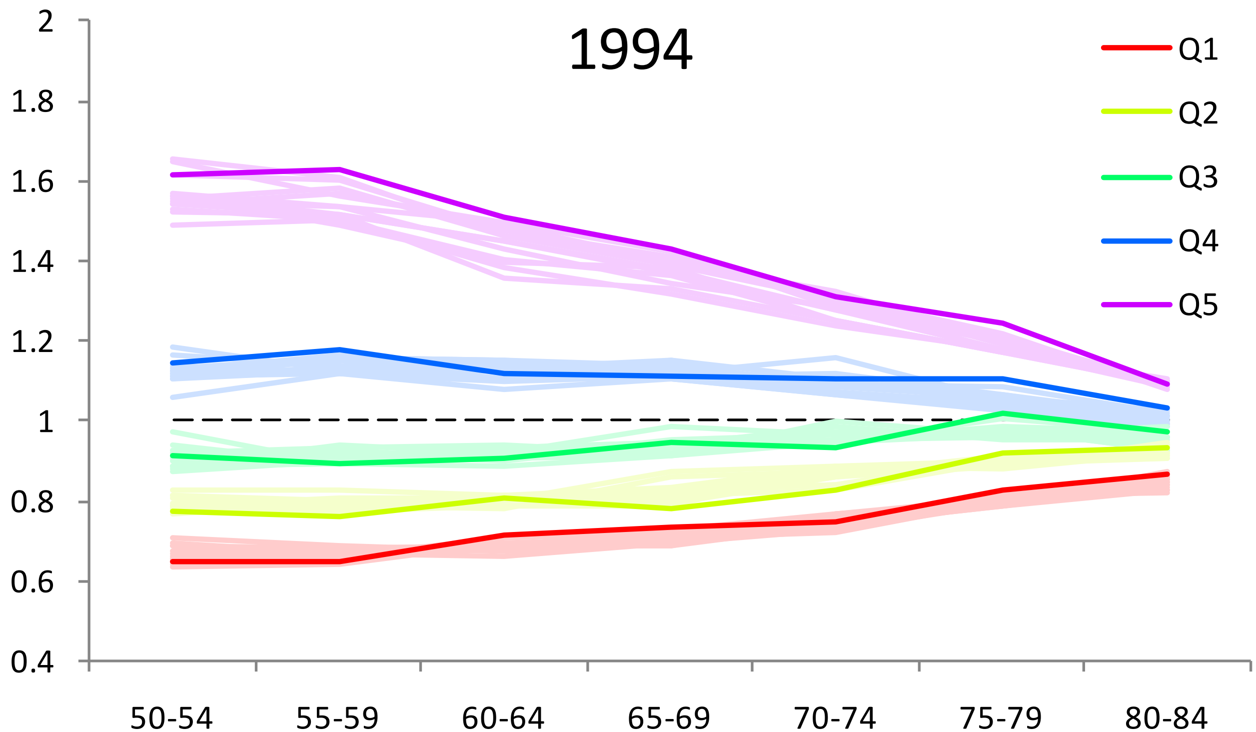
Modelling mortality differentials

Subpopulations model



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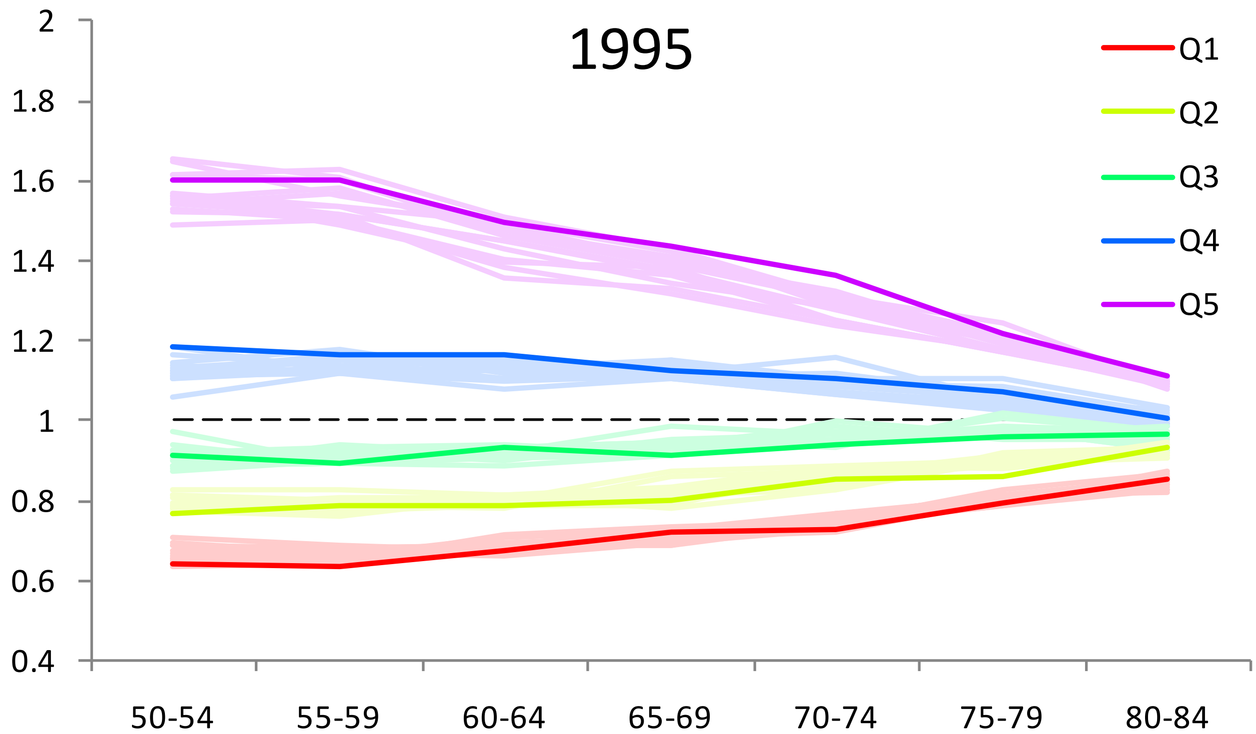
Modelling mortality differentials

Subpopulations model



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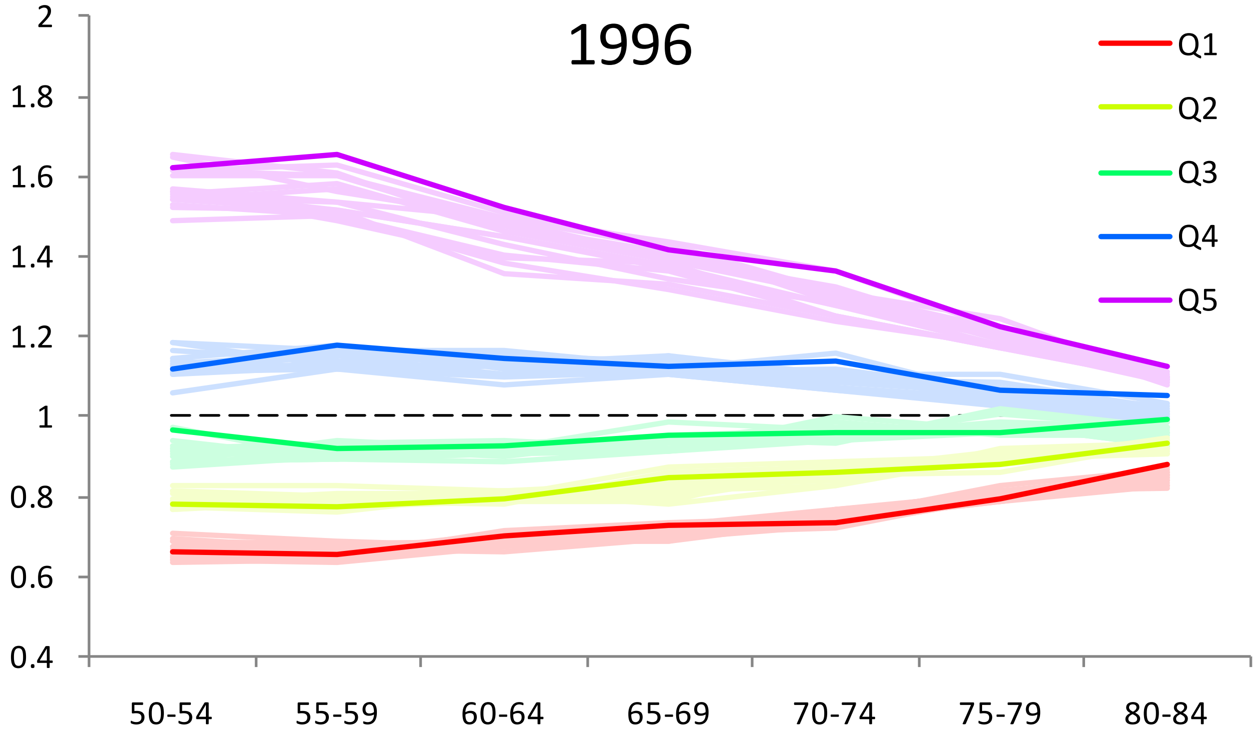
Modelling mortality differentials

Subpopulations model



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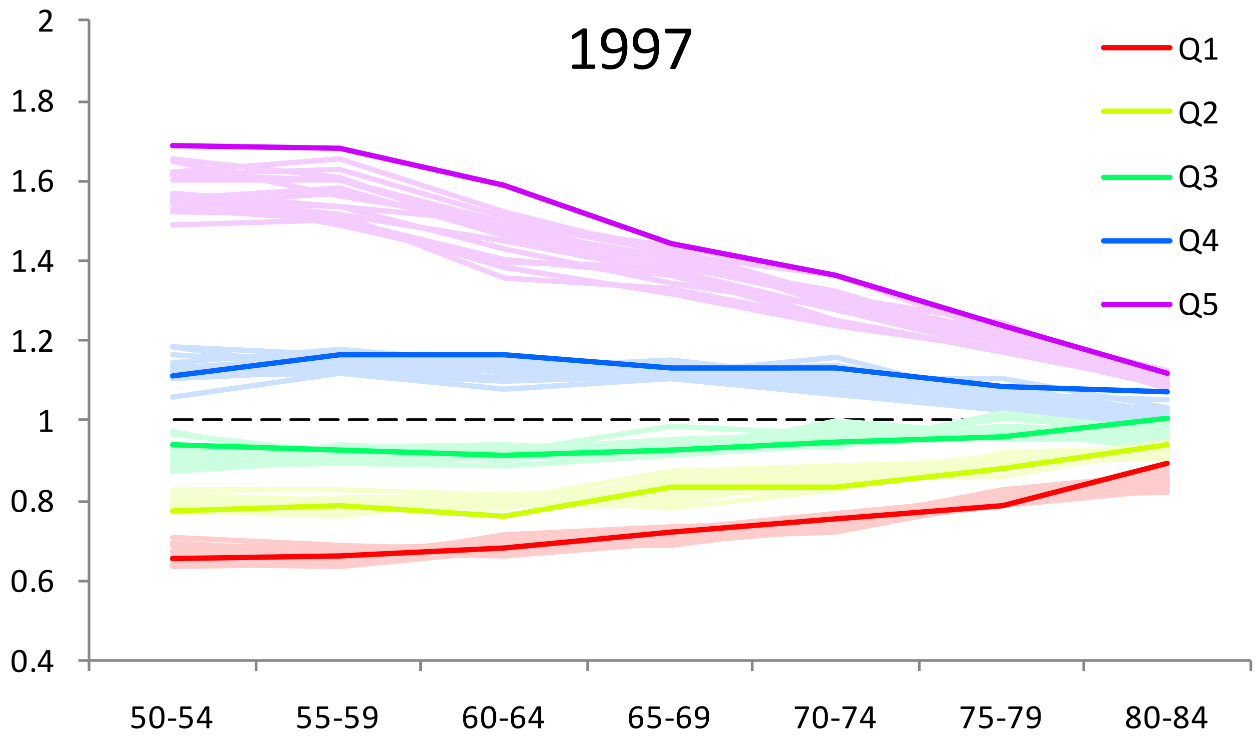
Modelling mortality differentials

Subpopulations model



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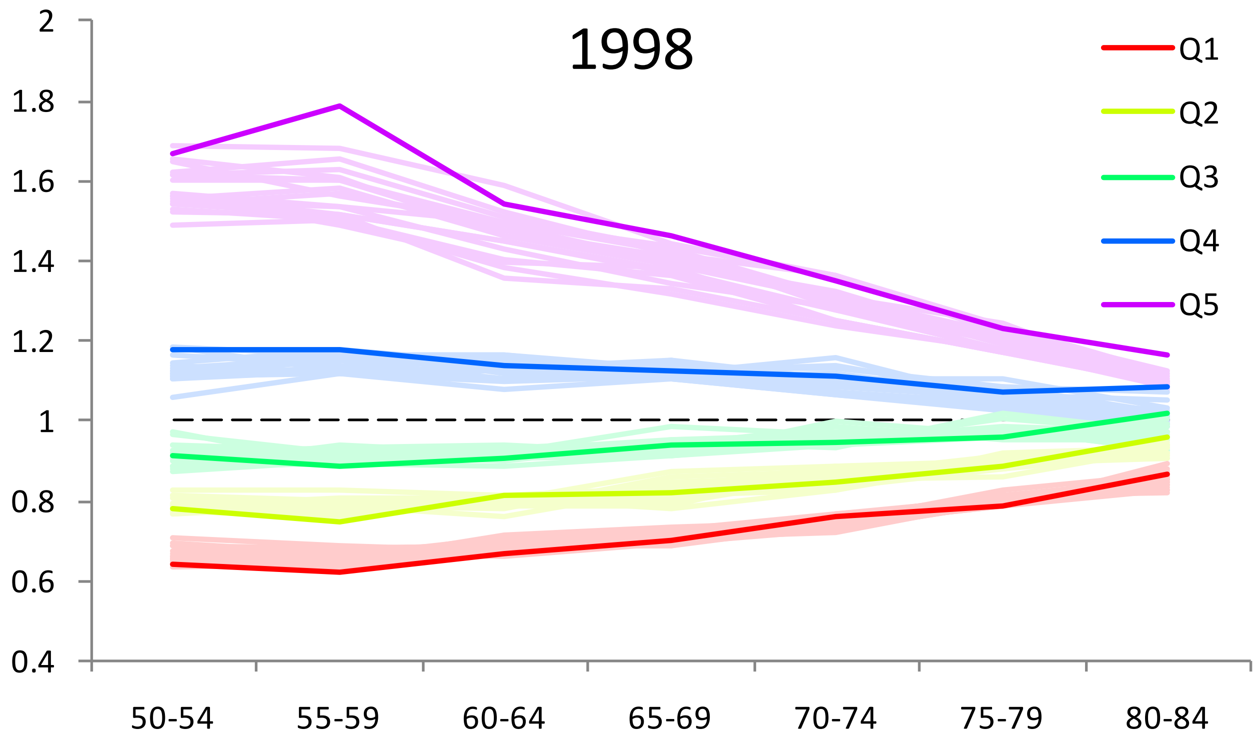
Modelling mortality differentials

Subpopulations model



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Modelling mortality differentials

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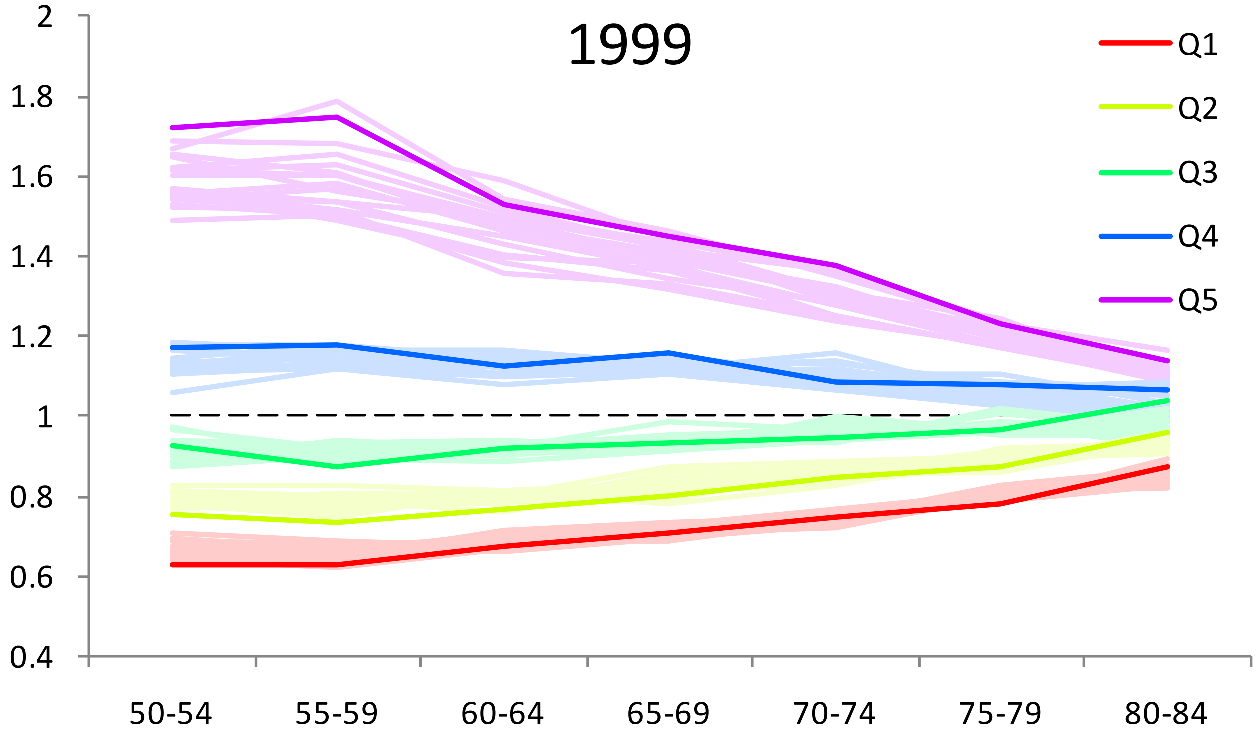

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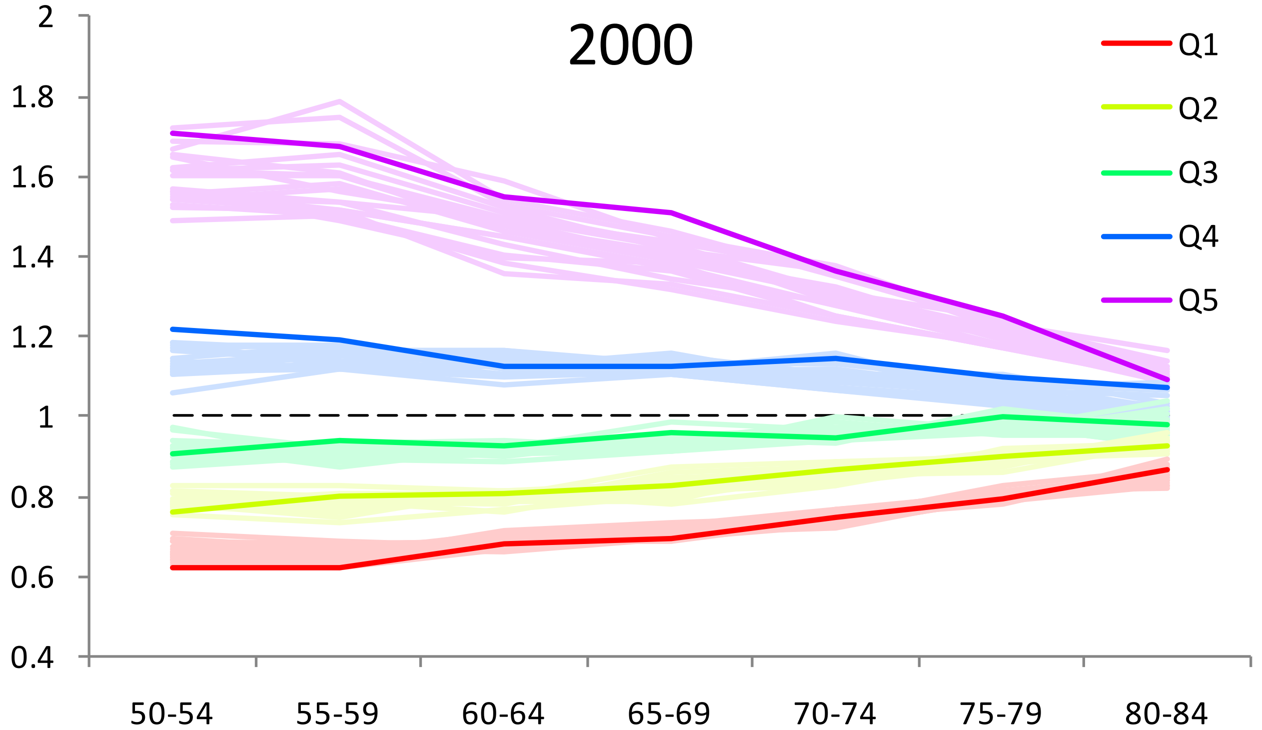
Modelling mortality differentials

Subpopulations model



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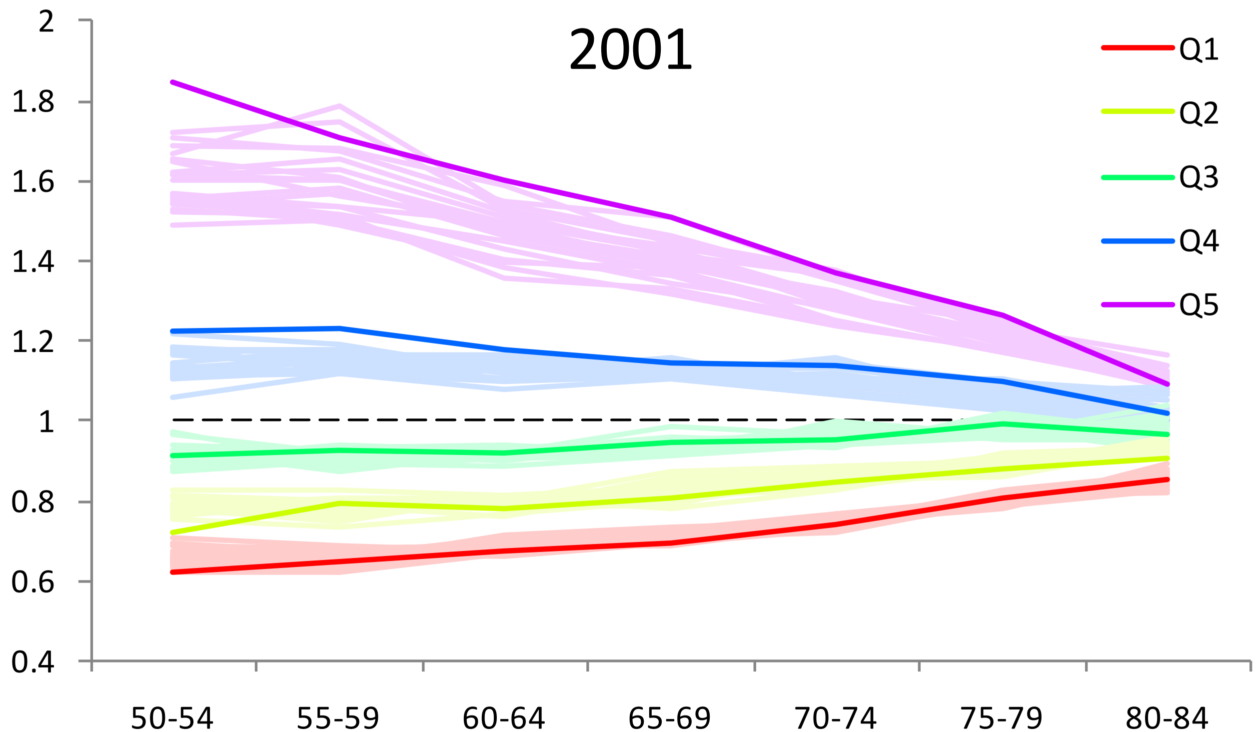
Modelling mortality differentials

Subpopulations model



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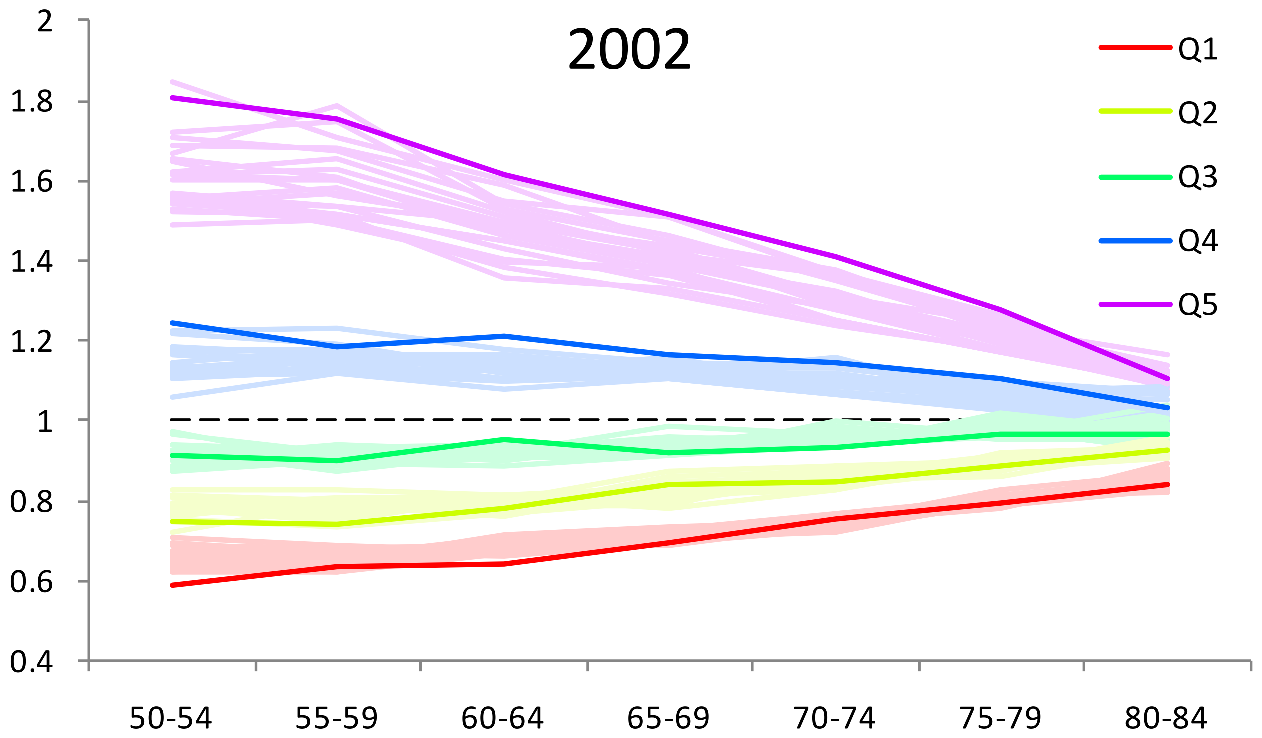
Modelling mortality differentials

Subpopulations model



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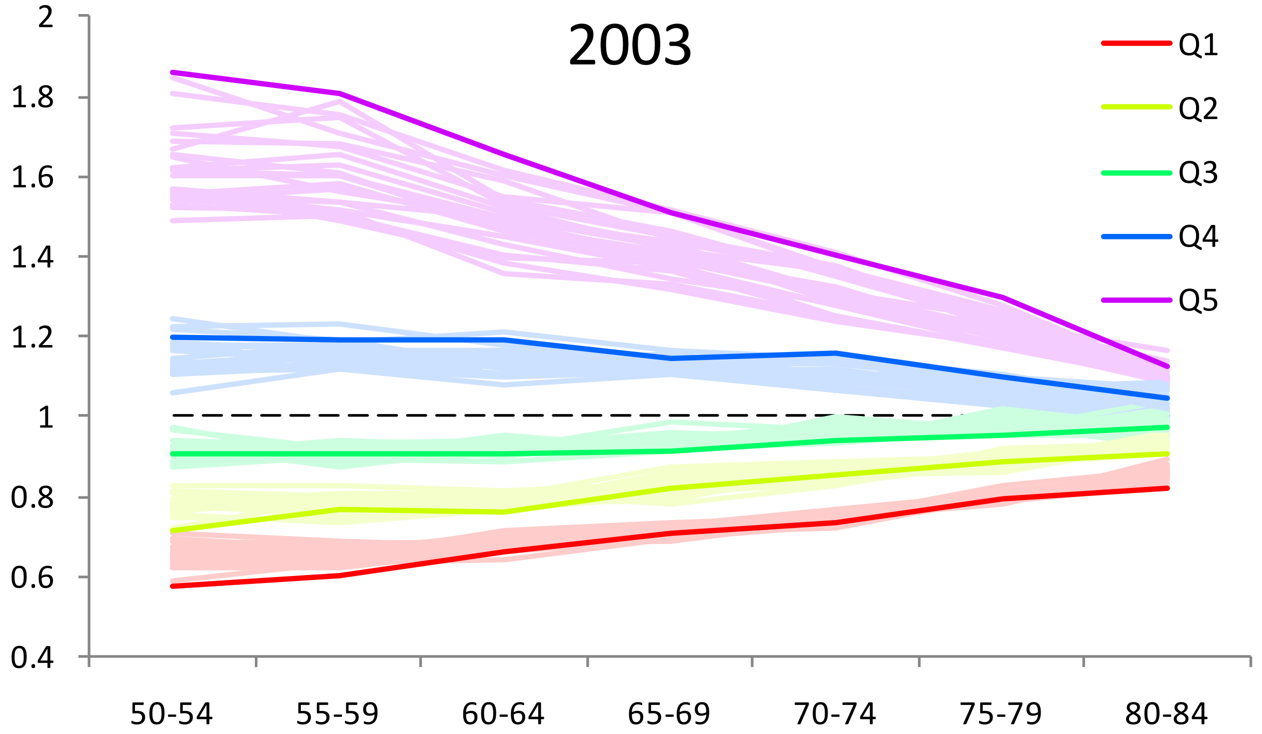
Subpopulations model



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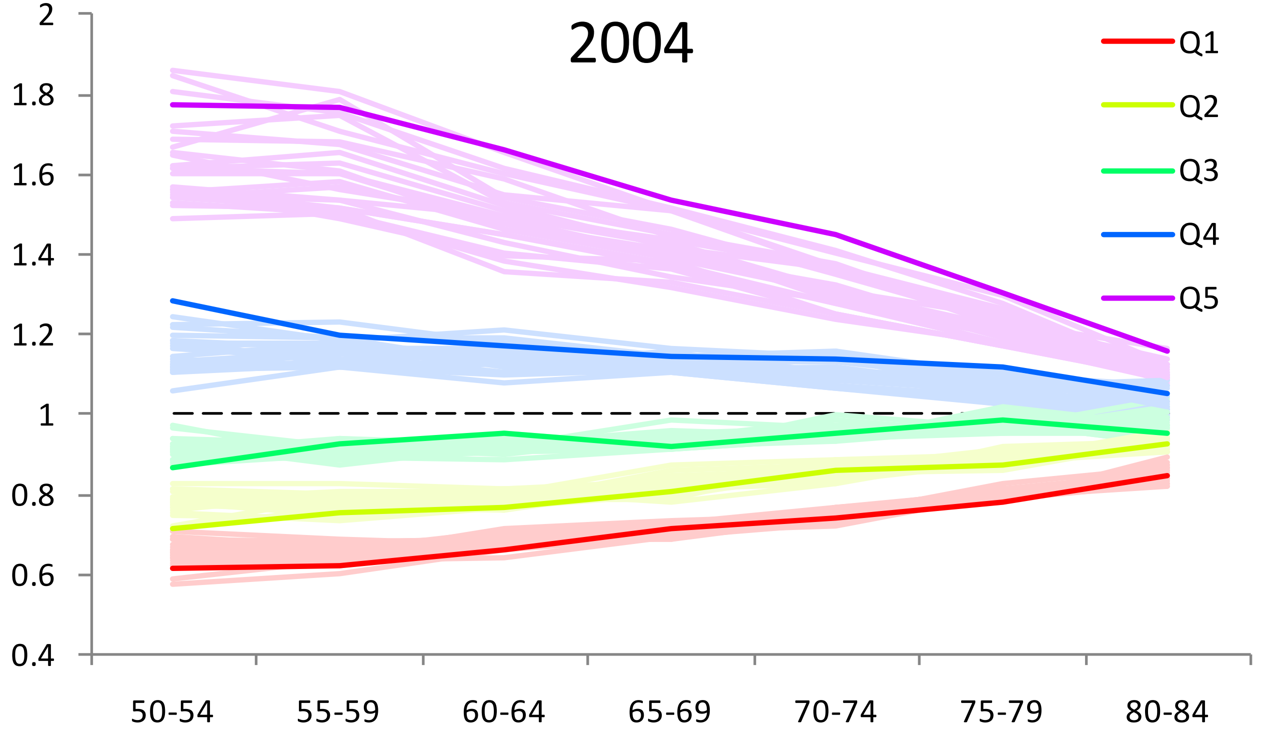
Modelling mortality differentials

Subpopulations model



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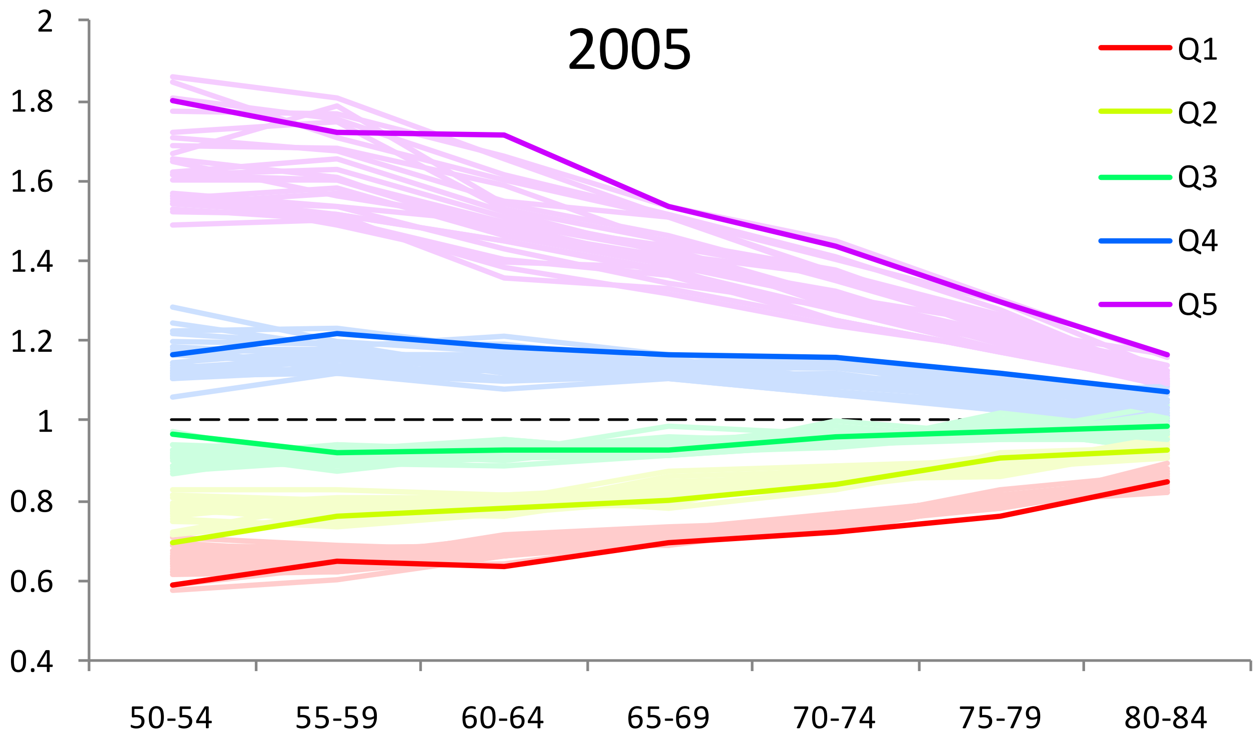
Modelling mortality differentials

Subpopulations model



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Modelling mortality differentials

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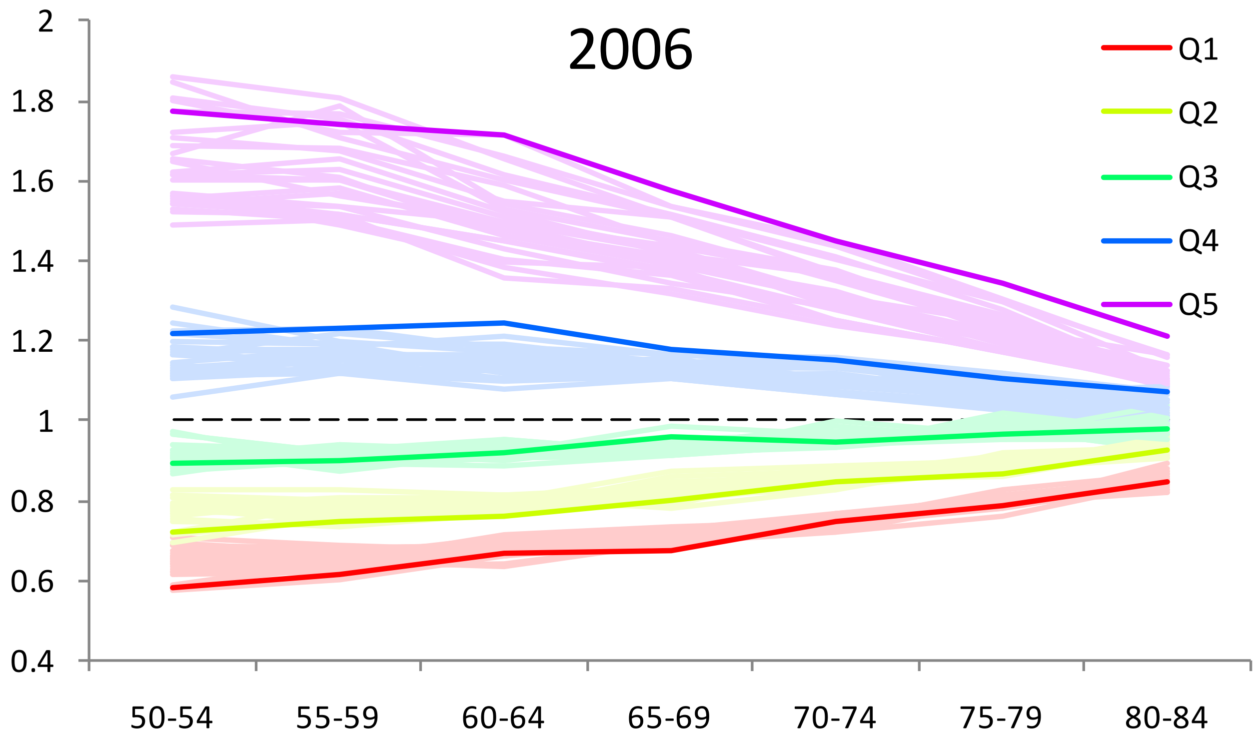

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Modelling mortality differentials

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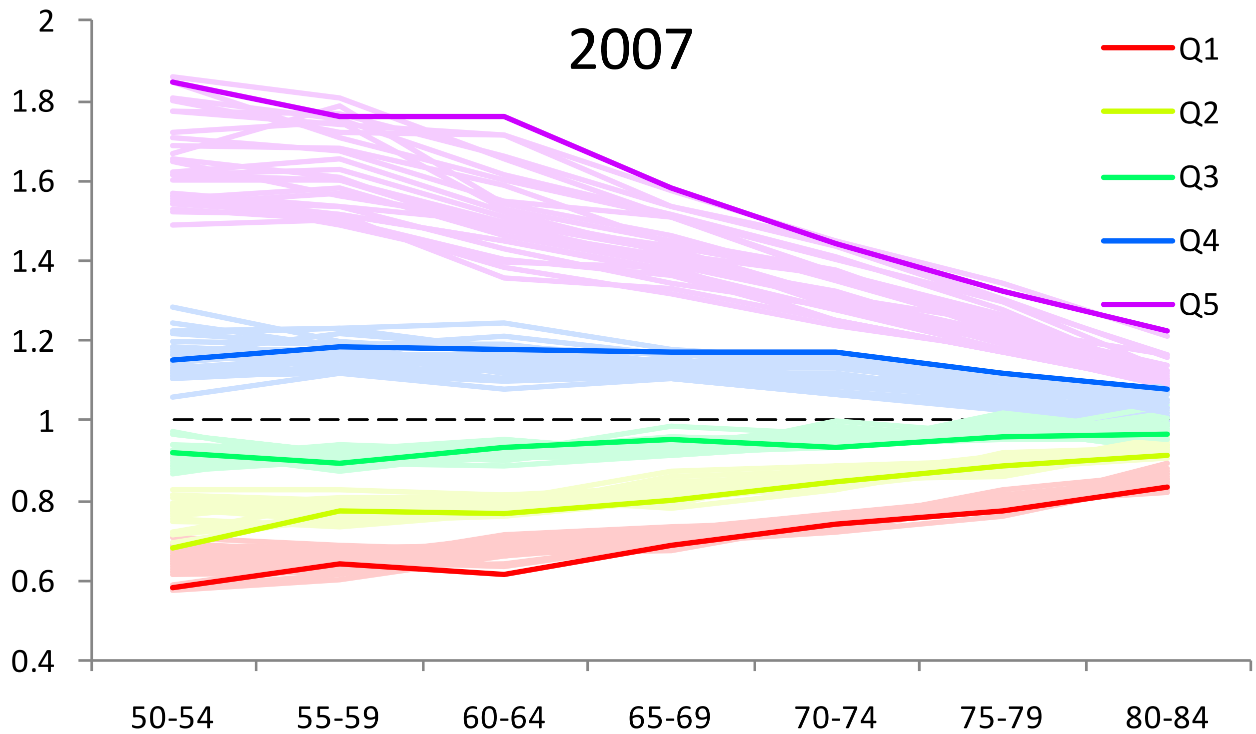

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Modelling mortality differentials

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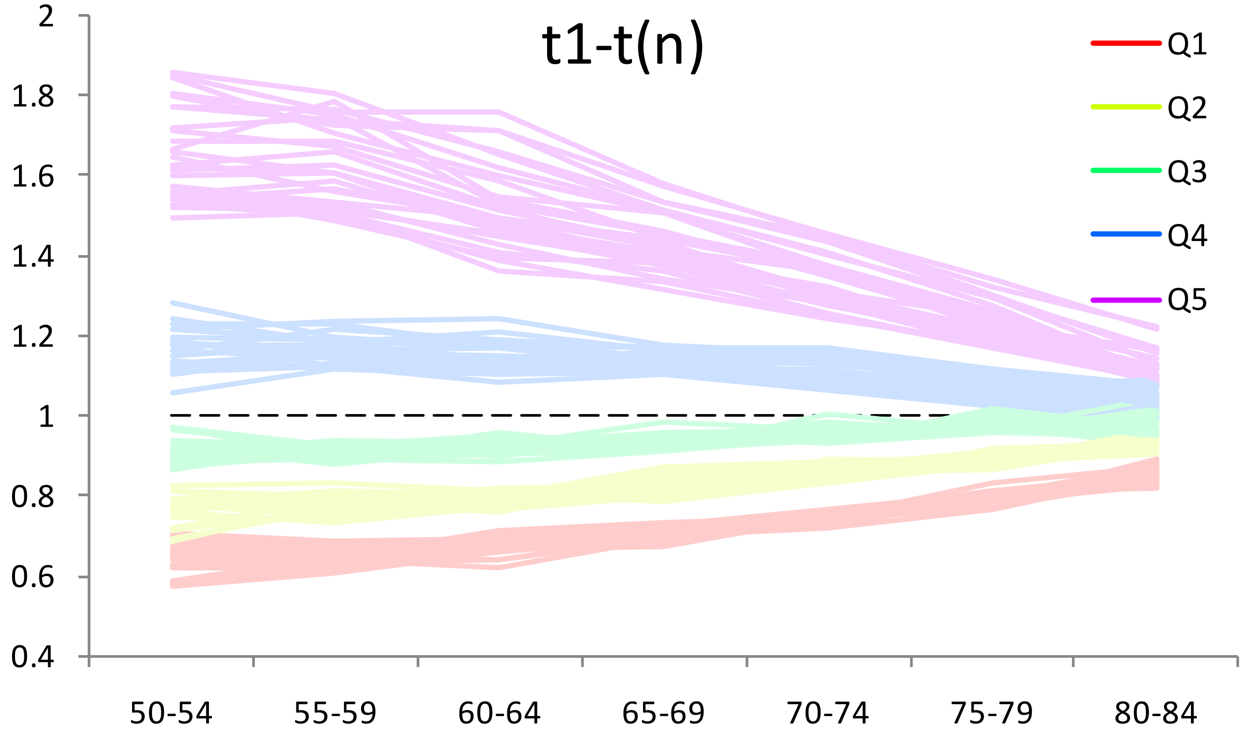

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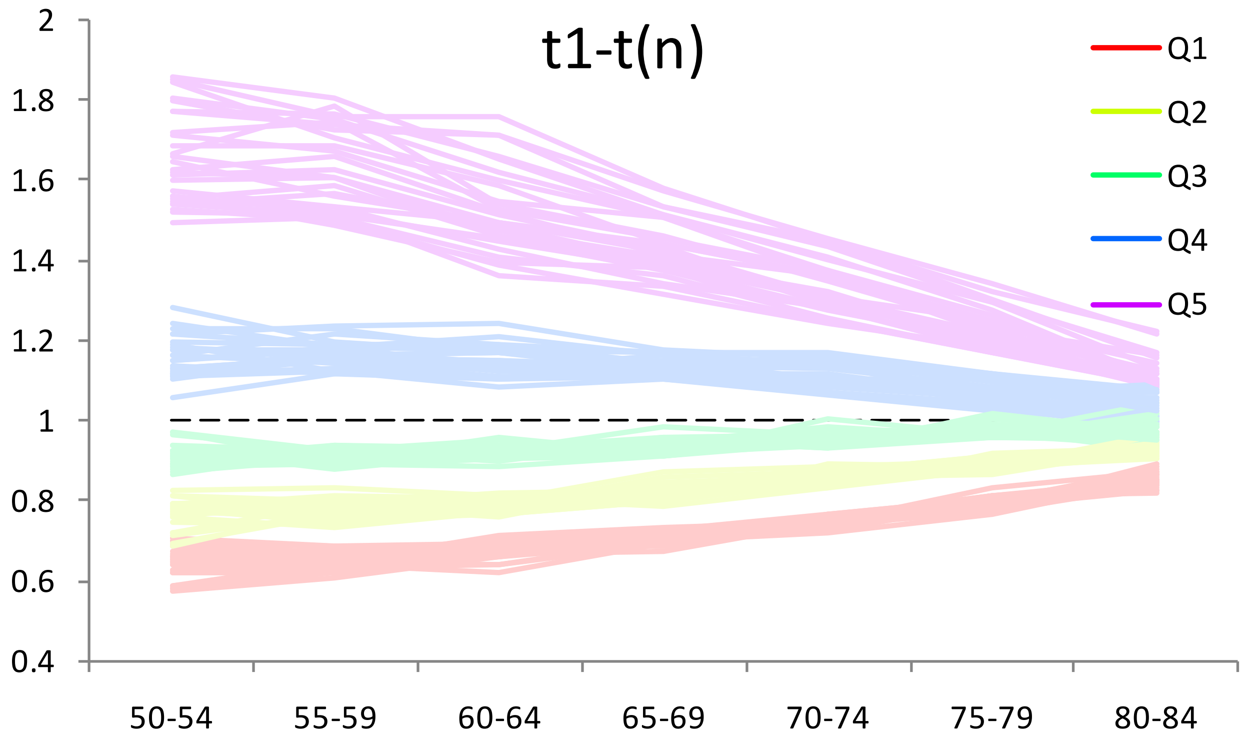
Modelling mortality differentials

Subpopulations model



$$ {}_nD_{xtg} \sim \text{Poisson}({}_ne_{xtg}\, {}_n\mu_{xtg}) $$

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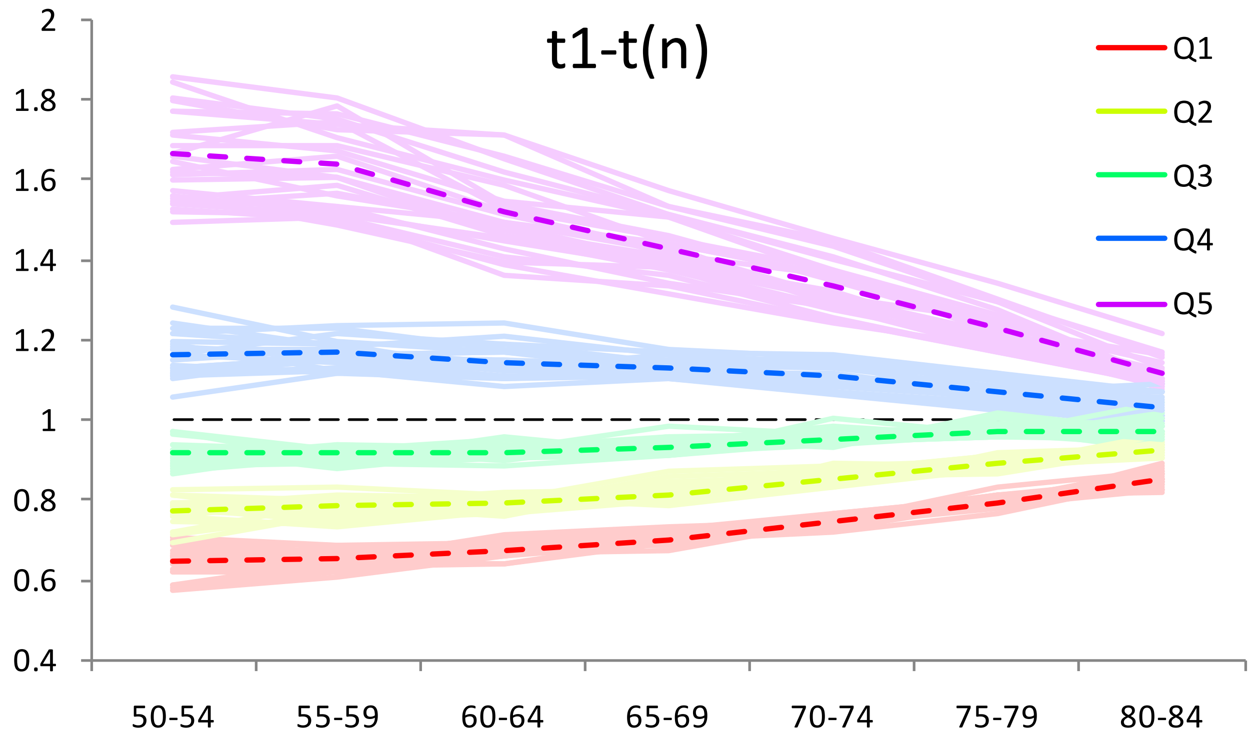
Modelling mortality differentials

Subpopulations model



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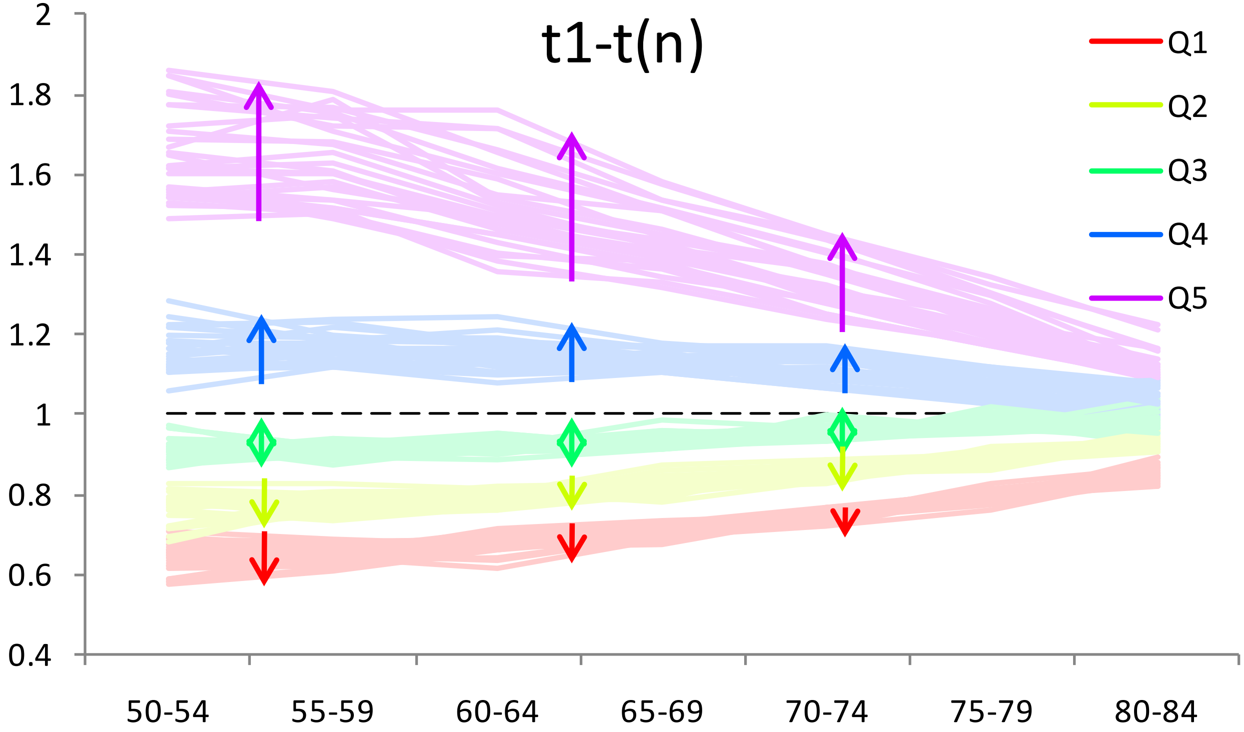
Modelling mortality differentials

Subpopulations model



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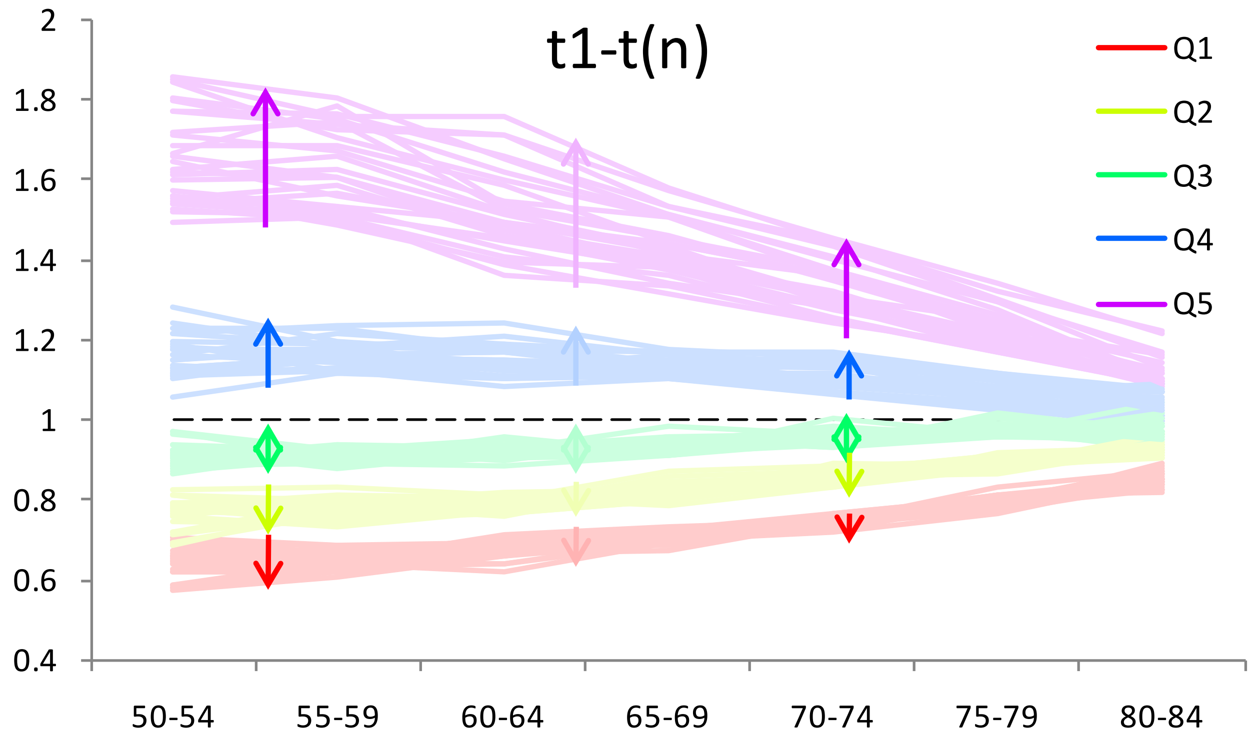
Modelling mortality differentials

Subpopulations model



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Modelling mortality differentials

Summing up

- Reference population

$$D'_{xt} \sim \text{Poisson}(e'_{xt}\mu'_{xt}) \quad \text{with} \quad \mu'_{xt} = \exp\left(\alpha'_x + \beta'_x\kappa'_t + \gamma'_{t-x}\right)$$

- Subpopulations

$${}_nD_{xtg} \sim \text{Poisson}({}_ne_{xtg}\,{}_n\mu_{xtg}) \quad \text{with} \quad {}_n\mu_{xtg} =_n \bar{\mu}'_{xt}\exp\left(\alpha_{xg} + \beta_x\kappa_{tg}\right),$$

where

$${}_n\bar{\mu}'_{xt} = \left(\prod_{i=0}^{n-1}\mu'_{x+i,t}\right)^{\frac{1}{n}} = \exp\left(\frac{1}{n}\sum_{i=0}^{n-1}\left(\alpha'_{x+i} + \beta'_{x+i}\kappa'_t + \gamma'_{t-x-i}\right)\right)$$

Modelling mortality differentials

Model comments and assumptions

- In the subpopulations model
  - The term α_{xg} quantifies level differentials in mortality
  - The terms β_x κ_{tg} quantify trend differentials in mortality
    - Positive trend in κ_{tg} $\leftrightarrow$ slower mortality improvements
    - Negative trend in κ_{tg} $\leftrightarrow$ faster mortality improvements
    - β_x indicate the magnitude of improvement differentials at each particular age
- The age-modulating parameter β_x is subgroup independent
  - This is convenient in the forecasting of mortality in socioeconomic subpopulations where an ordering of mortality levels is natural

Modelling mortality differentials

Model comments and assumptions (cont.)

- If subpopulation g_1 has historically had lower mortality than subpopulation g_2

$$\alpha_{x,g_1} < \alpha_{x,g_2} \quad \text{for all } x \in \mathcal{X}$$

- Then a sufficient condition for maintaining this ordering in the forecasted mortality rates, i.e.,

$$n\mu_{x,t_n+h,g_1} < n\mu_{x,t_n+h,g_2} \quad \text{for } h > 0$$

- Is

$$\beta_x > 1 \quad \text{and} \quad (\kappa_{t_n+h,g_1} - \kappa_{t_n,g_1}) > (\kappa_{t_n+h,g_2} - \kappa_{t_n,g_2}), h > 0$$

- Then the problem of preserving the mortality ordering among the subpopulations reduces to modelling appropriately the multivariate time index κ_{tg}

Modelling mortality differentials

Model comments and assumptions (cont.)

- The cohort effect is the same for all the subpopulations
- These assumptions are reasonable for socioeconomic subpopulations but might not be appropriate for multiple population of a different nature (e.g. countries)

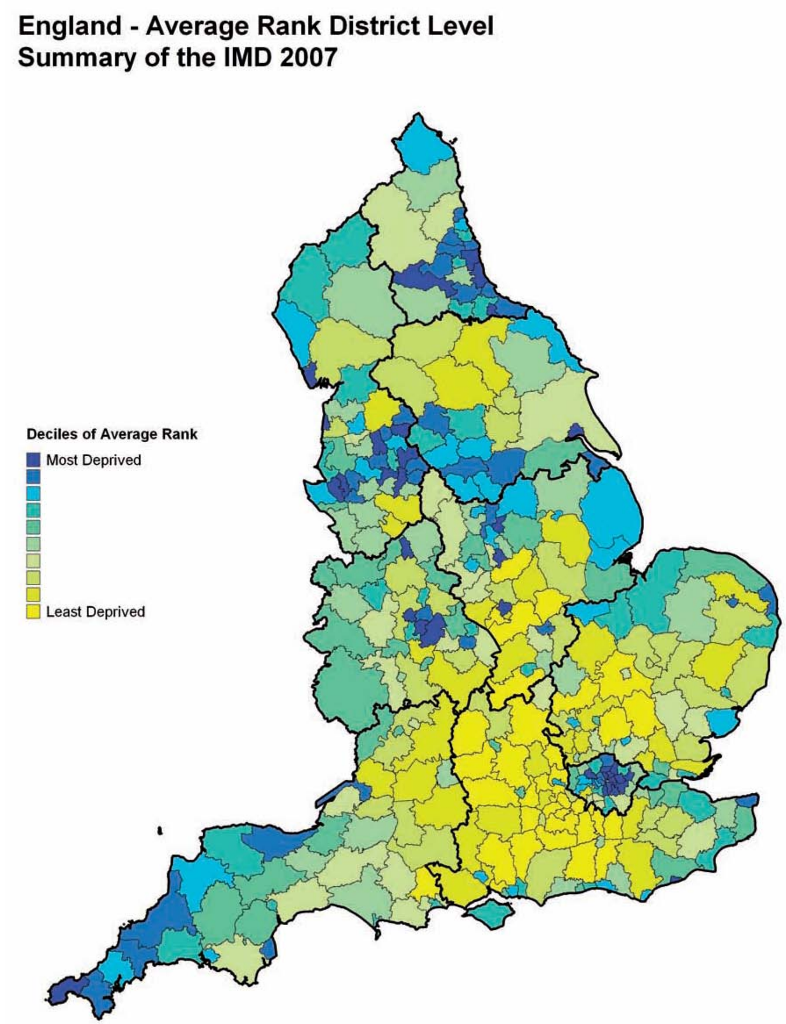
Agenda

- Motivation
- Literature review
- Modelling mortality differentials
- Case study: Mortality by deprivation in England
- Conclusions

Case study: Mortality by deprivation in England

Application data - IMD 2007

- We measure socio-economic conditions using the Index of Multiple Deprivation 2007 (IMD 2007), which combines indicators across 7 deprivation domains into a single deprivation rank
  - Income, employment, health, education, housing and services, crime, and living environment)
- Indicators relate to 2005
- The IMD 2007 is presented at Lower Layer Super Output Area (LSOA)
- SOAs are a geography for the collection and publication of small area statistics
- There are 32,482 LSOA in England with an average population of approximately 1,500 people



Source: Noble et al (2007)

Case study: Mortality by deprivation in England

Application data (Cont…)

Subpopulation data

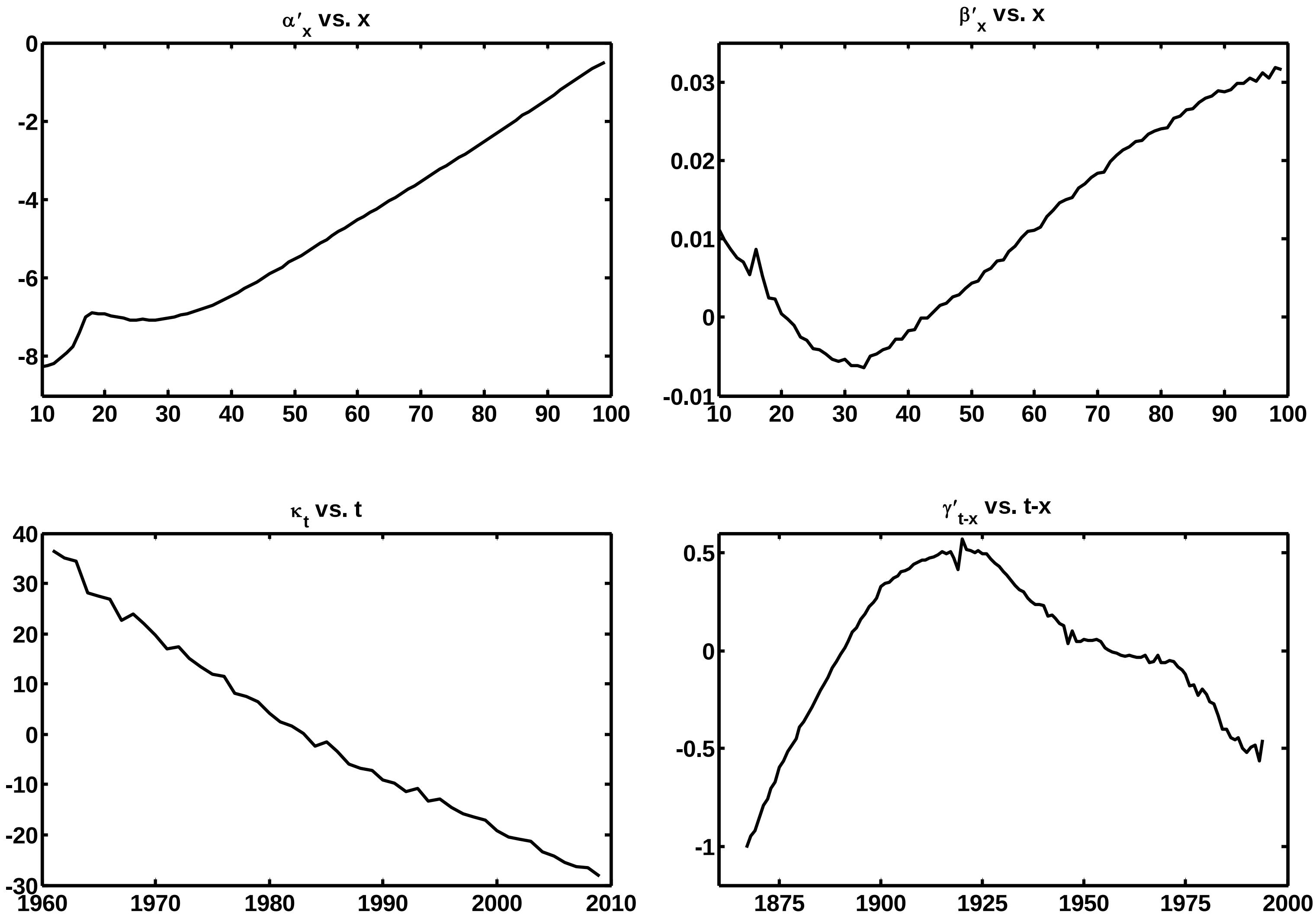
- England male population disaggregated by IMD quintile
- Ages:50-54,55-59,…,80-84
- Period: 1981-2007

Reference population data

- England and Wales male population (Human Mortality Database)
- Ages: 10-99
- Period: 1961-2009

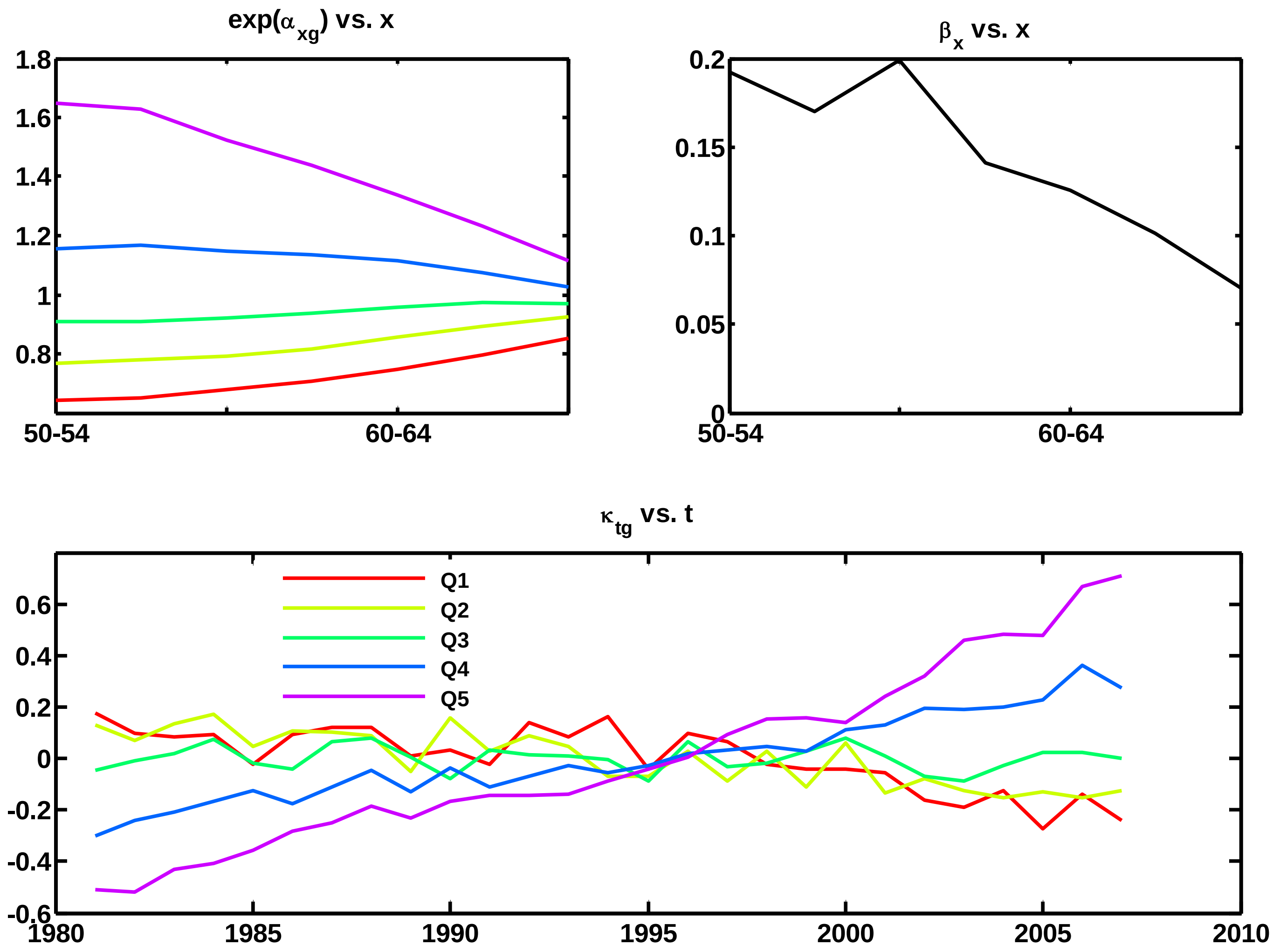
Case study: Mortality by deprivation in England

England and Wales male population parameters



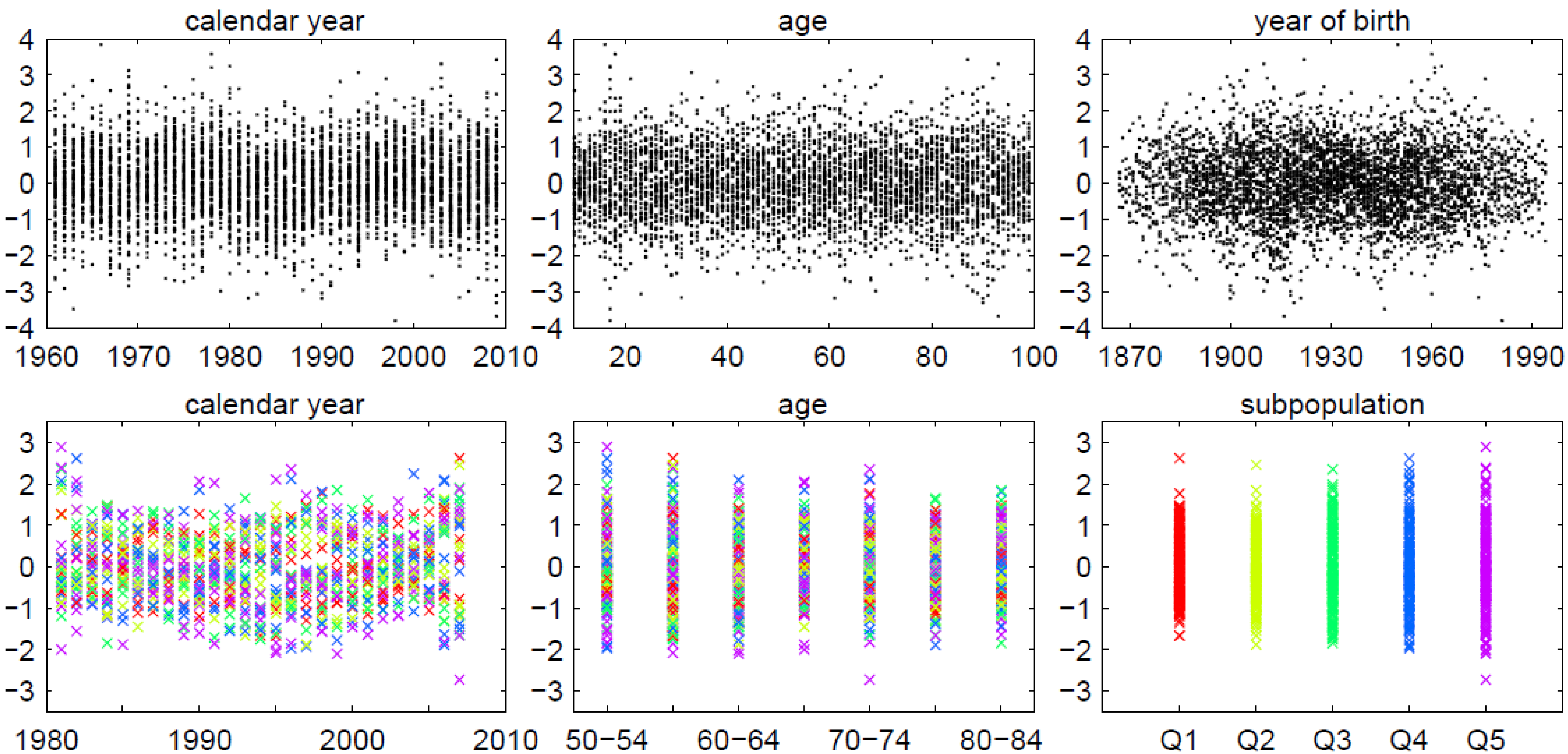
Case study: Mortality by deprivation in England

Male subpopulation parameters



Case study: Mortality by deprivation in England

Deviance residuals



Case study: Mortality by deprivation in England

Male population time index forecasts

- Reference population
  - Random walk with drift with adjusted starting point to tackle the curvature
  - Follow the procedure proposed by Denuit and Goderniaux (2005)

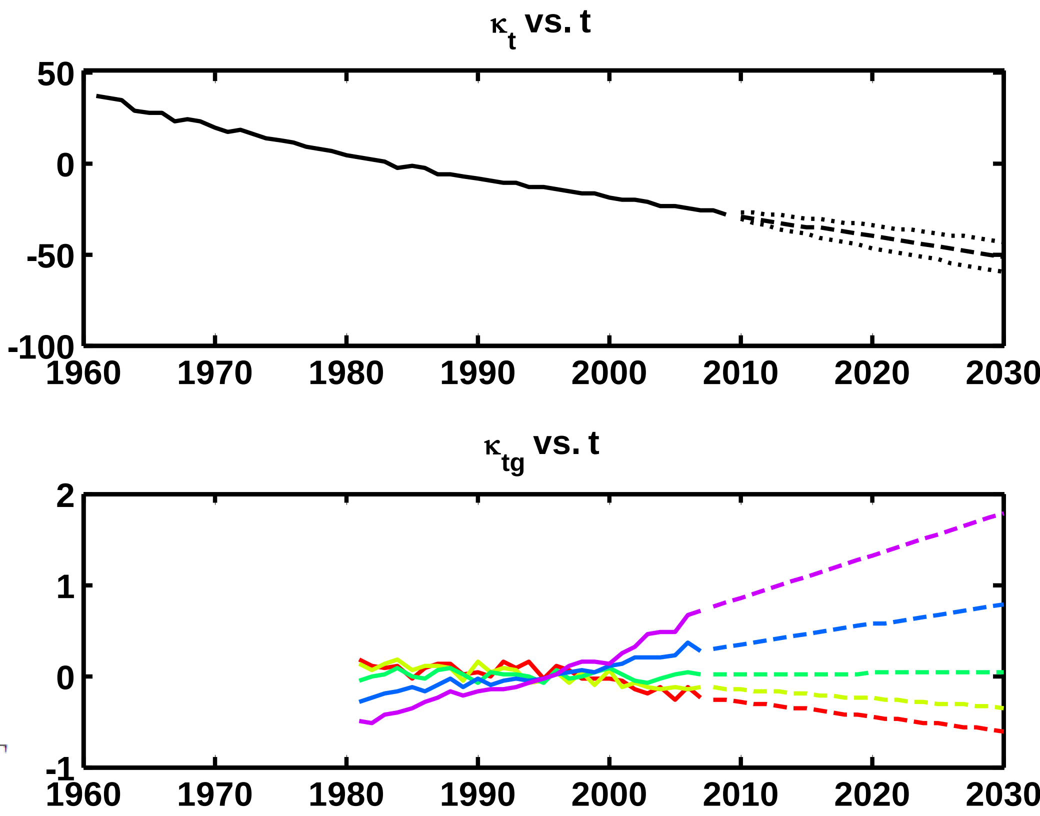
$$k'_t = d' + k'_{t-1} + \epsilon'_t$$

$$\epsilon'_t \sim \mathrm{N}(0, \sigma')$$

- Subpopulations
  - Multivariate random walk with drift
  - Capture the correlation between subpopulations

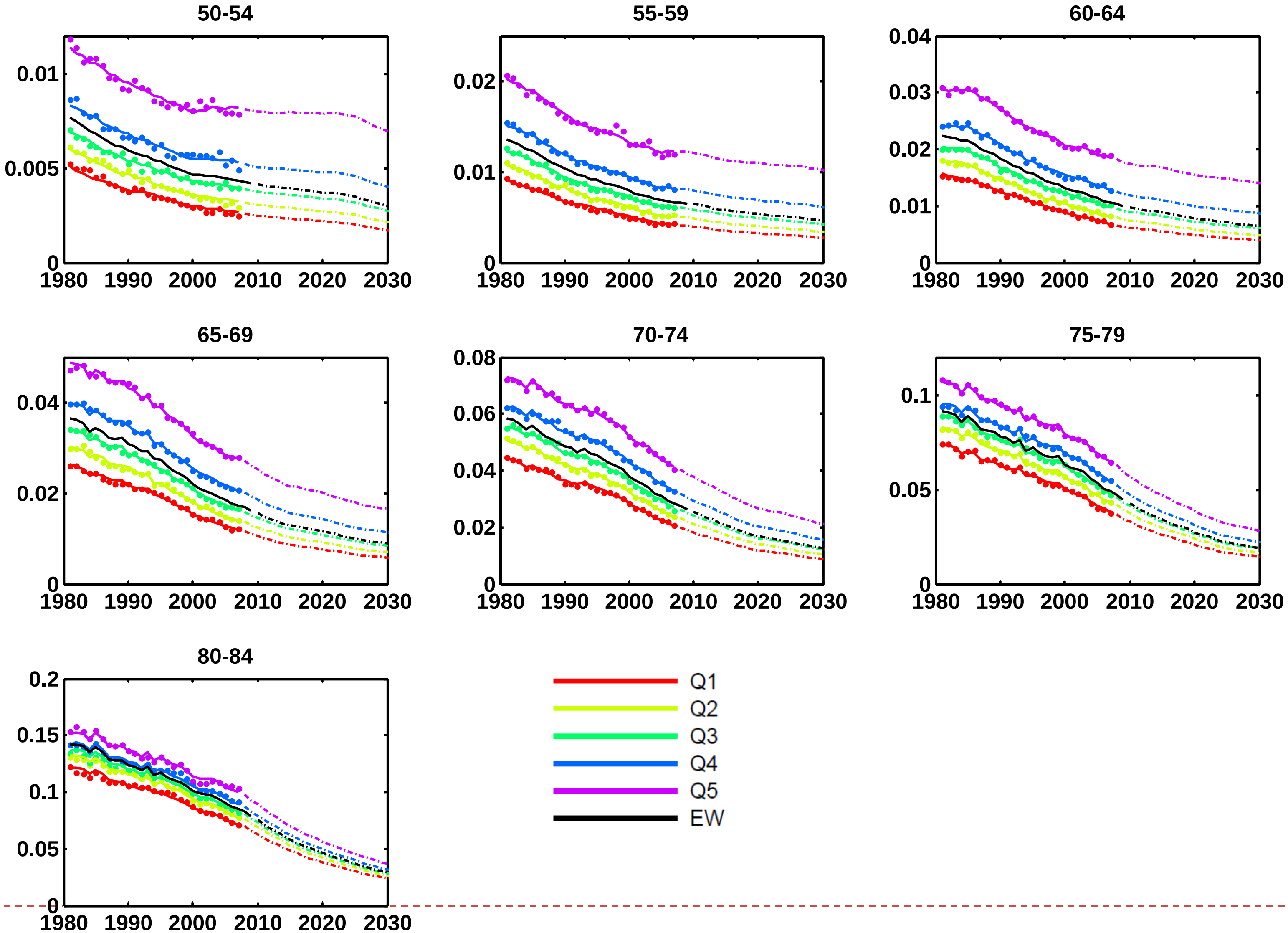
$$\mathbf{k_t} = \mathbf{d} + \mathbf{k_{t-1}} + \epsilon_\mathbf{t}$$

$$\begin{aligned} \mathbf{k_t} &:= (k_{t,g_1}, k_{t,g_2}, \ldots, k_{t,g_m})^T \\ \mathbf{d} &:= (d_1, d_2, \ldots, d_m)^T \\ \epsilon_\mathbf{t} &\sim \mathrm{N}(0, \Sigma) \end{aligned}$$



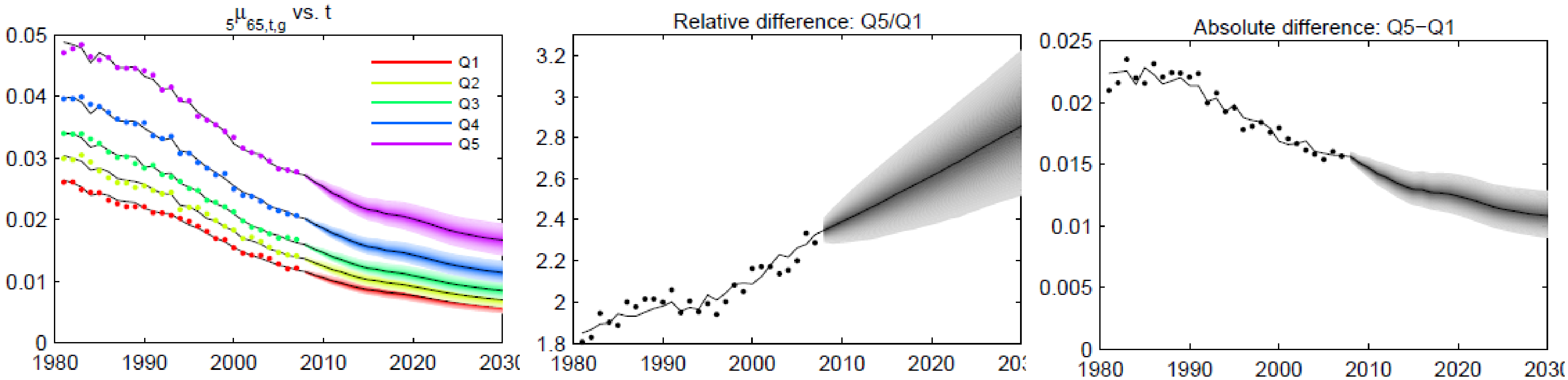
Case study: Mortality by deprivation in England

Male subpopulations death rates forecasts



Case study: Mortality by deprivation in England

Mortality rate differences – Male age 65-69



	1981	1995	2007	2020	2030
Q1	0.0261	0.0197	0.0122	0.0077	0.0058
Q2	0.0299	0.0220	0.0141	0.0092	0.0070
Q3	0.0340	0.0251	0.0168	0.0109	0.0085
Q4	0.0396	0.0308	0.0207	0.0143	0.0114
Q5	0.0471	0.0393	0.0278	0.0201	0.0166
Q5/Q1	**1.80**	**1.99**	**2.28**	**2.61**	**2.86**

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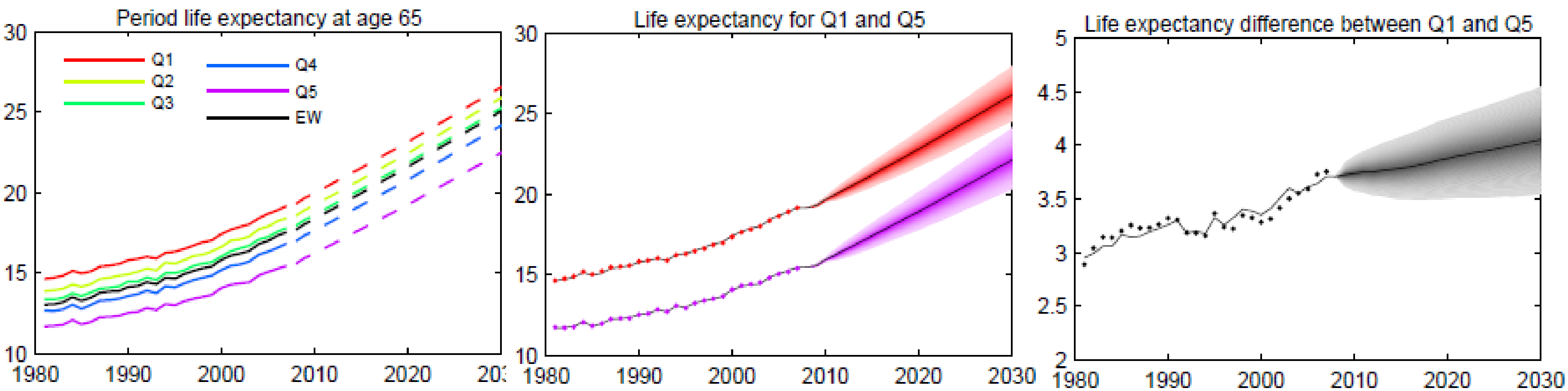
Period life expectancies at male aged 65



	1981	1995	2007	2020	2030
EW	13.1	14.6	17.54	21.6	25.0
Q1	14.6	16.3	19.1	23.1	26.5
Q2	13.9	15.6	18.4	22.4	25.9
Q3	13.4	15.0	17.8	21.8	25.2
Q4	12.7	14.1	16.8	20.8	24.2
Q5	11.7	12.9	15.4	19.3	22.5
Q5/Q1	**2.9**	**3.4**	**3.8**	**3.9**	**4.1**

Case study: Mortality by deprivation in England

Implications for life annuities

- We consider two different sets of assumptions for the calculation of cohort life expectancies and annuity rates (cohort trajectory)

1. Level and trend differences in mortality
   - Use the full forecasting model

$$_n\mu_{x,t_n+j,g} \; = \; _n\bar{\mu}'_{x,t_n+j} \exp\left(\widehat{\alpha}_{xg} + \widehat{\beta}_x \kappa_{t_n+j,g}\right)$$

2. Level differences in mortality but no trend differences
   - Level differences are fixed at their value in 2007
   - Mortality improvement for all the subpopulations follow the behaviour of the improvements in England and Wales

$$_n\mu_{x,t_n+j,g} \; = \; _n\bar{\mu}'_{x,t_n+j} \exp\left(\widehat{\alpha}_{xg} + \widehat{\beta}_x \kappa_{2007,g}\right)$$

Case study: Mortality by deprivation in England

Implications for life annuities

Level immediate annuity rates in 2007 at 4% rate as a percentage of England and Wales rate (cohort trajectory)

Males

		Level and trend					Level only				
Age	**EW**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**
60	15.55	6.1%	3.5%	1.0%	-3.4%	-10.3%	5.8%	3.3%	1.0%	-2.8%	-8.9%
65	13.53	6.9%	3.9%	1.1%	-3.5%	-10.4%	6.6%	3.7%	1.1%	-3.0%	-9.2%
70	11.37	7.4%	3.9%	1.1%	-3.4%	-10.0%	7.1%	3.8%	1.1%	-3.0%	-9.1%
75	8.93	7.7%	3.9%	1.1%	-3.0%	-9.0%	7.4%	3.8%	1.1%	-2.7%	-8.4%
80	6.58	7.0%	3.4%	1.3%	-2.0%	-6.8%	6.9%	3.4%	1.3%	-1.8%	-6.4%

Females

		Level and trend					Level only				
Age	**EW**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**
60	16.28	4.2%	2.3%	0.6%	-2.3%	-7.1%	3.9%	2.1%	0.6%	-1.7%	-5.8%
65 7.8%	14.59	4.6%	2.5%	0.5%	-2.4%	-7.6%	4.2%	9.0%	0.5%	-1.9%	-6.4%
70	12.44	5.1%	2.5%	0.5%	-2.4%	-7.8%	4.7%	2.3%	0.6%	-1.9%	-6.8%
75	9.99	5.0%	2.2%	0.5%	-2.2%	-7.3%	4.8%	2.0%	0.5%	-1.8%	-6.5%
80	7.42	4.4%	1.6%	0.5%	-1.5%	-5.7%	4.2%	1.5%	0.4%	-1.2%	-5.1%

- Significant variation of annuity rates with the level of deprivation
- Greater variation in males than females
- The variability by socioeconomic conditions can be greater than the variability by sex
- Although mortality differentials decrease with rising age their impact on annuity rates does not
- The impact of ignoring improvement differences is in general small
  - Except for younger ages in the most deprived quintile

Agenda

- Motivation
- Literature review
- Modelling mortality differentials
- Case study: Mortality by deprivation in England
- Conclusions

Conclusions

- Lee-Carter based approach for the modelling of mortality in a set of socioeconomic subpopulations
  - Quantification of level and trend differences in mortality
  - Forecast of subpopulation-specific mortality rates that preserve the inverse relationship socioeconomic variables and mortality
- Application in the analysis of the extent of mortality differentials across deprivation subgroups in England for the period 1981- 2007.
  - Clear inverse relationship between area deprivation and mortality
  - Widening (relative) mortality gap between more and less deprived areas
- Socioeconomic differences in mortality have a material impact on the valuation of annuities
  - This impact is still significant at old ages
  - Differentials in baseline mortality have a significant impact
  - Differentials in rates of improvement have a less significant impact
- Future work:
  - Modelling of mortality disaggregated by socioeconomic variables and cause of death

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- Thanks to Madhavi Bajekal from UCL for helping us gain access to the deprivation specific mortality data and for the very good discussions on SES differences in mortality
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Thank you!

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Reserve Slides

Case study: Mortality by deprivation in England

Application data - IMD 2007

- We measure socio-economic conditions using the Index of Multiple Deprivation 2007 (IMD 2007), which combines indicators across seven domains into a single deprivation rank

	Domain Weight
Income deprivation	22.5 %
Employment deprivation	22.5%
Health deprivation and disability	13.5%
Education, skills and training deprivation	13.5%
Barriers to housing and services	9.3%
Crime	9.3%
Living Environment deprivation	9.3%

Source: Noble et al (2007)

Case study: Mortality by deprivation in England

Limitations

- The IMD 2007 used to measure socio-economic status has as one of its components a measure of health deprivation and disability
- We have used an ecological measure of socio-economic deprivation instead of individual level socio-economic measures (Ecological Fallacy)
- It is plausible that healthier people will tend to move from more deprived areas to less deprived ones and that less healthy people remain at home, resulting in a potential bias toward higher mortality inequalities
- Our analysis is based on quintile classification of LSOAs using the IMD 2007 which refers to area indicators in 2005. We have assumed time stability in the membership of the LSOAs in the deprivation subgroups.