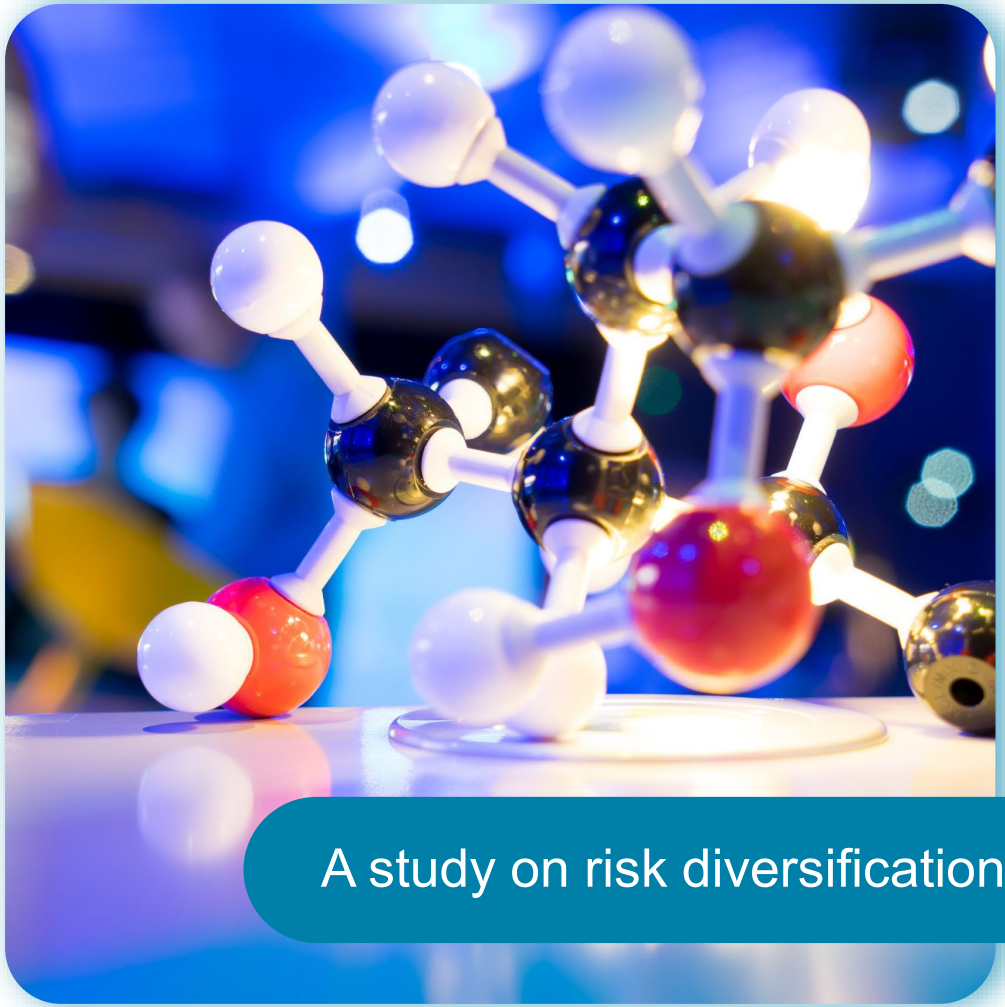


Prospective mortality modelling by cause of death

Agne Ulcinaite and Julien Tomas
SCOR – Sweden Re
07/09/2023

A study on risk diversification according to the dependency structure

LONGEVITY LONDON 2023 18

SCOR SWEDEN RE

Prospective mortality modelling by cause of death
Agenda

1 Context

2 Cause of death mortality: assumptions on the dependency structure

3 Prospective scenarios on Alzheimer's and dementia diseases

4 Impact on life expectancy

5 Impact on risks diversification

6 Conclusion

Context
Why do we need prospective scenarios ?

“What-if” type of scenarios are required by both regulators and internal stakeholders.

Regulators
Using stress test scenarios, regulators seek to:

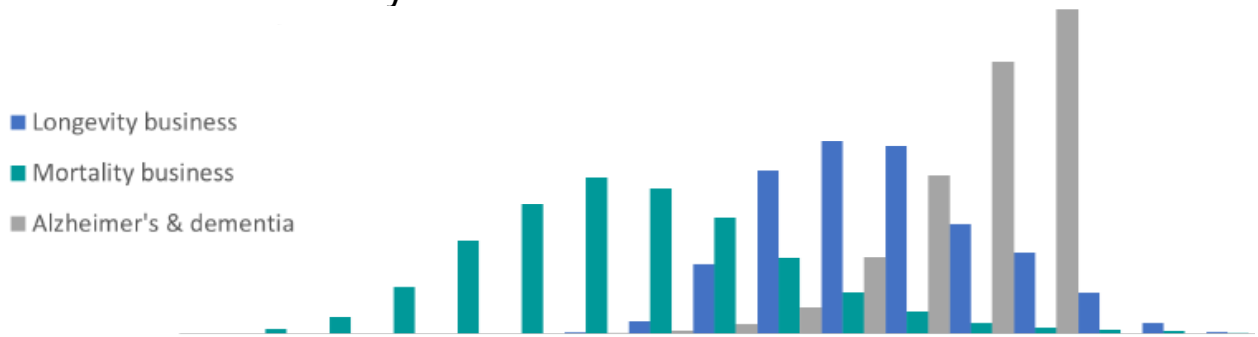
- validate internal model,
- identify probable crisis situations that would threaten viability of the company.

Internal stakeholders
Senior management is also particularly interested in probable scenario analysis which permits:

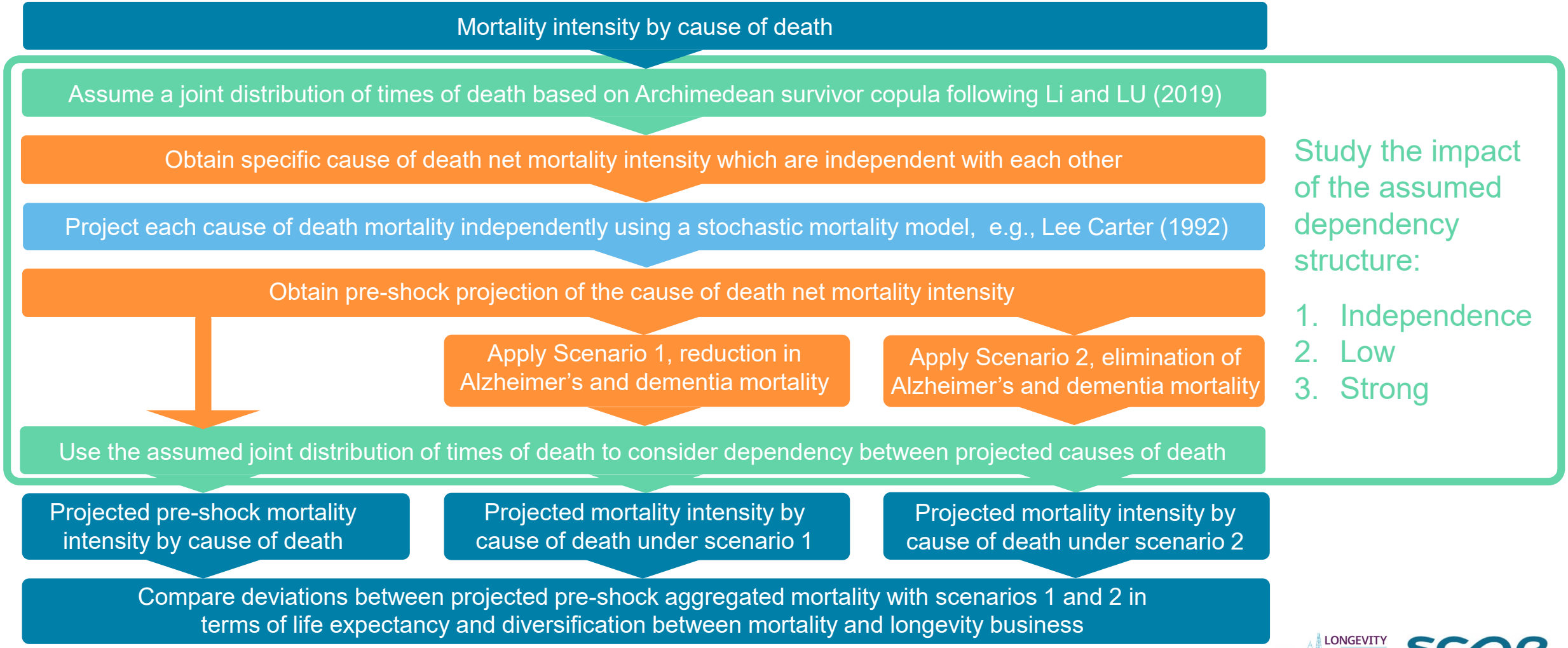
- assessing business resilience to shocks,
- supporting business acceptance and risk appetite decisions,
- evaluating portfolio diversification impact.

The goal of the scenario is to test diversification impact between mortality and longevity business.

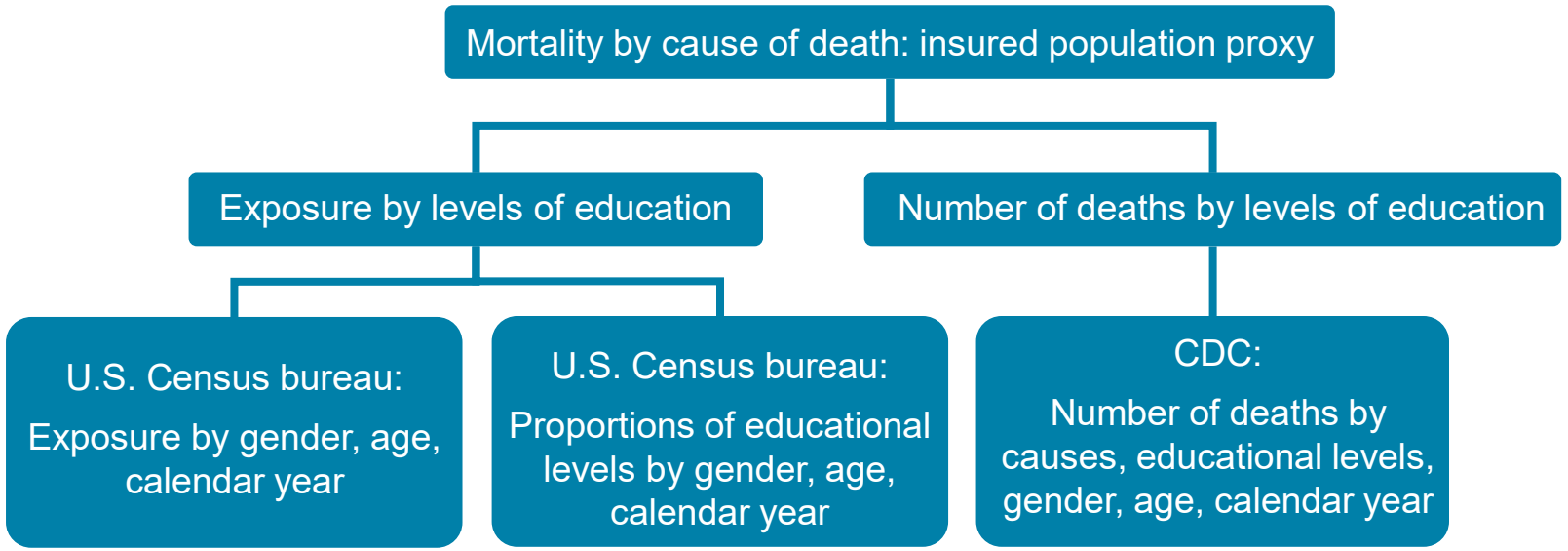
Scenario about future trends of Alzheimer’s and dementia diseases is likely to have important cumulative impact from both longevity and mortality books and not to totally offset one another.



Context
Illustration of the approach



Context
Application on U.S. cause of death data



Cause of deaths	ICD-10 codes
Alzheimer's and dementia	**F01, F03, G20-21, G30**
Neoplasms	C00-C97
Circulatory system diseases	IOO-I99
Respiratory diseases	J00-J98, U04
External causes	U01, V01-Y84
others	All others not included

Working with granular data :

- allows mortality modeling at a very granular level
- allows to build hypothetical scenarios by cause of death
- takes into account differences in the distribution of causes of death between different subpopulations
- allows expert judgment to be applied on future trends by cause

But it has some disadvantages:

- changes in classification
- arbitrary declaration of the primary cause at advanced ages
- time series are rather short
- trends different at some ages for a same cause
- dependency between causes (framework of competing risks) complexify the study

Mortality intensities and joint survival times
Crude mortality intensity

Each individual in a population is assumed to be exposed to m causes of death and may die from any one of these causes. The total lifetime of an individual, T, is given by the minimum of the m cause-specific lifetimes as:

$$T = \min(T_1, \dots, T_m).$$

In the competing risk framework, the observed cause of death is then the one corresponding to the minimum of the m stochastic lifetimes associated with the causes of death.

The all-causes (aggregate) mortality intensity is the instantaneous probability of death before time $t + u$ for an individual who already lived t years for small interval u:

$$\mu(t) = \lim_{u \to 0} \frac{\mathbb{P}(T \leq t + u | T > t)}{u}.$$

For a specific cause, the crude mortality intensity is :

$$\mu_j(t) = \lim_{u \to 0} \frac{\mathbb{P}(T \leq t + u, J = j | T > t)}{u},$$

and $\mu_j(t), j = 1, \dots, m$, sum up to the aggregate mortality intensity: $\mu_1(t) + \cdots + \mu_m(t) = \mu(t)$.

Mortality intensities and joint survival times
Net cause-specific intensities

The net survival function of cause T_j is the survival if the risks of death other than the cause j were removed,

$$S_j(t) = \mathbb{P}[T_1 > 0, \dots, T_j > t, \dots, T_m > 0] = \exp\left(-\int_0^t \lambda_j(s)ds\right),$$

where $\lambda_j(t)$ is the net cause-specific intensities of T_j. When studying a hypothetical scenario on a cause of death j, the net cause-specific intensities $\lambda_j(t)$ can be modified to reflect the excess or deficit mortality resulting from adverse events or future medical innovations affecting this specific cause. It is defined by

$$\lambda_j(t) = \lim_{u \to 0} \frac{\mathbb{P}(T_j \leq t + u | T_j > t)}{u} = -\frac{\mathrm{d}}{\mathrm{d}t} \log S_j(t)\,.$$

However, the cause-specific $\mathbb{P}(T_j \leq t + u | T_j > t)$ cannot be, in general, estimated from data as only $\mathbb{P}(T \leq t + u, J = j | T > t)$ is observed. In estimating the net mortality intensity, the joint distribution of the survival times $(T_1, \dots, T_m)$ denoted by $S(t_1, \dots, t_m)$ should then be considered:

$$S(t_1, \dots, t_m) = \mathbb{P}[T_1 > t_1, \dots, T_m > t_m].$$

The joint distribution of the survival times is related to the crude cause-specific mortality intensities:

$$\mu_j(t) = -\frac{\vartheta}{\vartheta t_j} \log \mathbb{P}[T_1 > t_1, \dots, T_m > t_m] \,|_{t_1 = \dots = t_m = t}\,. \tag{1}$$

Modeling mortality scenarios using Archimedean survivor copula
Li and Lu (2019)

The approach assumes that the survival times $(T_1, \dots, T_m)$ have a joint Archimedean survivor copula. The joint distribution writes:

$$\mathbb{P}[T_1 > t_1, \dots, T_m > t_m] = \psi\left(\psi^{-1} \circ S_1(t_1) + \cdots + \psi^{-1} \circ S_m(t_m)\right), \qquad \forall t_1, \dots, t_m > 0,$$

where the symbol $\circ$ represents the composition of functions and ψ the generator function. In the numerical applications, the Clayton copula is used.

The Clayton copula is obtained by assuming $\psi(t) = (1+t)^{-1/\theta}$ where θ is a parameter that captures the dependence. The higher the value of θ, the stronger positive dependence between the survival times. When θ approaches 0, the copula reduces to the independent copula. In a Clayton copula, the joint distribution of the survival times is

$$S(t_1, \dots, t_m) = \left[S_1(t_1)^{-\theta} + \cdots + S_m(t_m)^{-\theta} - m + 1\right]^{-1/\theta}. \tag{2}$$

If the joint survivor copula is Archimedean with generator ψ, Li and Lu (2019) have shown that the net survival function can be determined by the copula and the crude cause-specific mortality intensities:

$$S_j(t) = \psi\left[-\int_0^t \frac{\exp\left(-\int_0^t \sum_{i=1}^m \mu_i(u)\,\mathrm{d}u\right)}{\psi' \circ \psi^{-1} \circ \exp\left(-\int_0^t \sum_{i=1}^m \mu_i(u)\,\mathrm{d}u\right)} \mu_j(s)\mathrm{d}s\right], \qquad \forall j = 1, \dots, m. \tag{3}$$

Modeling mortality scenarios using Archimedean survivor copula
Li and Lu (2019)

The procedure of estimating the net mortality intensities and applying modeling mortality scenarios is:

1. The crude mortality intensities $\mu_{j,c,t}$ for each cause of death j, cohort c and calendar year t are obtained by

$$\mu_{j,c,t} = \frac{D_{j,c,t}}{E_{j,c,t}}, \qquad \forall j = 1, \dots, m,$$

   where $D_{j,c,t}$ and $E_{j,c,t}$ are the corresponding number of death and exposure, respectively.

2. The marginal intensities are derived from the net survival functions $S_{j,c}(t)$:

$$\lambda_{j,c,t} = -\log \frac{S_{j,c}(t)}{S_{j,c}(t-1)},$$

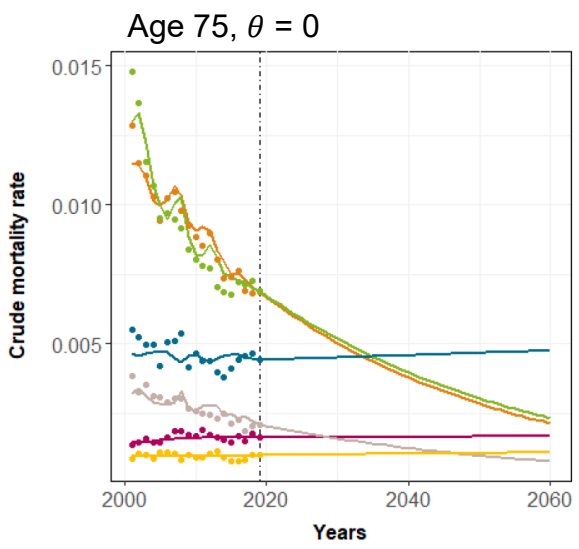
   where the marginal survival function $S_{j,c}(t)$ are obtained from the crude intensity of each cohort using Equation (1).

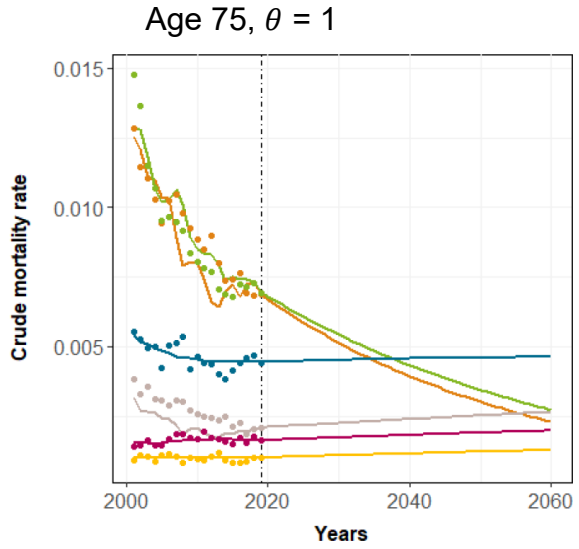
3. The Lee and Carter (1992) model is used to forecast the pre-shock marginal intensities for each cause of death separately.

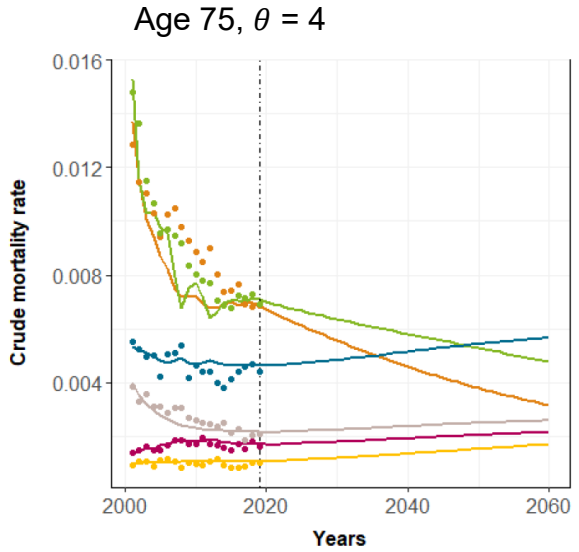
4. Scenarios 1 and 2 are applied on the marginal Alzheimer's and dementia mortality intensity.

5. Lastly, after projecting the net intensities and applied a shock of the net Alzheimer's and dementia mortality intensity, the reverse reasoning is applied to recover the corresponding post-shock crude intensities using Equations (2) and (3). The latter are then used to obtain the aggregate future mortality improvement resulting from the scenario.

Cause of death mortality: assumptions on the dependency structure
Comparison in terms of residual life expectancy







Residual life expectancy (years)				
Age 55	Year 2001	Year 2019	Year 2040	Year 2060
$\theta = 0$	26	29	30.9	32.1
$\theta = 1$	26	29	30.1	30.7
$\theta = 4$	26	29	29.3	29

Age 75	Year 2001	Year 2019	Year 2040	Year 2060
$\theta = 0$	10.1	11.8	13	13.9
$\theta = 1$	10.1	11.8	12.3	12.4
$\theta = 4$	10.1	11.8	11.6	10.8

Prospective scenarios on Alzheimer's and dementia diseases
Basis of the scenarios

Early risk identification

- ✓ Neuroimaging, applications of deep learning, and other AI methods
- ✓ Genetic profiling,
- ✓ Identification of new biomarkers,
- ✓ Improving identification of functional and cognitive performance

Progress in prevention measures

- ✓ Interventions enhancing or maintaining the cognitive reserve
- ✓ Interventions targeting modifiable risk factors for dementia

Progress in treatments

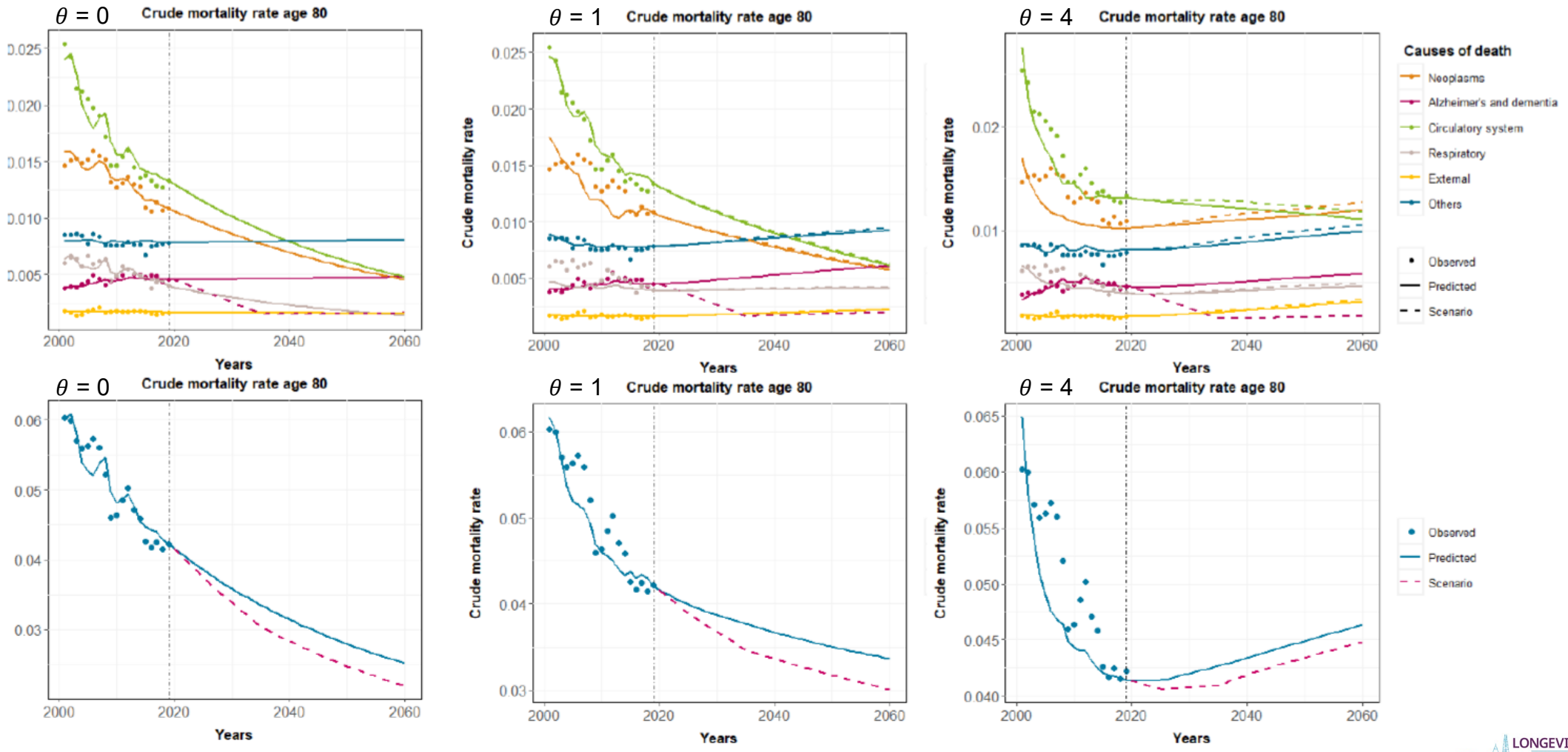
- ✓ Tau-directed therapies
- ✓ Anti-neuro inflammatory drugs
- ✓ Antioxidants,
- ✓ Stem cell therapies,
- ✓ Drugs' repositioning and repurposing

Scenario 1: A reduction in Alzheimer's and dementia mortality due to success in delaying onset and slowing deterioration Mortality decreases by 66% over the next 15 years. After mortality remains at 33% of its pre-shock estimate.

Scenario 2: An elimination of Alzheimer's and dementia as a cause of loss of autonomy and mortality over the next 5 years.

Prospective scenarios on Alzheimer's and dementia diseases

Scenario 1 and assumptions on the dependency structure



Prospective scenarios on Alzheimer's and dementia diseases
Impact on life expectancy

Residual life expectancy					
Âge	Dependence	Cas	Année 2019	Année 2040	Année 2060
55	$\theta = 0$	Scénario central	29	30.9	32.1
		Scénario 1	29	31.4	32.7
		Δ (mois)	-	5.9	7.3
	$\theta = 1$	Scénario central	29	30.1	30.7
		Scénario 1	29	30.6	31.2
		Δ (mois)	-	5	6.4
	$\theta = 4$	Scénario central	29	29.3	29
		Scénario 1	29	29.5	29.1
		Δ (mois)	-	2.2	1.7
75	$\theta = 0$	Scénario central	11.8	13	13.9
		Scénario 1	11.8	13.5	14.5
		Δ (mois)	-	5.7	7
	$\theta = 1$	Scénario central	11.8	12.3	12.5
		Scénario 1	11.8	12.7	13
		Δ (mois)	-	4.7	6
	$\theta = 4$	Central scenario	11.8	11.6	10.9
		Scénario 1	11.8	11.7	11
		Δ (mois)	-	1.7	1.1

Prospective scenarios on Alzheimer's and dementia diseases
Impact on mortality and longevity risks diversification

A model point of each portfolio:

- Mortality (age 55)
- Longevity (age 75)
- No geographical difference and no portfolio size characteristics
- Compare PV claims over 40 years of projection (fixed rate 1,5%)
- Annuity and face amounts are fixed so that PVs are equal in central scenario

Independent causes ($\theta = 0$)			
Business	Central scenario	Δ Scenario 1	Δ Scenario 2
(1) Mortality claims	2000	-53	-83
(2) Longevity claims	2000	+33	+82
Total (1)-(2)	0	-20	-1

Clayton's copula, $\theta = 1$			
Business	Central scenario	Δ Scenario 1	Δ Scenario 2
(1) Mortality claims	2000	-37	-57
(2) Longevity claims	2000	+28	+68
Total (1)-(2)	0	-9	+9

Clayton's copula, $\theta = 4$			
Business	Central scenario	Δ Scenario 1	Δ Scenario 2
(1) Mortality claims	2000	-10	-14
(2) Longevity claims	2000	+14	+30
Total (1)-(2)	0	+4	+16

Conclusion

- Scenarios on major life risks are requested by regulators as well as by internal stakeholders, such as risk management.
- Working with cause of death mortality allows the construction of hypothetical scenarios on one or more specific causes.
- Survival Archimedean copula is used to take into account the dependence structure between causes of death.
- The assumed dependence structure impacts the diversification

Conclusion

Advantages and disadvantages of the approach

Advantages

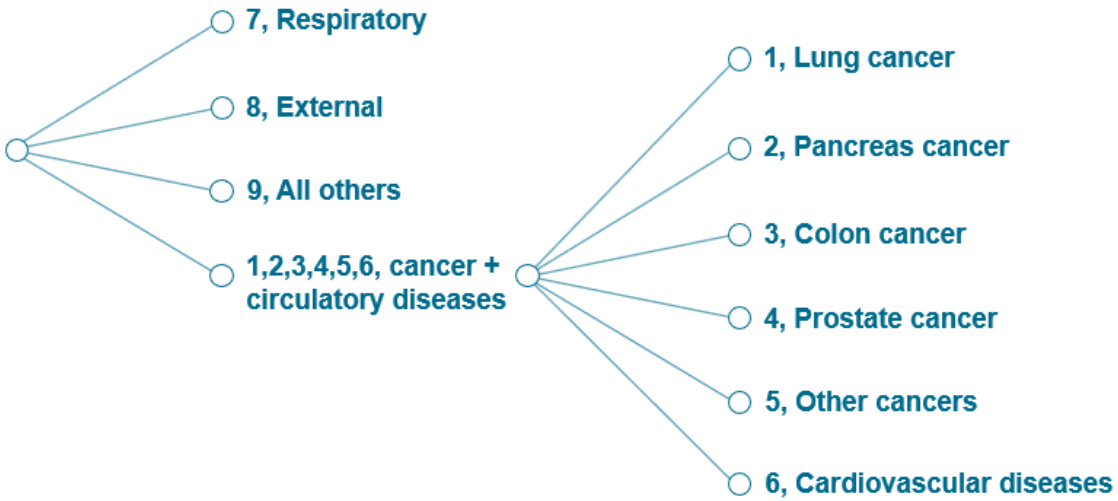
- ✓ Competing risks framework
- ✓ Allows to build hypotheticals scenarios and to evaluate their impacts on different lines of business
- ✓ Take into account improvements in mortality between cohorts and intra-cohort dependence between different causes
- ✓ Explicit expression between intensities of crude and net mortality using survival copulas

Disadvantages

- ✓ Working with cause of death data is complicated
- ✓ Unique dependency structure between all the causes
- ✓ Classic models, like Lee-Carter, may not be suitable to describe mortality trends by cause due to larger volatility
- ✓ Difficulty of empirically estimating the parameter θ

Conclusion
Future developments

- Hierarchical dependence structure using hierarchical Archimedean copula introduces a dependency with several levels and can also be asymmetrical. For example, it could allow to have a stronger dependence between cancers and cardiovascular diseases and a weaker one between this group of causes and the other ones.



- State space models can be advantageous for projecting the times series in a dependent way.
- Estimate empirically the copula parameter

References

- G. Buffet, M. Fysekidis, J. Tomas, and A. Ulcinaite. Alzheimer's disease and dementia footprint scenario: A research report series volume 1 – introduction. Assessing Risk for Alzheimer's Disease and Dementia | SCOR, 2022.
- R. Lee and L. Carter. Modeling and forecasting U.S. mortality. Journal of the American Statistical Association, 87(419):659 671, 1992.
- H. Li and Y. Lu Modeling cause of death mortality using hierarchical Archimedean copula. Scandinavian Actuarial Journal, 3 247 272 2019
- J. Tomas, and A. Ulcinaite. Alzheimer's disease and dementia footprint scenario: A research report series volume 2 – Scenarios modelling and implication to mortality Alzheimer's Disease and Dementia Footprint Scenario | SCOR , 2023.
- A. Ulcinaite. Prospective mortality by cause of death modeling. https://www.institutdesactuaires.com/docs/mem/4da83757d8c32c4f37480bae39e9df89.pdf, 2023.

Assumptions influencing the results

- The magnitude of the shock on Alzheimer's and dementia mortality and its horizon is determined by expert judgment following discussions with medical experts. These scenario assumptions are not the only factors influencing the resulting post-shock aggregate future mortality. What are other assumptions could be influencing the results?
- The within-cohort dependence among the causes of death in the copula framework is another parameter set by expert judgment. The current modeling assumes a small dependency between competing risks. Conversely, having a total dependence would mean that all the deaths in a cohort saved from dying of Alzheimer's and dementia would be redistributed to the other causes at the exact same time of death, leading to no gain in life expectancy.
- The pre-shock Alzheimer's and dementia mortality forecast at high ages is also influencing the outcome. Due to its recent increase, the model projects this upward trend allowing for large impacts for both scenarios. Generally speaking, the larger the increase is, the larger the number of deaths saved from dying of Alzheimer's and dementia, and the larger the potential impact of an improvement scenario.
- The pre-shock mortality projection of the other causes at high ages also affects the result. To the extent other causes, such as neoplasms or cardiovascular diseases, have a high mortality, the impact of a shock on Alzheimer's and dementia mortality would be relatively small. Individuals would die of neoplasms or cardiovascular diseases shortly after being saved from Alzheimer's and dementia.
- Finally, the shape of the mortality at very high ages, i.e., the completion assumption of the mortality table, influences the outcome as it defines the survival time of individuals saved from dying of Alzheimer's and dementia.

Cause of death mortality: assumptions on the dependency structure
Relationship between θ and Kendall's τ

The Kendall's τ is a commonly used ranking correlation measure which in this case captures the correlation between the causes specific time at death, i.e. T_j.

It can be shown that the Kendall's τ correlation for a Clayton survival copula is

$$\tau = \frac{\theta}{2+\theta}.$$

- $\theta = 0$ is equivalent to $\tau = 0$. It corresponds to assuming independence between the competing causes of death.
- $\theta < 0$ is equivalent to $\tau < 0$. It corresponds to a negative correlation between the causes specific time at death. This scenario is rarely used.
- $\theta = 1$ is equivalent to $\tau = 1/3$. It corresponds to assuming that the correlation between two causes specific time at death is $1/3$.
- $\theta = 4$ is equivalent to $\tau = 2/3$. It corresponds to assuming that the correlation between two causes specific time at death is $2/3$.
- As θ increases, τ approaches 1. It implies a stronger dependence between two causes specific time at death.


SCOR
The Art & Science of Risk