Max Planck Odense Center on the Biodemography of Aging
University of Southern Denmark

Projecting Maximum Country Life Expectancy using Provincial Data Only

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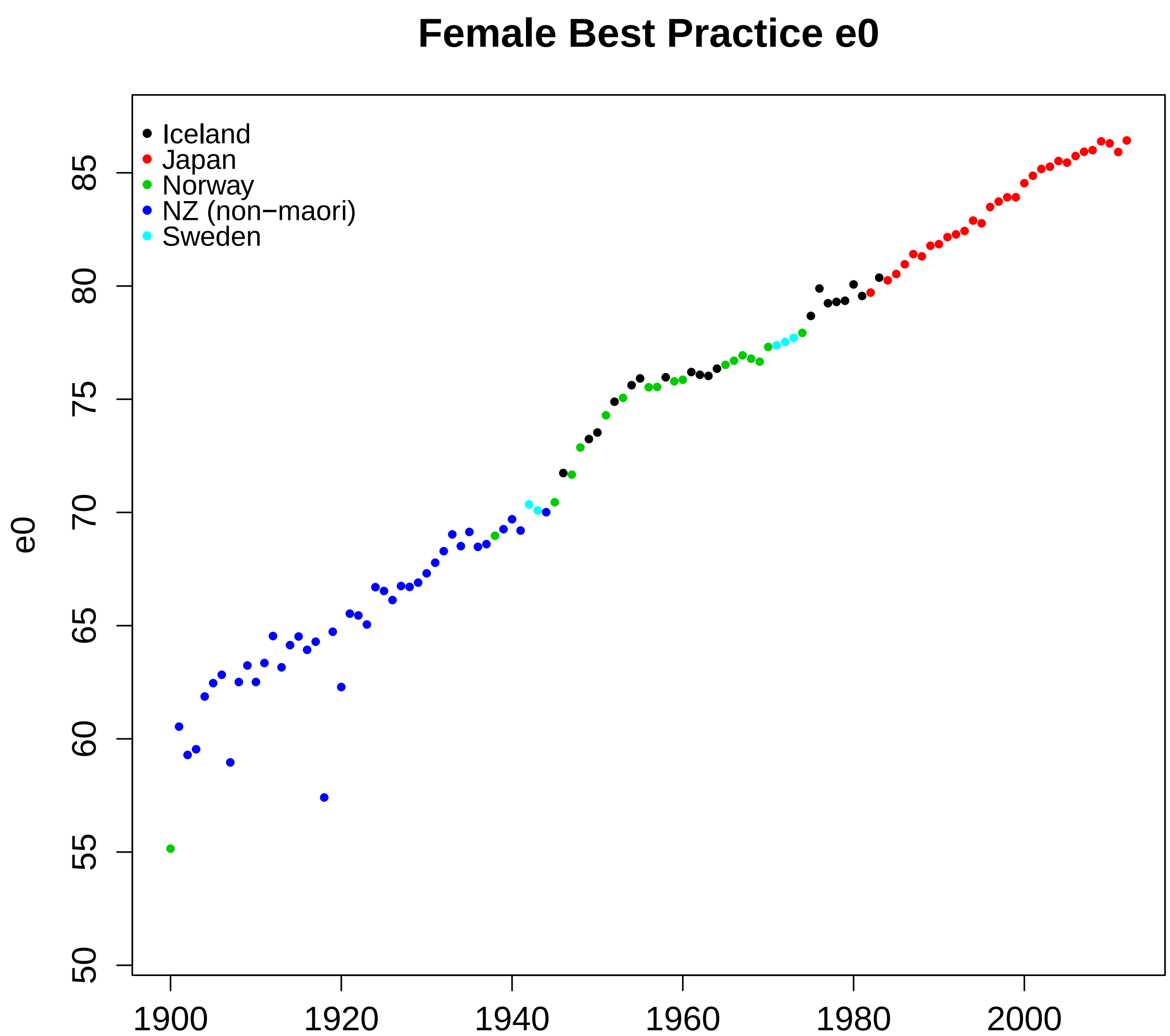
Twelfth International Longevity Risk and Capital Markets Solutions Conference

Overview

- What is Global Best practice Life expectancy?
- Extreme Value Theory in brief and its relation to Best practice Life expectancy
- Regional Best practice Life expectancy and inference

Females e_0



Female Best Practice e0
Iceland
Japan
Norway
NZ (non−maori)
Sweden
e0
50
55
60
65
70
75
80
85
1900
1920
1940
1960
1980
2000

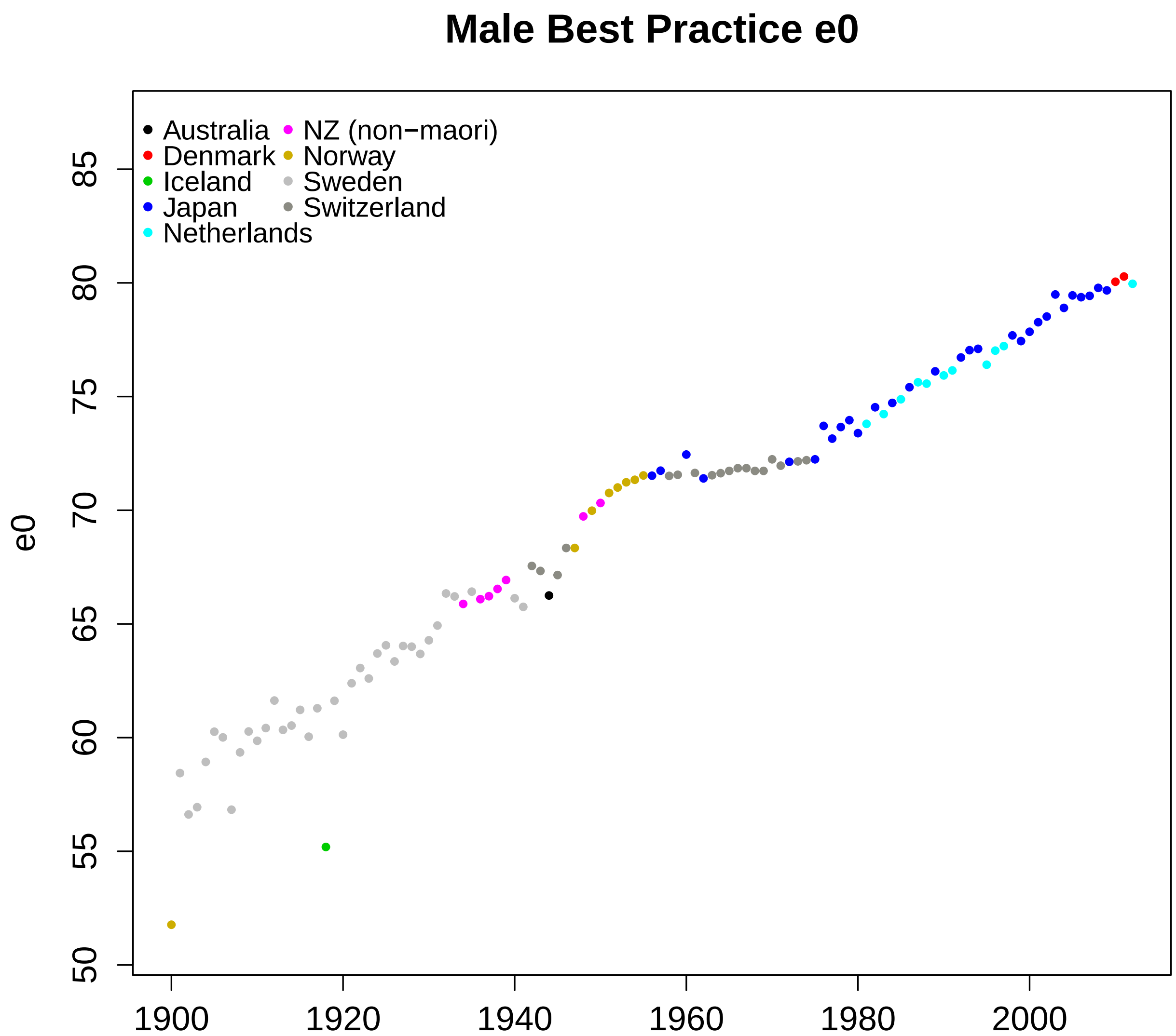
Global Best Practice Life Expectancy

- Global Best Practice Life Expectancy (BPLE) is the maximum life expectancy observed among nations at a given age.
- At birth, has been increasing almost linearly - beginning in Scandinavia c. 1840 - at about 3 months per year (Oeppen and Vaupel, 2002).
- Life expectancy trends may fit better than individual-country trends in age-standardized (log) death rates (White, 2002).

Global Best Practice Life Expectancy

- Nations experience more rapid life expectancy gains when they are farther below BPLE and tend to converge towards BPLE (Torri and Vaupel, 2012).
- It is sensible to consider national mortality trends in a larger international context rather than individual projections (Lee, 2006; Wilmoth, 1998).

Males e_0



Male Best Practice e0
Australia
Denmark
Iceland
Japan
Netherlands
NZ (non−maori)
Norway
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Switzerland
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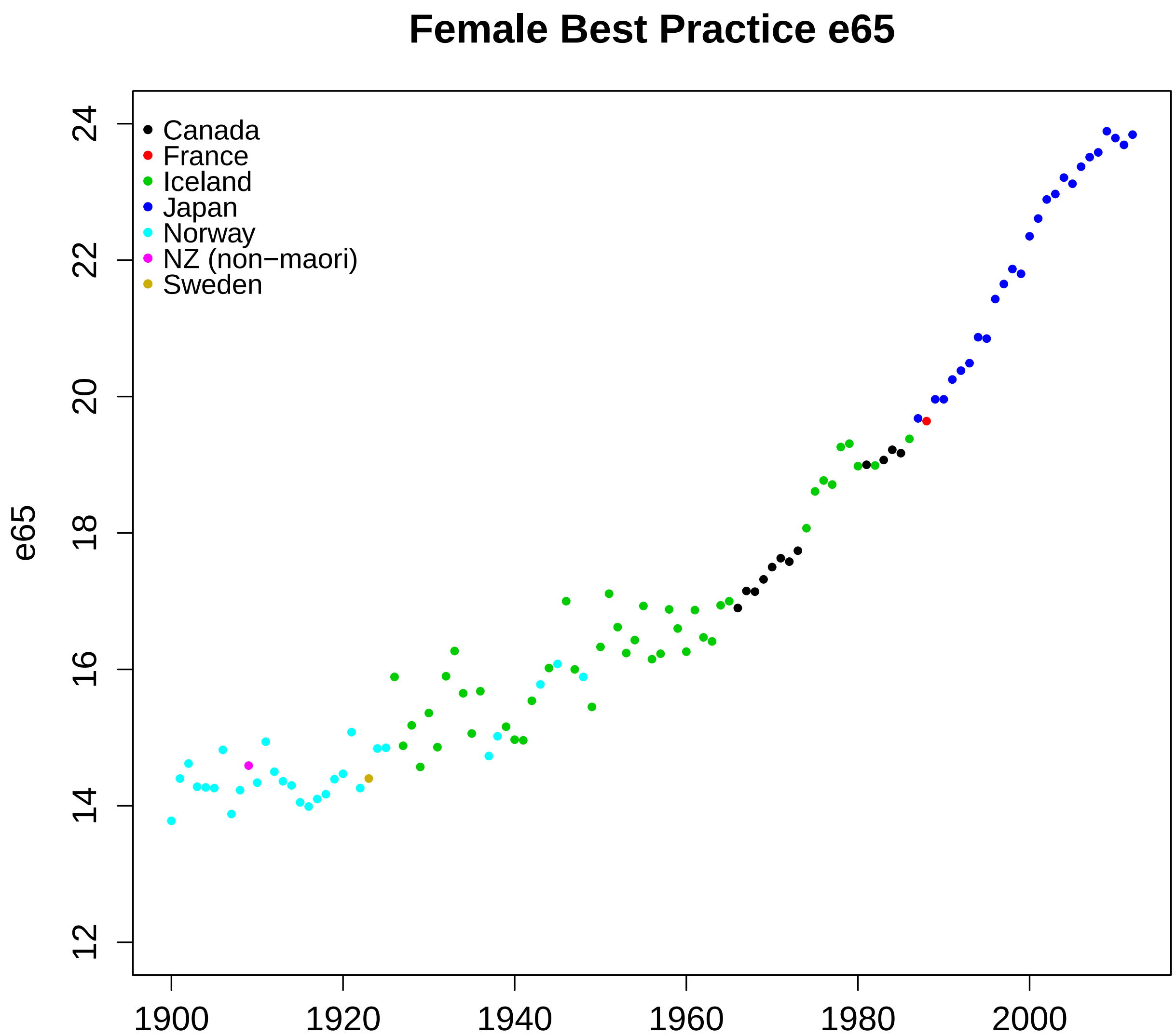
Females e_{65}


Female Best Practice e65
Canada
France
Iceland
Japan
Norway
NZ (non−maori)
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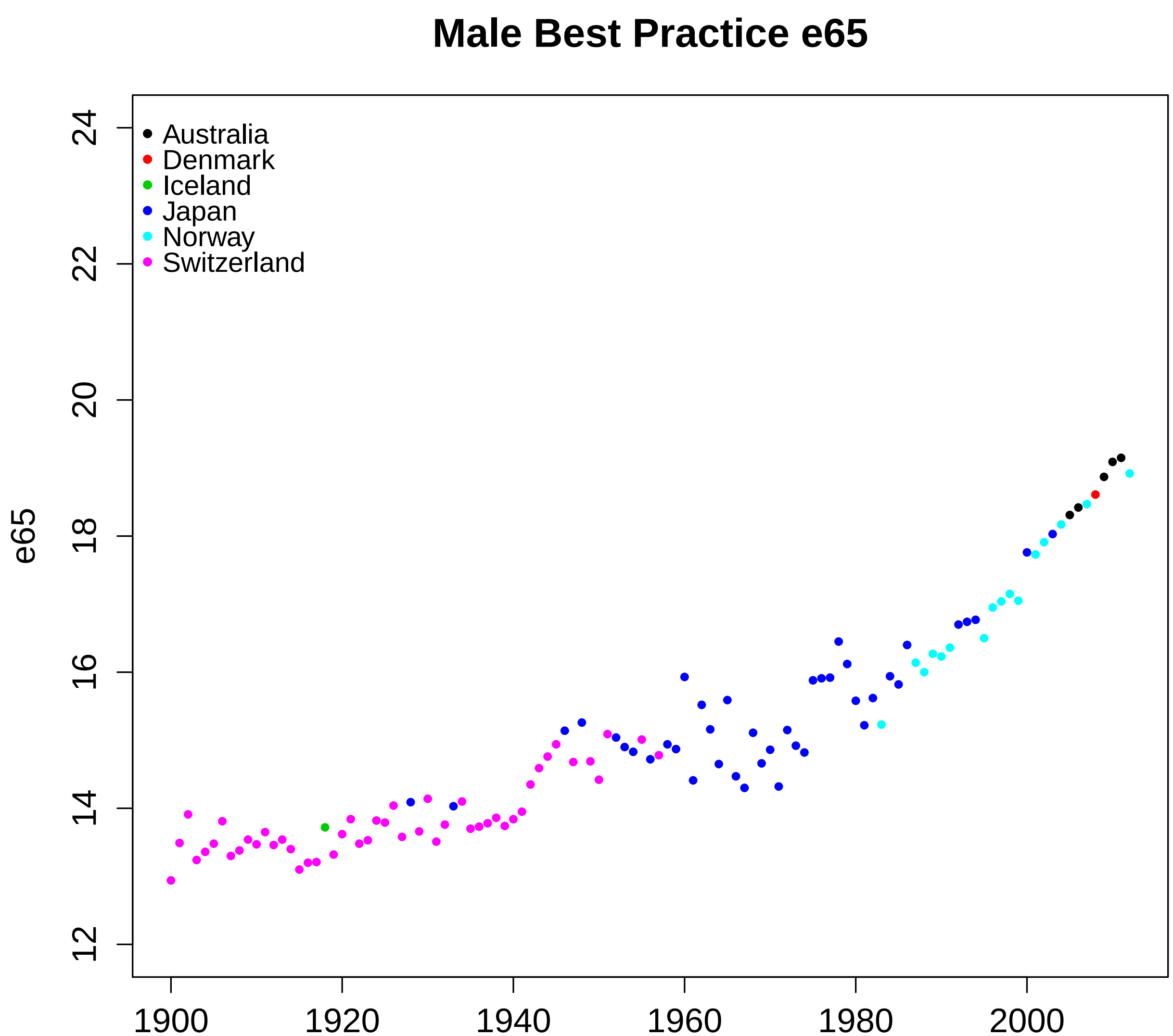
Males e_{65}


Male Best Practice e65
e65
Australia
Denmark
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Breakpoints

- Vallin and Meslé (2009) expanded on work of Oeppen and Vaupel and argued that BPLE trend may comprise multiple segments
- Each segment corresponds to distinct health transition phases

Breakpoints

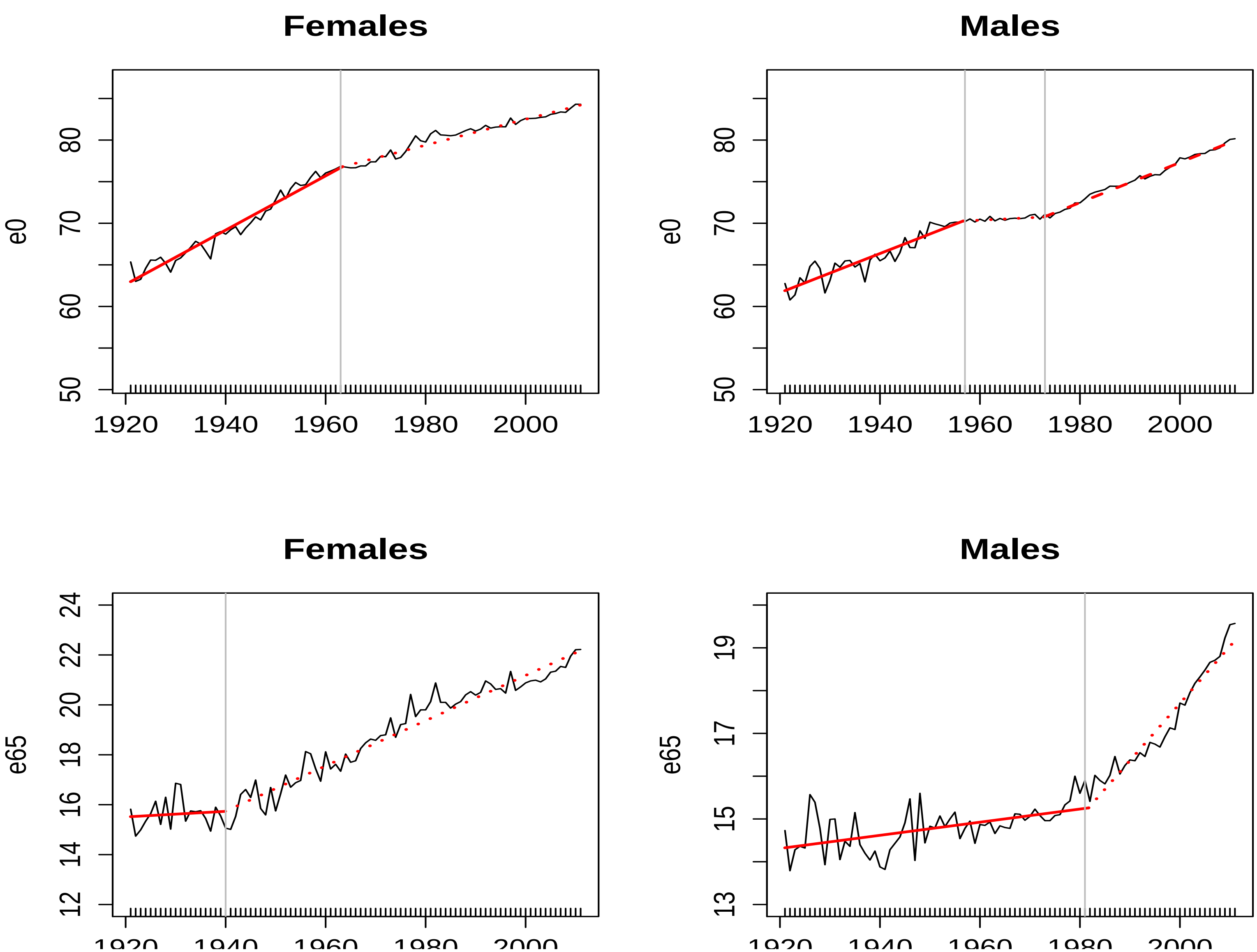


Figure: Breakpoints in the trend of the highest life expectancies at

Can we model Global BPLE?

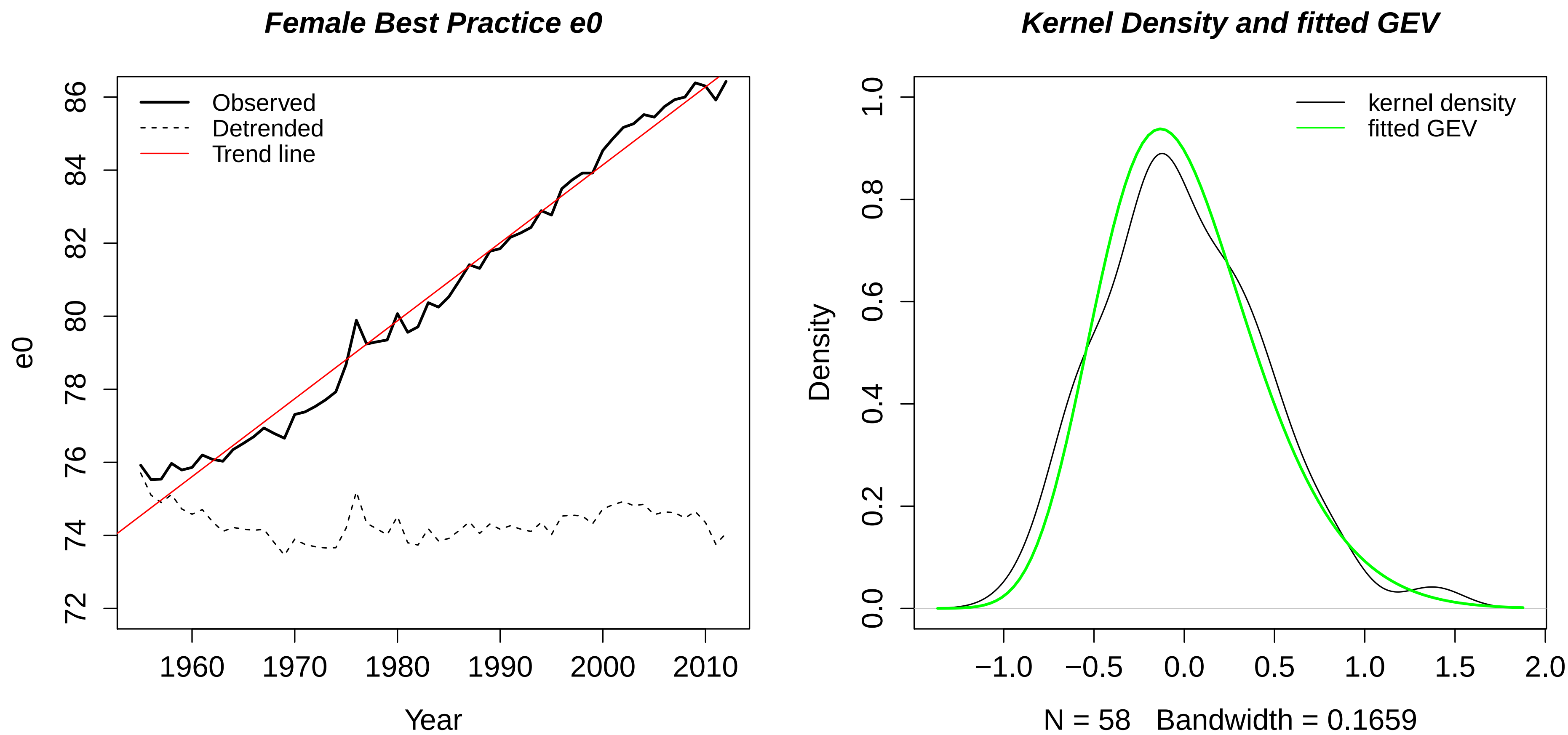


Figure: Left panel: raw and detrended data. Right panel: kernel density and fitted GEV distribution.

Can we model Global BPLE?

Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independent, identically distributed random variates all having a common distribution function $F(x)$.

Let $M_n = \max\{X_1, X_2, \ldots, X_n\}$.

The distribution of the maxima, M_n, converges (for large n) to the Generalized Extreme Value (GEV) Distribution.

Theoretical motivation

Extremal Types theorem

If there exists sequences of constants $\{a_n > 0\}$ and $\{b_n\}$, such that as $n \to \infty$,

$$P\left(\frac{M_n - b_n}{a_n} \leq z\right) \to G(z) \qquad (1)$$

where $G(z)$ is a non-degenerate distribution function, then G **must** be a member of the Generalized Extreme Value (GEV) family of distributions (Fisher and Tippett, 1928; Gnedenko, 1943).

Theoretical motivation

Extremal Types theorem

- This is a remarkable result because regardless of the underlying distribution, the distribution of the maxima (or minima) converges to one of the Generalized Extreme Value family of distributions.
- Can maximum period life expectancies be approximately modeled as a GEV?

The Generalized Extreme Value Distribution

$$G(z) = \exp\Big\{ - \Big[1 + \xi(\frac{z-\mu}{\sigma})\Big]_{+}^{\frac{-1}{\xi}} \Big\} \quad (2)$$

where $b_{+} = max(0, b)$. The situation where $\xi = 0$ is not defined in (2), but taken as the limit as $\xi \to 0$, given by

$$G(z) = \exp\Big\{ -\exp\Big[-(\frac{z-\mu}{\sigma}) \Big] \Big\}. \quad (3)$$

The Generalized Extreme Value Distribution

- μ is the location parameter
- σ is the scale parameter
- ξ is the shape parameter, which determines the tail behaviour

  $\xi > 0$: polynomial tail decay and the Fréchet Distribution
  $\xi = 0$: exponential tail decay and the Gumbel Distribution
  $\xi < 0$: bounded upper finite end point and the Weibull Distribution

Inference

Quantiles

Inverting the GEV distribution function:

$$z_p = \mu - \frac{\sigma}{\xi}\left[1 - \{-log(1-p)\}^{-\xi}\right],$$

where p is the tail probability and $G(z_p) = 1 - p$

Return Levels

- Simply a different way of thinking about the quantiles.
- If data are annual the $(1-p)$th quantile would be exceeded on average once every $1/p$ years.

Regional Best Practice Life Expectancy?

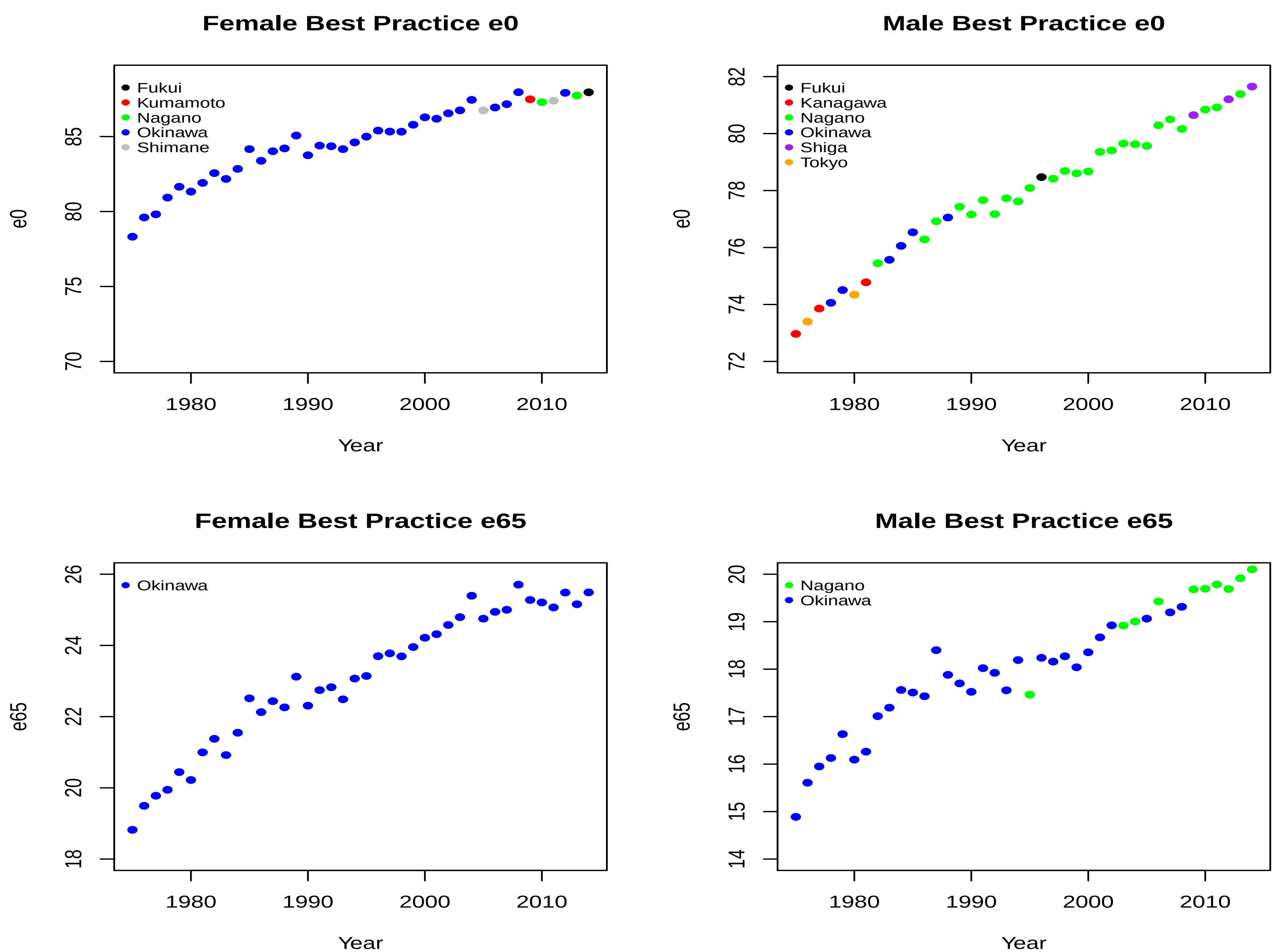
THE IDEA

- Can the notion of BPLE be extended to regions smaller that the global whole?
- If we find BPLE over an arbitrary region - itself comprised of smaller subregions - would there also be a regular temporal evolution e.g. strong (piecewise) linear trends?
- What sort of inferences can we perform?

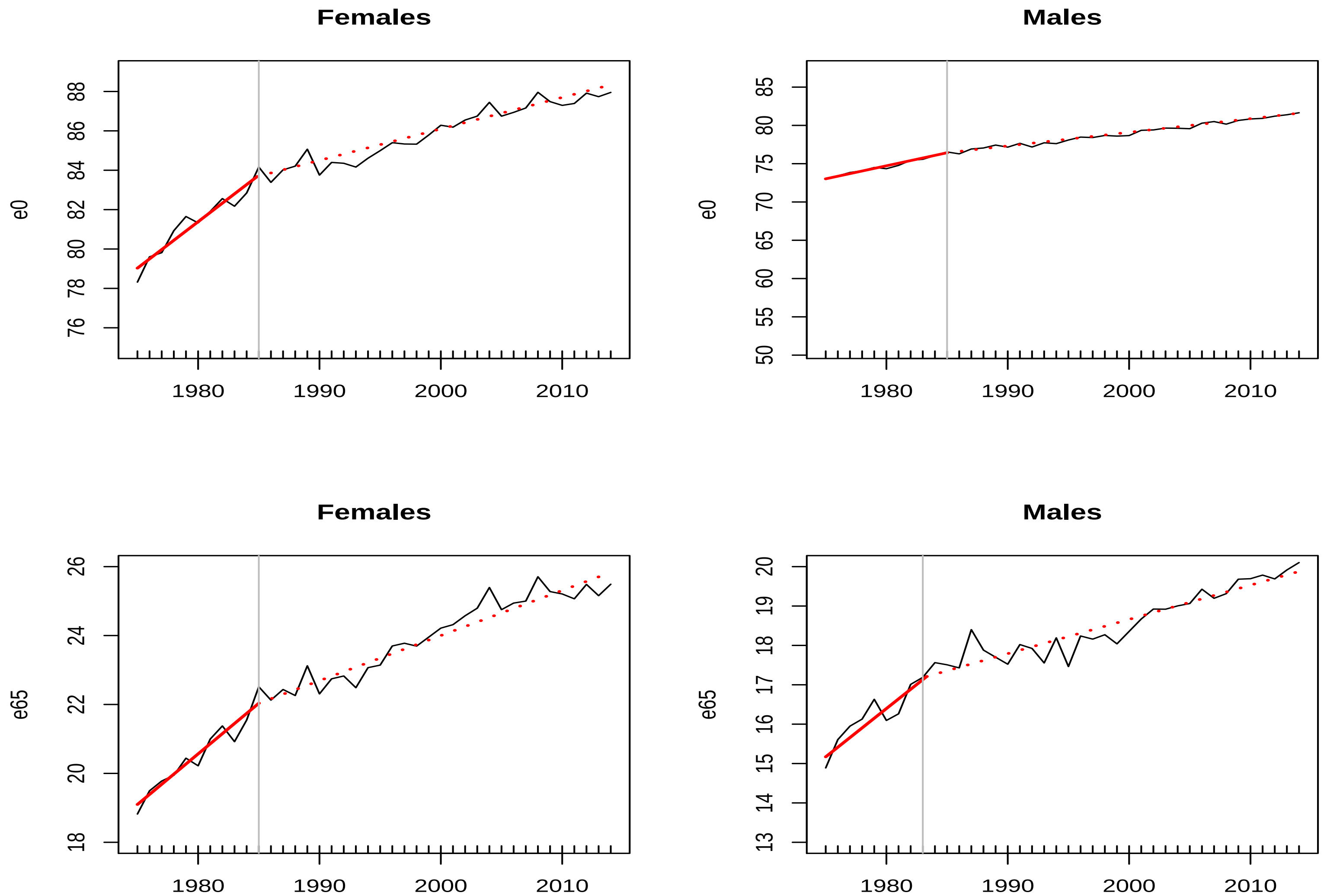
Data

- Canadian Human Mortality Database (CHMD)
  - Life expectancy data broken down by province
  - Covers period from 1921 to 2011 (but Newfoundland from 1949)
- Japanese Mortality Database (JMD)
  - Life expectancy data broken down by prefecture
  - Covers period from 1975 to 2014

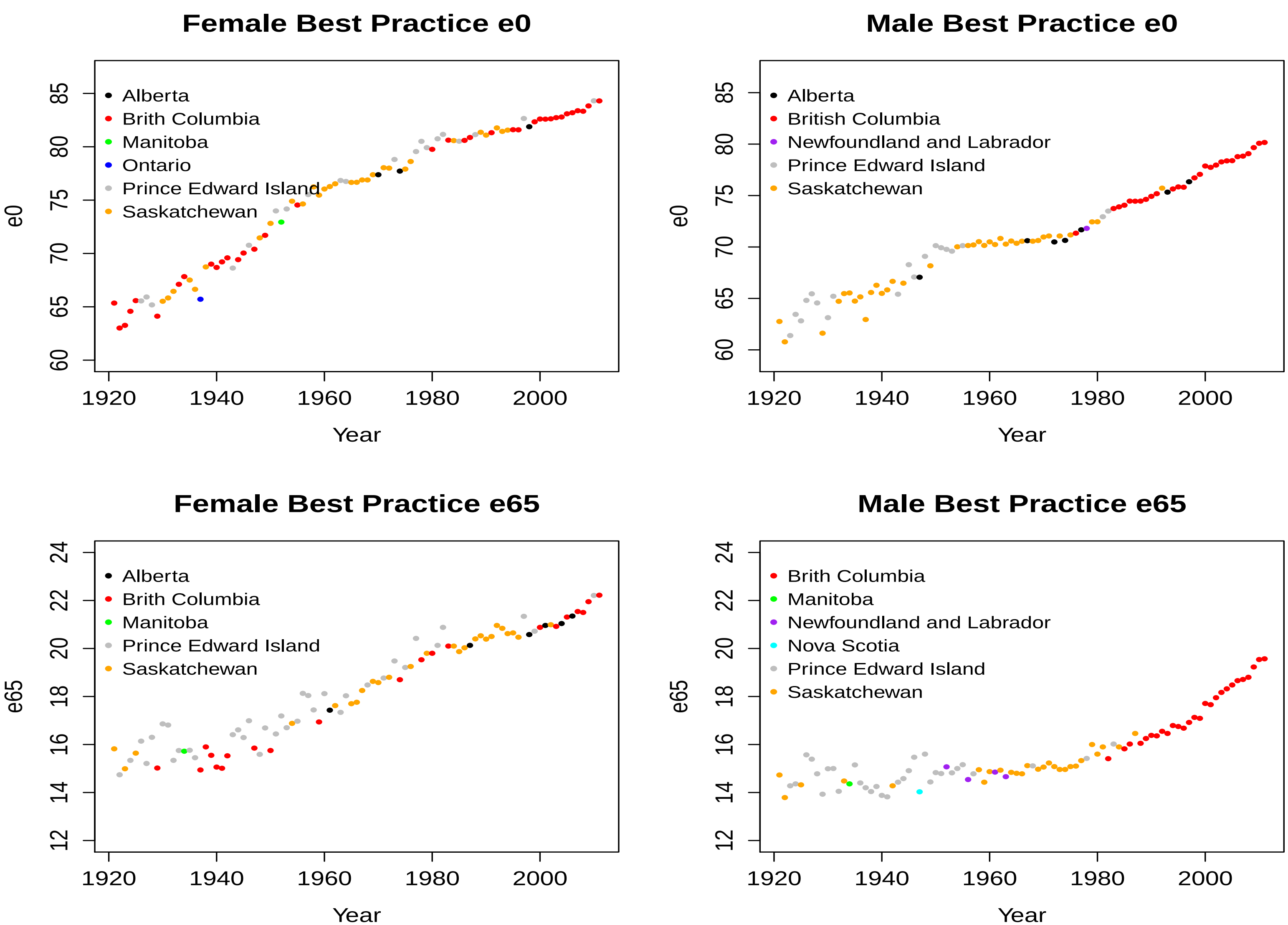
Japan - Maximum e_x by Prefecture



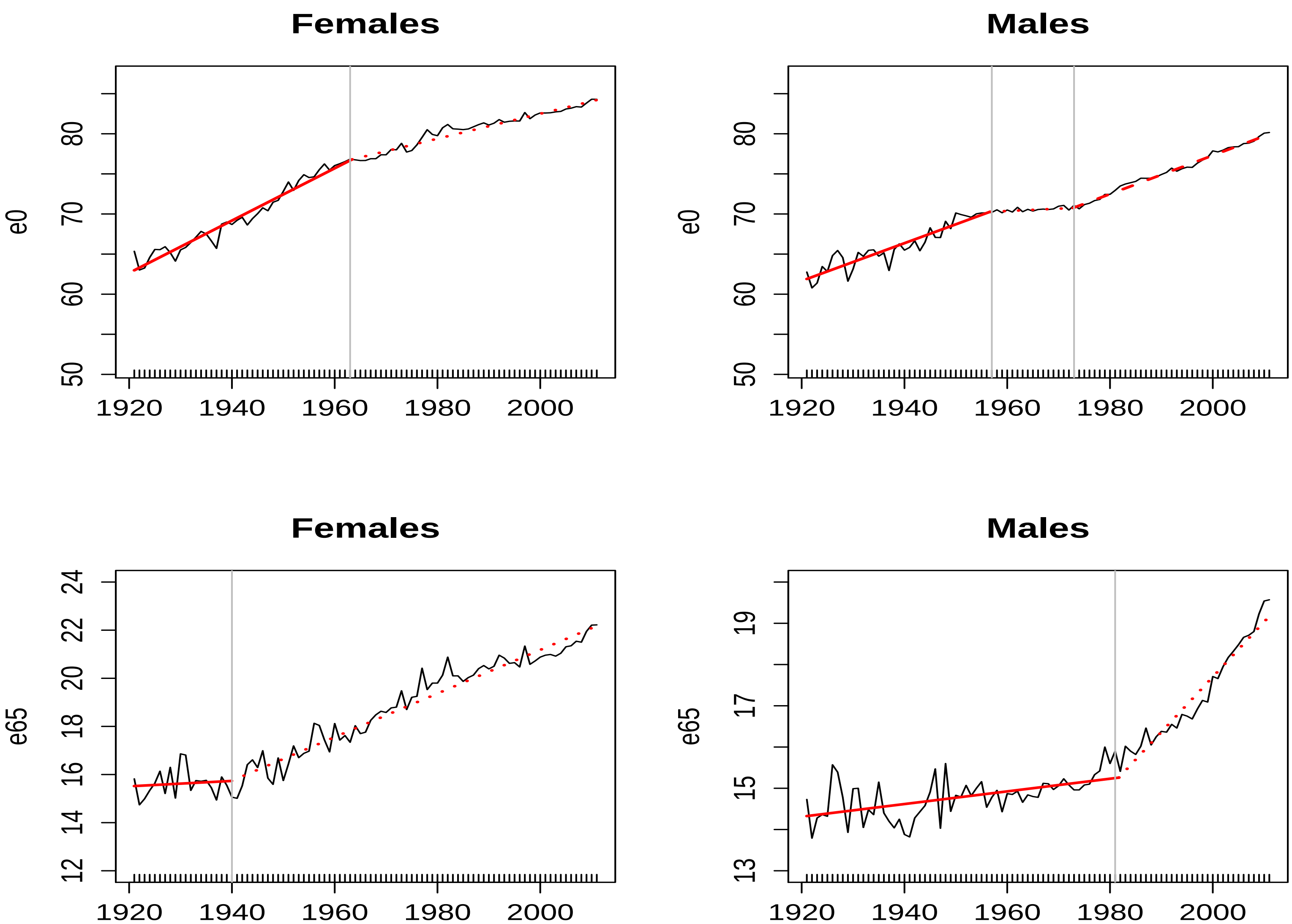
Japan - Maximum e_x by Prefecture



Canada - Maximum e_x by Province



Breakpoints in Canadian e_x



The Model

Time-dependent GEV model to annual maximum provincial e_x:

$$GEV(\mu_t, \sigma_t, \xi_t) \quad with \quad \mu_t = \beta_0 + \beta_1 t;\ \sigma_t = \sigma;\ \xi_t = \xi$$

$$G(z_t) = \exp\left\{ -\left[1 + \xi\left(\frac{z - (\beta_0 + \beta_1 t)}{\sigma}\right)\right]^{\frac{-1}{\xi}} \right\}$$

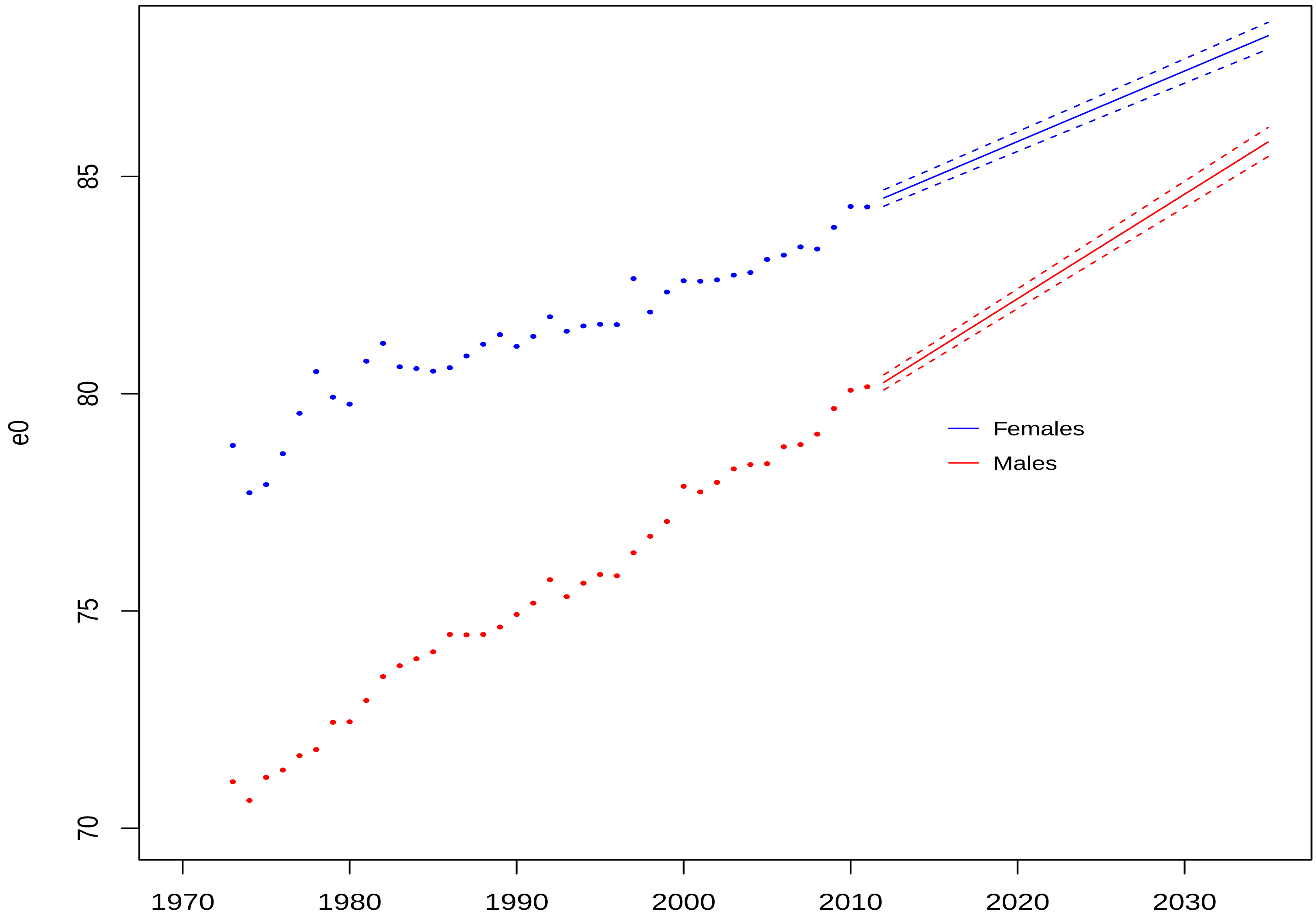
- Other forms of time dependence possible but linearity in μ reasonable and parsemonious choice.
- Parameters estimated jointly using maximum likelihood

Parameter estimates - Canada

	Neg. Likelihood	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}$	$\hat{\xi}$
Female e_0	30.8	76.4(0.11)	0.16 (0.003)	0.37 (0.030)	0.10
Male e_0	3.9	70.6 (0.10)	0.24 (0.004)	0.27 (0.03)	-0.34 (0.15)
Female e_{65}	45.2	15.4 (0.11)	0.09 (0.002)	0.42 (0.040)	-0.15 (0.08)
Male e_{65}	-3.60	15.1 (0.11)	0.13 (0.006)	0.27 (0.04)	-0.29 (0.14)

Table: Maximized negative log-likelihoods, parameter estimates and standard errors (in parentheses) of the Block Maxima Model

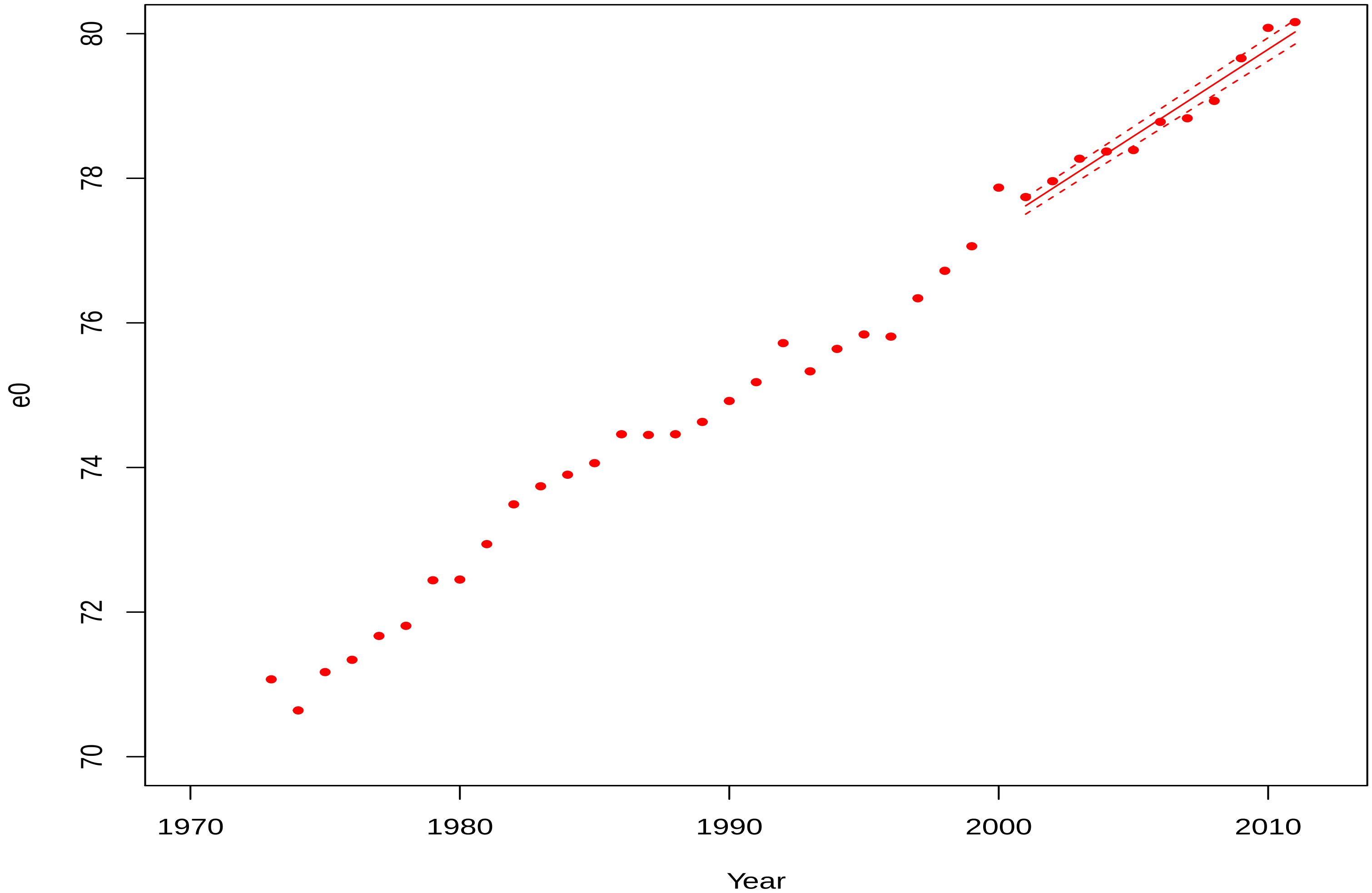
Projected e_0 - Canada



Probability statements - Canada

Year	$P(e_{0,f}^{max} > 87.5)$	$P(e_{0,f}^{max} > 89)$
2030	0.44	0.02
2035	0.99	0.11

Forecast Performance e.g.Canadian Males e_0



Comments

- Classical theory assumes that life expectancies between subregions are independent but there are dependencies
- Should have "large enough" number of subregions
- Data wastage?

Ongoing work: added sophistication

- Relax linearity assumption for time dependence to allow any time-varying shape for GEV parameters
- More flexible Dynamic Linear Model for time-varying parameters for forecasting
- Flexible GLM type framework for modelling
  - Vector Generalized Linear Models (Yee & Hastie, 2003), or
  - Generalized Additive Models for Location, Scale and Shape (Rigby & Stasinopoulos, 2001, 2005)

Take Aways

- Method can be used to make inferences about future maximum life expectancy for a region e.g a country using sub-regional information only e.g. states/ provinces/ prefectures
- Since a probability distribution is fitted, it is straightforward to obtain probabilities
- Ancillary benefit: if provinces with maxima also have high proportion of population then projecting median gives a workable estimate of overall country e_x
- Underlying theoretical model assumptions may be hard to achieve in practice but acid test is usually good assessment of empirical fit

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