LONGEVITY RISK OF PENSIONS IN CZECH REPUBLIC

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1. INTRODUCTION

The contribution deals with some aspects of ***longevity risk*** in the framework of the ***pension system in the Czech Republic***:

(1) ***Sustainability of pensions*:** one investigates

- ***which regular annuity spending (spending rate)*** is admissible
- for a ***given pension account***
- with the ***given investment efficiency*** and
- under the ***given tolerance***.

(2) ***Generation Life Tables*** **(*GLT*):**

- in the pension context one should apply special Life Tables with projections distinguishing among ***particular generations*** **(*cohorts*)**;
- moreover, these Tables should be adjusted in a special way to be ***suitable for pensions***.

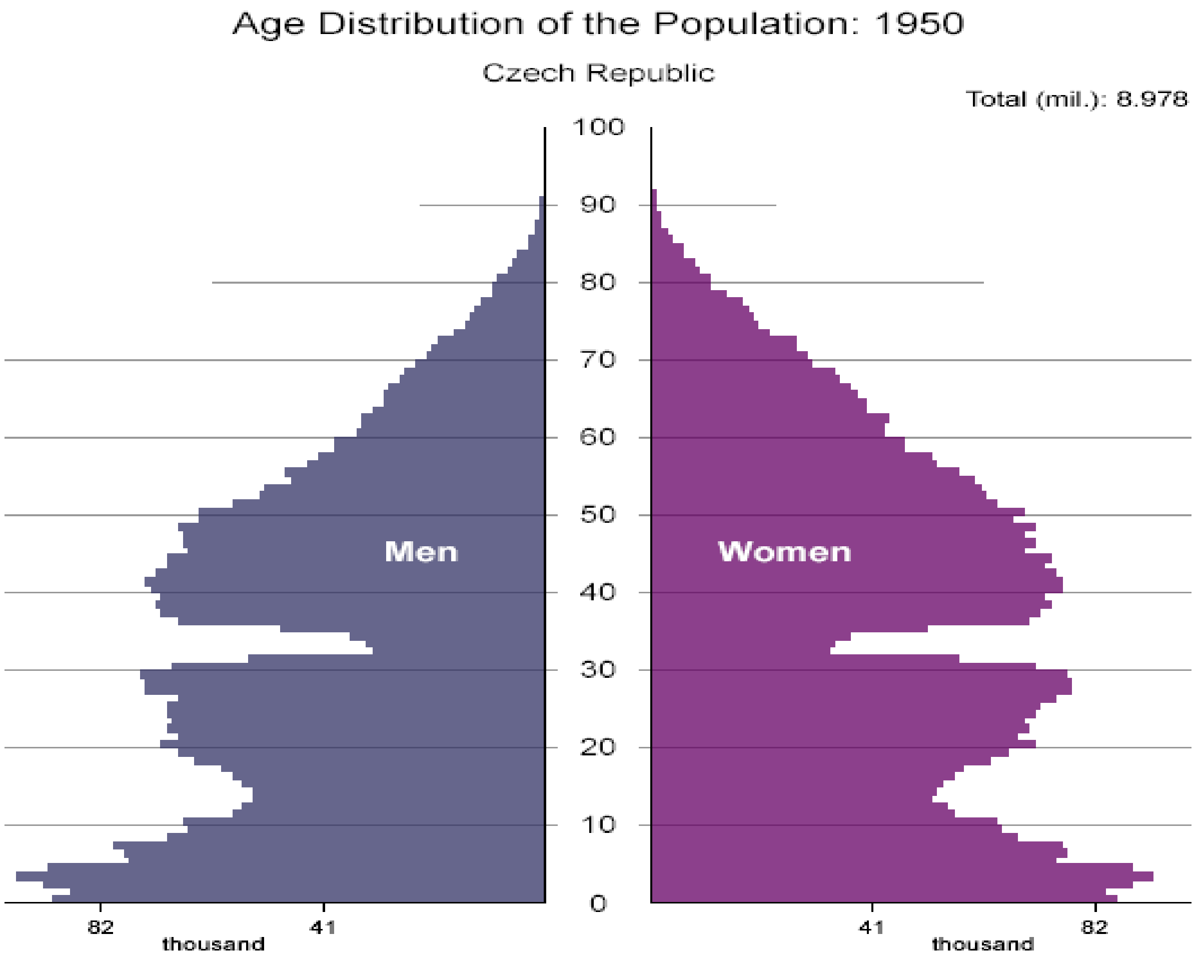
(3) ***Unisex Life Tables*** **(*ULT*):**

- due to various ***anti-discrimination laws*** concerning the insurance the ULT should be constructed for the Czech population.

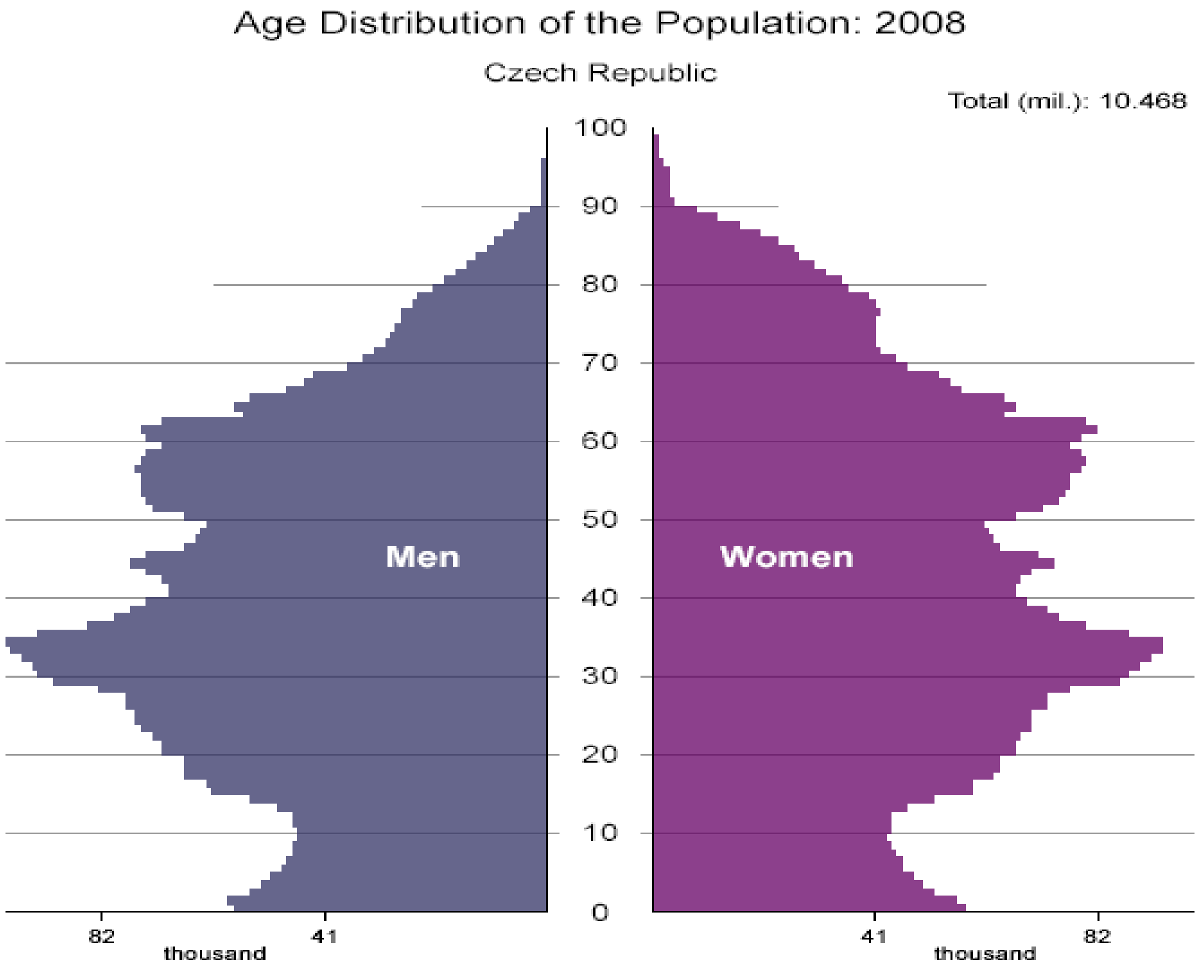
Pension system in Czech Republic:

- after 1989 (the end of the epoch of socialism) in 1996 very up-to-date ***state pension system*** (the ***first pillar*** comparable e.g. with Germany):
  - public
  - mandatory
  - PAYG
  - DB (with a small fixed component) and
- insignificant ***private pension funds*** (the ***third pillar***) with the ***state support*** (more „public saving system“ than pension system)
- unfortunately after 2000 the ***demographic situation in the Czech Republic has become critical:***

Comparison of the age distribution of the Czech population in 1950:

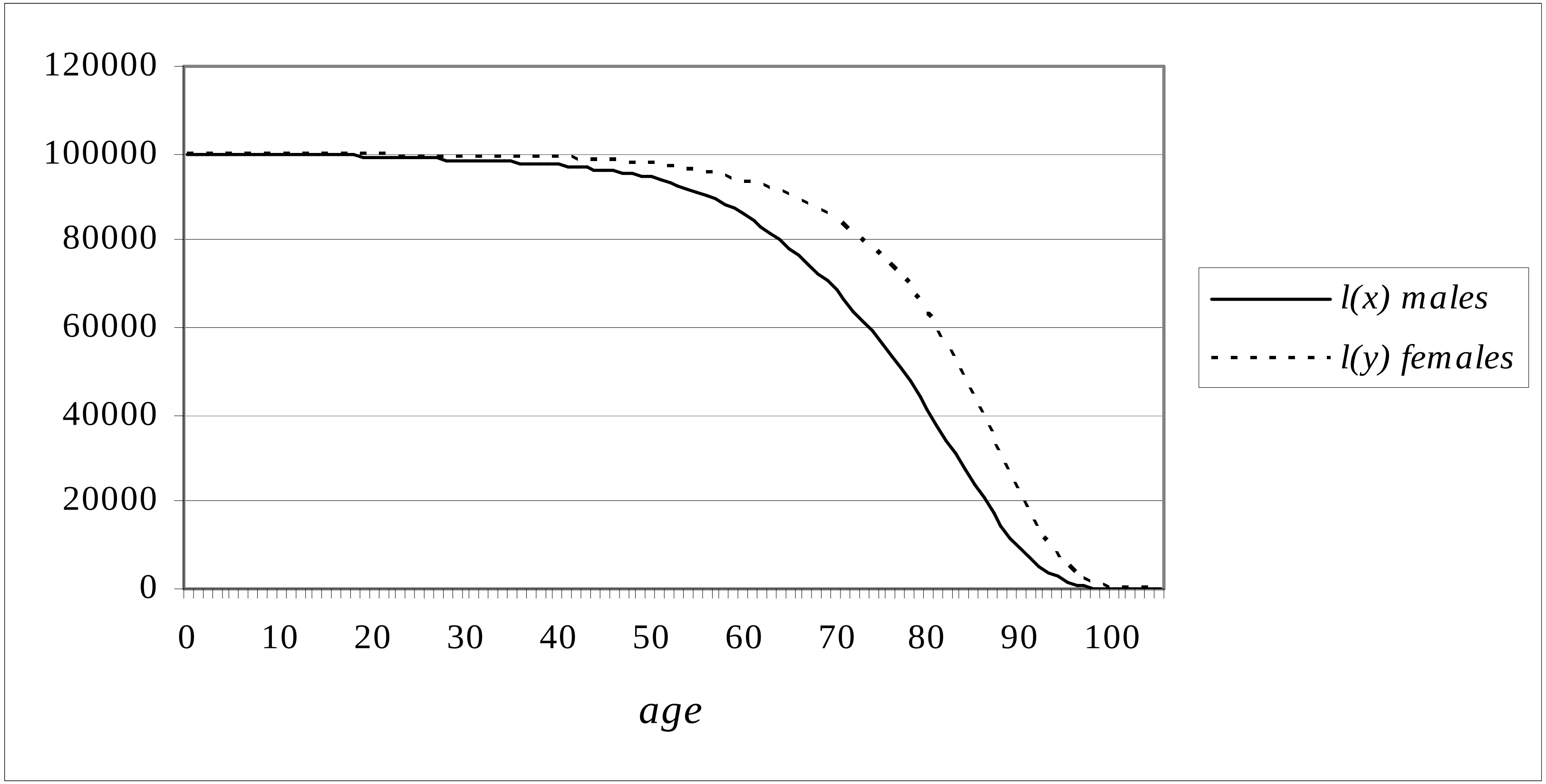


Age distribution of the Czech population in 2008:



Source: Eurostat (2008)

Numbers of survivors till particular ages from the Life Tables of the Czech Republic in 2010:



Source: Czech Statistical Office (2010)

– the state pension system and its mandatory costs become ***critical for the state budget***, see e.g. the enormous ***volume of benefits in 2011:***

- population in Czech Republic: 10 546 000
- ***number of registered retirees***: 2 873 000
- ***number of pension annuities***: 3 501 000 (plus 70 000 to abroad)
  - number of old-age pensions: 2 340 000
  - number of disability pensions: 445 000
  - number of survivor's pensions: 716 000
- ***mean monthly old-age pension***: 10 552 CZK

  male: 11 700 CZK, female: 9 584 CZK
- ***mean monthly salary***: 24 319 CZK

- ***mean age of retirees***: 68 years
- ***number of contributors***: 5 040 000
- ***number of contributors/1 retiree***: 1.75
- ***number of contributors/1 old-age retiree***: 2.15

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Pension reform since 2013 is urgent introducing:

- the ***second pillar*** with opt-out;
- new progressive solutions involving ***commercial life insurance*** (e.g. DC with buyout by private insurance companies);
- ***actuarial analyses*** are supposed.

2. SUSTAINABILITY OF PENSIONS

- ***how much to "save" annually*** during the ***accumulation phase*** and ***how much to "spend" annually*** during the ***decumulation phase***;
- ***many random aspects*** → the best approach is the one applied in modern finance, namely the ***Value-at-Risk*** (***VaR***) =
  = ***the highest loss which can occur with a given (tolerance)***;
- in the pension context: ***probability of sustainable pension*** =
  = ***the probability that the retired person will not be 'ruined' before the moment of death***

Assumption:

- ***DC pension plans***;
- the ***contributions*** to the system are ***defined in advance*** by a percentage of the participant's salary;
- ***accumulated capital w*** on the participant's account in the age of retirement → decumulated by corresponding annual pension payments;
- the ***investment risk*** is fully ***on the side of the participants*** of the pension plan (not on the side of the pension provider);

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- two aspects of ***random character*** should be at least considered:

(1) ***Randomness of interest rates r(t)*:**

- for investment of the capital from the participant's account;
- originally in the age of retirement the participant holds $S_0 = w$;
- this type or randomness can be modeled by means of ***geometric Brown motion***:

$$S_t = S_0 \cdot \mathrm{e}^{B_t^{(\mu,\,\sigma)}} = S_0 \cdot \mathrm{e}^{\mu \cdot t + \sigma \cdot B_t},$$

μ is the ***drift***

σ is the ***volatility***

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− in particular, S_t is ***log-normally distributed***:

$$\ln S_t \sim N(\ln S_0 + \mu \cdot t, \sigma^2 t);$$

with the ***mean value***:

$$\mathrm{E}(S_t) = S_0 \cdot \mathrm{e}^{\left(\mu + \frac{\sigma^2}{2}\right) \cdot t} = S_0 \cdot \mathrm{e}^{\nu \cdot t};$$

and the ***median value***:

$$\mathrm{M}(S_t) = S_0 \cdot \mathrm{e}^{\mu \cdot t}.$$

(2) ***Randomness of the future lifetime T_x of an individual aged x:***

− can be modeled in the simplest case by the ***exponential law of mortality***:

$$ {}_t p_x = \exp\left\{ -\int_x^{x+t} \lambda_x \, \mathrm{d}s \right\} = \mathrm{e}^{-\lambda_x \cdot t}, $$

λ_x is the ***instantaneous force of mortality***;

↓

− the ***expected remaining lifetime*** (the ***life expectancy***) ***at age x***:

$$ e_x = \mathrm{E}(T_x) = \frac{1}{\lambda_x}; $$

− the ***median remaining lifetime at age x***:

$$M(T_x) = \frac{\ln(2)}{\lambda_x}.$$

↓

(3) ***Combination of randomness of r(t) and T_x:***

– ***the present value PV_x of the standard pension*** (which pays unit annual payments in continuous time):

$$PV_x = \int_0^{T_x} e^{-(\mu \cdot t + \sigma \cdot B_t)}\, dt;$$

↓

– the ***probability of ruin*** (the ***probability of unsustainable pension***):

$$P(PV_x > w) = P\left(\int_0^{T_x} e^{-(\mu \cdot t + \sigma \cdot B_t)}\, dt > w\right),$$

$w > 0$ is the sum on the participant's account at the age of retirement x;

↓

– it can be approximated as

$$\mathrm{P}\left(PV_x > w\right) \sim \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{1/w} z^{\alpha-1} \exp\left(-\frac{z}{\beta}\right) \mathrm{d}z =$$

$$= 1 - \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{w} y^{-(\alpha+1)} \exp\left(-\frac{1}{\beta y}\right) \mathrm{d}y$$

where

$$\alpha = \frac{2\mu + 4\lambda_x}{\sigma^2 + \lambda_x} - 1, \quad \beta = \frac{\sigma^2 + \lambda_x}{2}$$

and $\Gamma(\alpha)$ is the gamma function $\Gamma(\alpha) = \int_0^{\infty} z^{\alpha-1} \mathrm{e}^{-z} \, \mathrm{d}z$.

Results for the Czech Republic:

(1) ***Financial data***:

− ***technical interest rate*** 2.5 % used for insurance calculations according to the legislative in the Czech Republic in year 2012;

− ***more scenarios*** both for the drift μ and the volatility σ using the technical interest rate 2.5 % as one of possibilities.

(2) ***Longevity data***:

− ***Life Tables*** (LT) for males and females in the Czech Republic in 2010.

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The probability of ruin for various

- ***retirement ages*** 55, 60, …, 85;
- ***spending rates*** $1/w$ (e.g. *the spending rate 0.06 → a pension account of 1 000 000 will pay 60 000 annually, i.e. 5 000 monthly*);
- ***investment drifts and volatilities***.

Probability of ruin for various retirement ages and spending rates with fixed μ = 1 % and σ = 5 % - males:

Male	***Spending rate* 1 / *w*:**									
x	***0.01***	***0.02***	***0.03***	***0.04***	***0.05***	***0.06***	***0.07***	***0.08***	***0.09***	***0.10***
55	0.6 %	4.0 %	10.8 %	20.2 %	31.0 %	41.9 %	52.2 %	61.5 %	69.5 %	76.2 %
60	0.4 %	2.7 %	7.5 %	14.4 %	22.9 %	32.1 %	41.3 %	50.1 %	58.2 %	65.5 %
65	0.2 %	1.7 %	4.8 %	9.7 %	15.9 %	23.0 %	30.5 %	38.2 %	45.7 %	52.8 %
70	0.1 %	1.0 %	2.9 %	5.9 %	10.0 %	15.0 %	20.6 %	26.6 %	32.7 %	38.9 %
75	0.1 %	0.5 %	1.5 %	3.1 %	5.5 %	8.4 %	12.0 %	15.9 %	20.3 %	24.8 %
80	0.0 %	0.2 %	0.6 %	1.4 %	2.5 %	4.0 %	5.9 %	8.1 %	10.6 %	13.4 %
85	0.0 %	0.1 %	0.2 %	0.5 %	1.0 %	1.6 %	2.4 %	3.4 %	4.5 %	5.9 %

↓

– e.g. under a ***conservative investment strategy*** (***μ = 1 %*** and ***σ = 5 %***) a ***male*** (***retirement age of 65***, ***spending rate 0.06***): the unsustainable pension with ***probability*** ***23.0 %*** (a ***female***: ***32.1 %***).

Probability of ruin for various retirement ages and spending rates with fixed μ = 2.5 % and σ = 5 % - males:

Male	***Spending rate* 1 / *w*:**									
x	***0.01***	***0.02***	***0.03***	***0.04***	***0.05***	***0.06***	***0.07***	***0.08***	***0.09***	***0.10***
55	0.1 %	1.4 %	5.0 %	10.9 %	18.9 %	28.2 %	37.9 %	47.5 %	56.4 %	64.4 %
60	0.1 %	1.0 %	3.6 %	8.0 %	14.1 %	21.5 %	29.6 %	37.9 %	46.2 %	54.0 %
65	0.1 %	0.7 %	2.5 %	5.5 %	9.9 %	15.4 %	21.8 %	28.6 %	35.6 %	42.6 %
70	0.0 %	0.5 %	1.6 %	3.5 %	6.5 %	10.2 %	14.7 %	19.8 %	25.3 %	31.0 %
75	0.0 %	0.3 %	0.9 %	2.0 %	3.7 %	5.9 %	8.7 %	12.0 %	15.7 %	19.7 %
80	0.0 %	0.1 %	0.4 %	1.0 %	1.8 %	3.0 %	4.4 %	6.3 %	8.4 %	10.7 %
85	0.0 %	0.0 %	0.2 %	0.4 %	0.7 %	1.2 %	1.9 %	2.7 %	3.7 %	4.8 %

↓

– e.g. under the ***legislative investment strategy*** (***μ = 2.5 %*** and ***σ = 5 %***) a ***male*** (***retirement age of 65***, ***spending rate 0.06***): the unsustainable pension with ***probability*** ***15.4 %*** (a ***female***: ***21.5 %***).

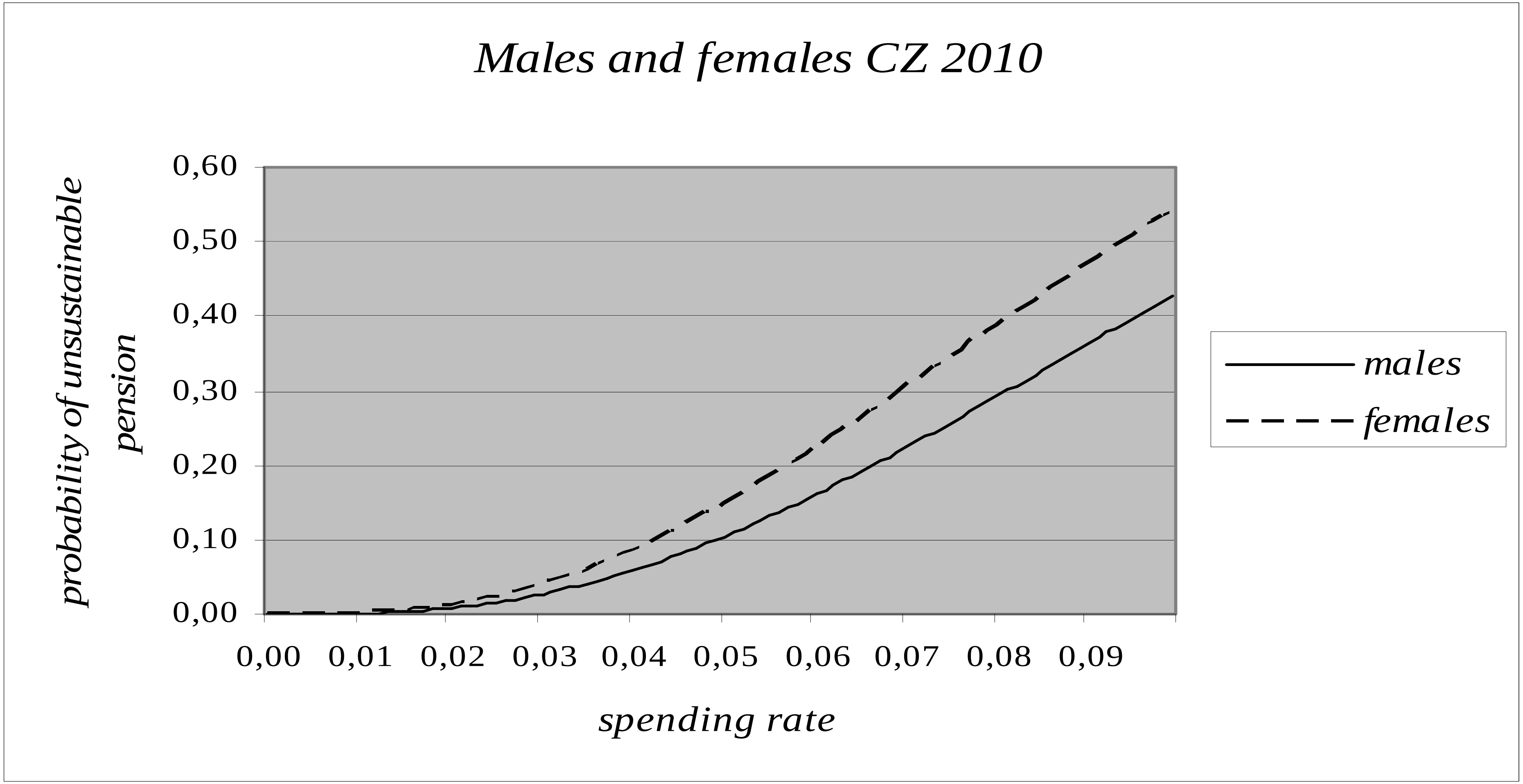
Probability of ruin for various retirement ages and spending rates with fixed μ = 5 % and σ = 10 % - males:

Male	***Spending rate* 1 / *w*:**									
x	***0.01***	***0.02***	***0.03***	***0.04***	***0.05***	***0.06***	***0.07***	***0.08***	***0.09***	***0.10***
55	0.0 %	0.6 %	2.2 %	5.5 %	10.4 %	16.7 %	24.0 %	31.9 %	40.0 %	47.9 %
60	0.0 %	0.4 %	1.7 %	4.2 %	8.0 %	13.0 %	19.0 %	25.6 %	32.7 %	39.8 %
65	0.0 %	0.3 %	1.3 %	3.1 %	5.9 %	9.7 %	14.3 %	19.6 %	25.4 %	31.4 %
70	0.0 %	0.2 %	0.9 %	2.1 %	4.0 %	6.7 %	10.0 %	13.9 %	18.3 %	23.0 %
75	0.0 %	0.1 %	0.5 %	1.3 %	2.4 %	4.1 %	6.2 %	8.7 %	11.7 %	14.9 %
80	0.0 %	0.1 %	0.3 %	0.7 %	1.3 %	2.2 %	3.3 %	4.7 %	6.4 %	8.4 %
85	0.0 %	0.0 %	0.1 %	0.3 %	0.6 %	1.0 %	1.5 %	2.2 %	3.0 %	3.9 %

↓

– e.g. under an ***effective investment strategy*** (μ = ***5 %*** and σ = ***10 %***) a ***male*** (***retirement age of 65***, ***spending rate 0.06***): the unsustainable pension with ***probability 9.7 %*** (a ***female***: ***13.0 %***).

The probabilities of unsustainable pension (μ = 2.5 % and σ = 5 %) with increasing spending rate:



Maximal spending rates 1/w admissible with a given tolerance of 1 % (i.e. one tolerates the probability of ruin of 1 %) with fixed σ = 5 % - - males:

Male x	***Drift μ:***									
	0 %	***0.5 %***	***1 %***	***1.5 %***	***2 %***	***2.5 %***	***3 %***	***4 %***	***5 %***	***6 %***
55	0.857 %	1.023 %	1.201 %	1.387 %	1.583 %	1.786 %	1.998 %	2.441 %	2.909 %	3.398 %
60	1.053 %	1.221 %	1.397 %	1.582 %	1.774 %	1.973 %	2.179 %	2.609 %	3.061 %	3.533 %
65	1.315 %	1.483 %	1.659 %	1.841 %	2.030 %	2.225 %	2.425 %	2.842 %	3.279 %	3.734 %
70	1.686 %	1.855 %	2.030 %	2.210 %	2.396 %	2.586 %	2.782 %	3.186 %	3.607 %	4.044 %
75	2.265 %	2.435 %	2.609 %	2.787 %	2.970 %	3.156 %	3.346 %	3.738 %	4.143 %	4.561 %
80	3.177 %	3.348 %	3.521 %	3.698 %	3.878 %	4.061 %	4.246 %	4.626 %	5.016 %	5.416 %
85	4.686 %	4.857 %	5.030 %	5.205 %	5.383 %	5.562 %	5.743 %	6.112 %	6.489 %	6.874 %

↓

- e.g. under a ***conservative investment strategy*** (***μ = 1 %*** and ***σ = 5 %***) and a ***low tolerance of 1 %*** → the ***maximal spending rate*** for a ***male*** (***retirement age of 65***): ***1.659 %*** (i.e. ***16 590 annually from 1 000 000***)

Maximal spending rates 1/w admissible with a given tolerance of 10 % (i.e. one tolerates the probability of ruin of 10 %) with fixed σ = 5 % - - males:

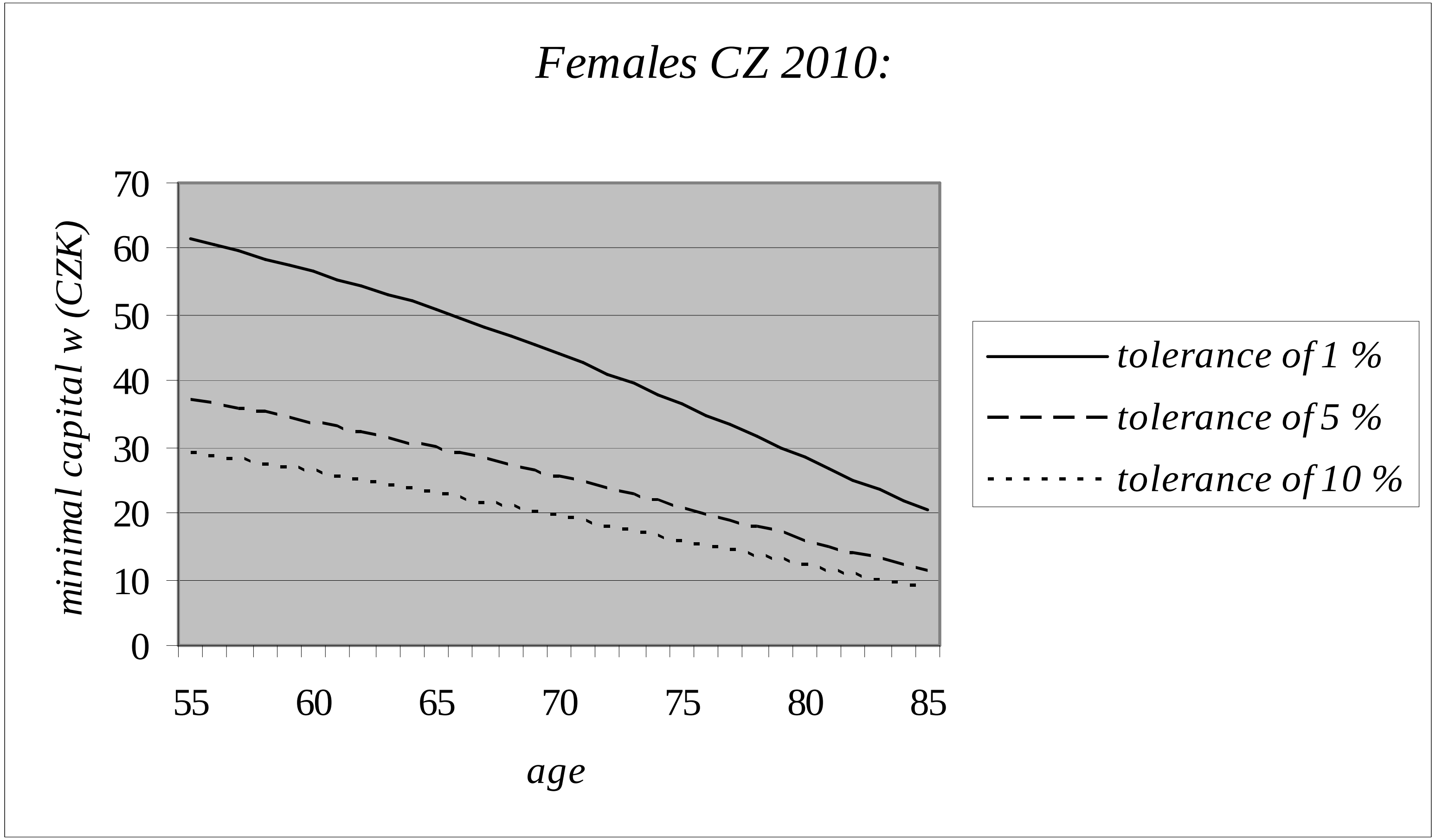
Male	***Drift μ:***									
x	***0 %***	***0.5 %***	***1 %***	***1.5 %***	***2 %***	***2.5 %***	***3 %***	***4 %***	***5 %***	***6 %***
55	2.284 %	2.587 %	2.897 %	3.214 %	3.536 %	3.864 %	4.198 %	4.877 %	5.573 %	6.282 %
60	2.783 %	3.086 %	3.396 %	3.711 %	4.031 %	4.356 %	4.686 %	5.357 %	6.043 %	6.741 %
65	3.445 %	3.749 %	4.058 %	4.371 %	4.690 %	5.012 %	5.338 %	6.001 %	6.676 %	7.364 %
70	4.384 %	4.688 %	4.996 %	5.309 %	5.625 %	5.944 %	6.267 %	6.921 %	7.587 %	8.262 %
75	5.849 %	6.153 %	6.461 %	6.772 %	7.086 %	7.403 %	7.722 %	8.368 %	9.023 %	9.686 %
80	8.156 %	8.462 %	8.769 %	9.079 %	9.391 %	9.705 %	10.021	10.658	11.303	11.955
85	11.969	12.275	12.582	12.891	13.201	13.513	13.826	14.457	15.092	15.733

↓

– e.g. under the ***legislative investment strategy*** (***μ = 2.5 %*** and ***σ = 5 %***) and a ***high tolerance of 10 %*** → the ***maximal spending rate*** for a ***male*** (***retirement age of 65***): ***5.012 %*** (i.e. ***50 120 annually from 1 000 000***)

Minimal capital for unit life annuity with given tolerance (μ = 2.5 % and σ = 5 %) – male:

Minimal capital for unit life annuity with given tolerance (μ = 2.5 % and σ = 5 %) – female:



3. GENERATION LIFE TABLES FOR PENSION SYSTEM

- special Life Tables with projections distinguishing among ***particular generations*** **(*cohorts*)**;
- these Tables must be adjusted in a special way to be ***suitable for pensions***.

Generation LT in Czech Republic should respect two facts typical just for pensions**:**

(1) ***decreasing trend*** of the rates of mortality in time;

(2) ***selection*** of pension portfolios.

− in the ***Current*** (or ***Period***) ***LT***:

q_x depends only on the age x (e.g. q_{65} from such Tables in 2010 concerns the male generation 1945);

− in the ***Generation LT***:

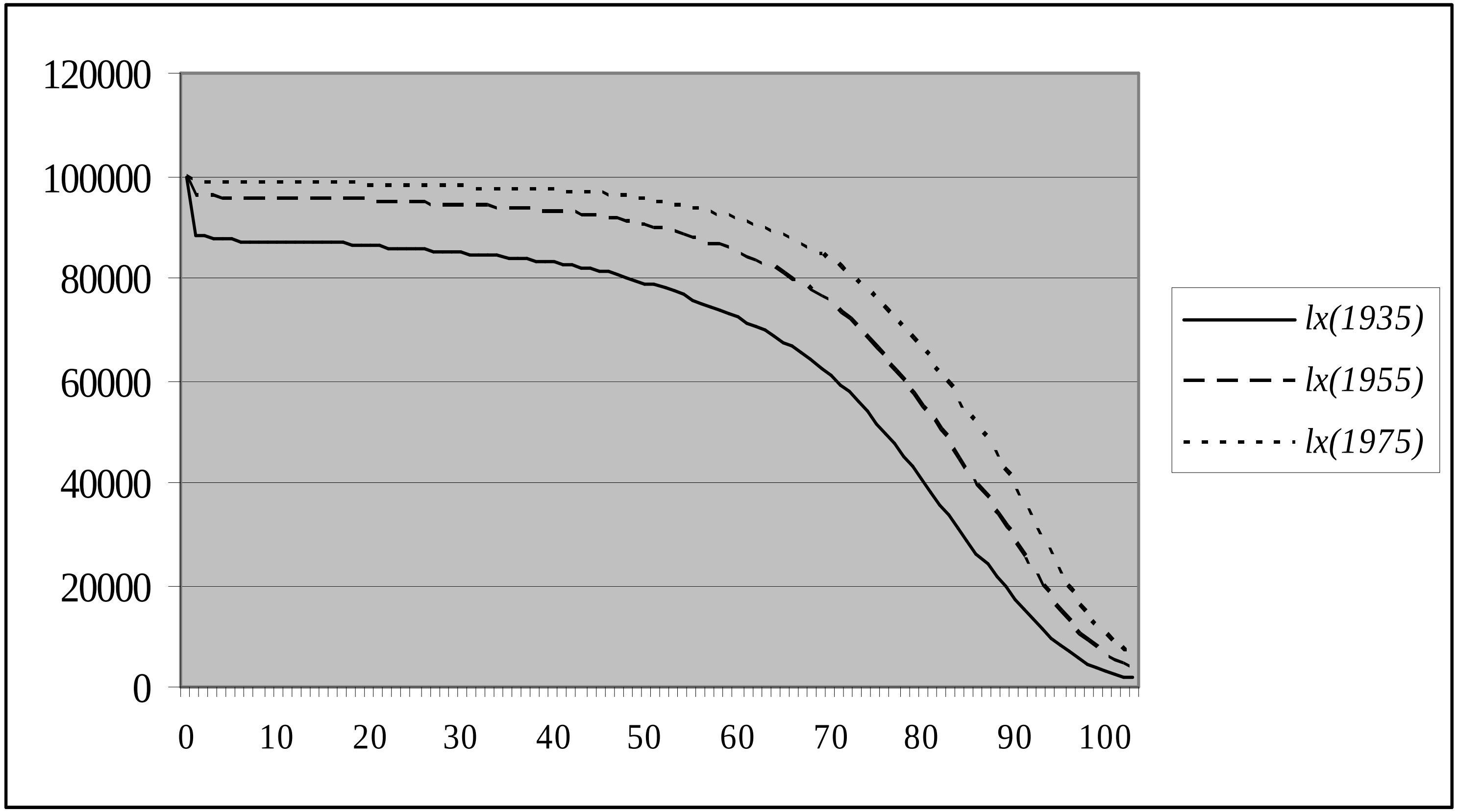
q_x^{τ} depends also on the year of birth (therefore the probability discussed above should be denoted as q_{65}^{1945}):

- this principle must be respected also for *multiyear* probabilities, e.g.

$$ {}_5p_{65}^{1945} = p_{65}^{1945} \cdot p_{66}^{1945} \cdot p_{67}^{1945} \cdot p_{68}^{1945} \cdot p_{69}^{1945} $$

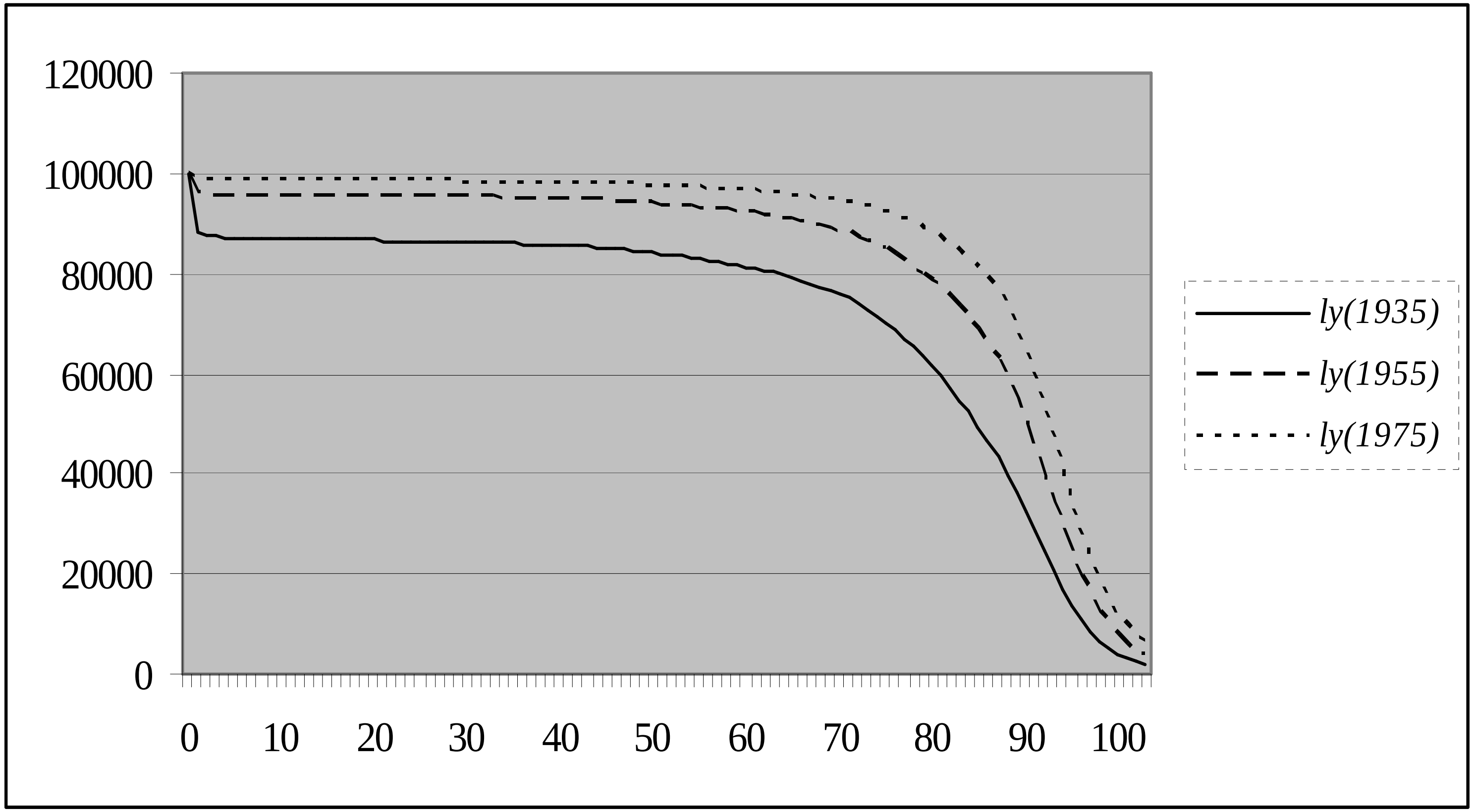
- the application of the Generation LT can be recommended just for the ***Czech Republic*** with a ***high longevity risk in the pension system***.

Numbers of survivors l_x for generations 1935, 1955 a 1975 according to the Generation LT for the Czech Republic - male:



Source: Cipra (1998)

Numbers of survivors l_x for generations 1935, 1955 a 1975 according to the Generation LT for the Czech Republic - female:



Source: Cipra (1998)

Annual payment for life annuity using Generation and Current LT:

Monthly payment for life annuity (the pension account of 1 000 000 in age 60)						
Pension from year:	***Male in age 60 by means of:***			***Female in age 60 by means of:***		
	$e_{60}=\ddot{a}_{60}(0\%)$	$\ddot{a}_{60}(4\%)$	$\ddot{a}_{60}(6\%)$	$e_{60}=\ddot{a}_{60}(0\%)$	$\ddot{a}_{60}(4\%)$	$\ddot{a}_{60}(6\%)$
1996	3 810	5 859	7 011	3 153	5 119	6 250
2000	3727	5 775	6 928	3 076	5 039	6 171
2005	3 626	5 675	6 829	2 990	4 948	6 081
2010	3 534	5 581	6 737	2 913	4 867	6 001
2015	3 446	5 492	6 649	2 844	4 795	5 930
2020	3 364	5 408	6 566	2 782	4 730	5 867
2025	3 287	5 329	6 488	2 728	4 673	5 810
2030	3 215	5 254	6 414	2 679	4 621	5 759
Current	5 128	7 111	8 235	4 087	6 010	7 106

Source: Cipra (1998)

Applying the Generation LT the differences are not negligible from the ***financial point of view*****:**

↑

– the monthly payments for pensions paid off from the age 60 achieved ***in various years*** if the pension account in the age 60 is 1 000 000;

– ***three alternative calculation formulas***:

– $1\,000\,000 / 12 \cdot e_{60}$

– $1\,000\,000 / 12 \cdot \ddot{a}_{60}(4\ \%)$

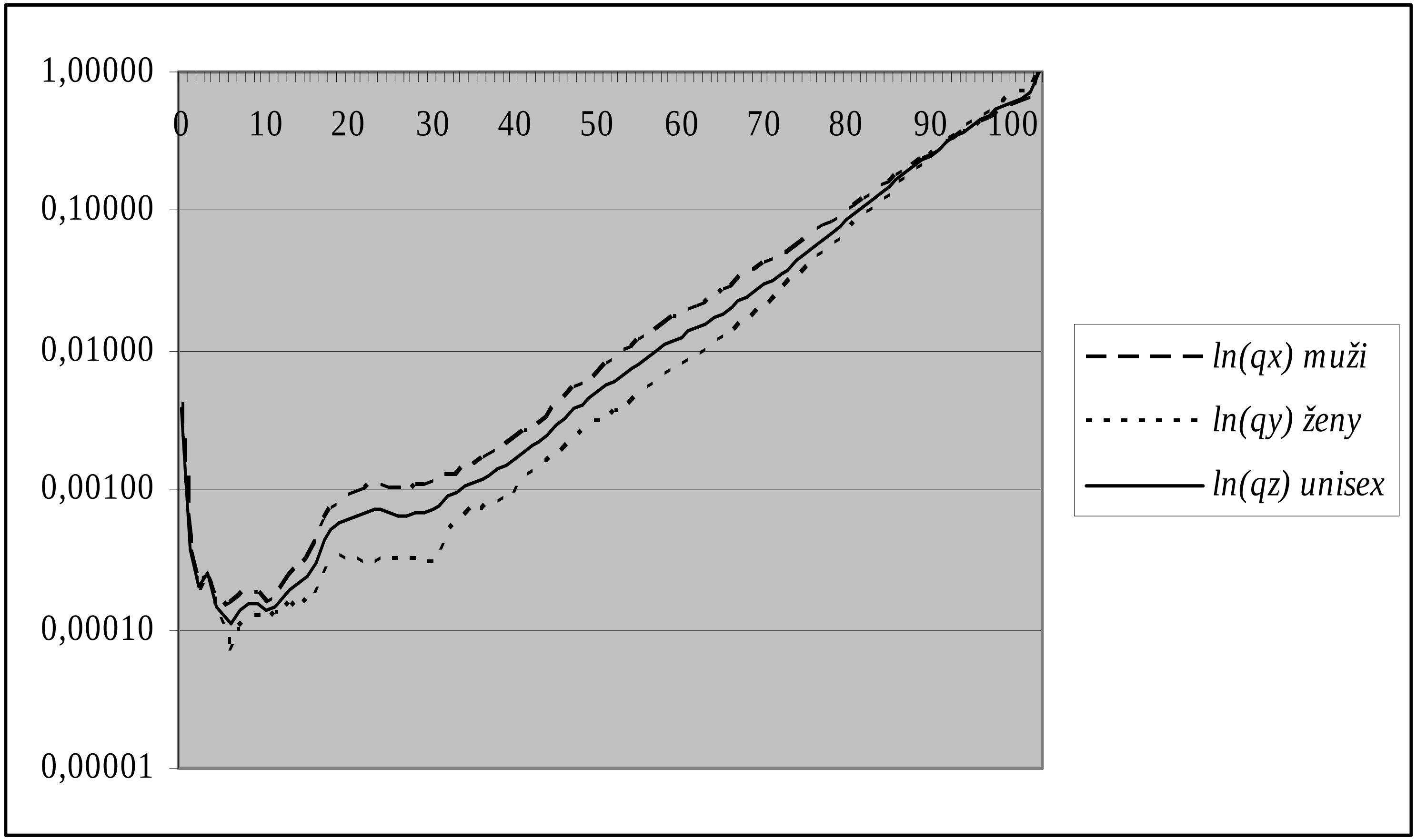
– $1\,000\,000 / 12 \cdot \ddot{a}_{60}(6\ \%)$

- e.g. for the ***interest rate 4 %*** p.a. and for the ***male generation of the age 60 in 2010***:
  - ***Current LT in 2010***: the monthly annuity payment is ***even 7 111***;
  - ***Generation LT***: the monthly annuity payment is ***only 5 581***.

4. UNISEX LIFE TABLES FOR PENSION SYSTEM

- ***Unisex LT*** can be looked upon as a reaction to the movement against the discrimination using the gender as a calculation parameter in pension systems;
- the ***probabilities of death*** q_x according to the ***Unisex LT for the Czech Republic in 2003*** confirm the expectation that the ***unisex mortality is approximately an average of the male and female mortality:***

Probabilities of death q_x according to the Unisex LT for the Czech Republic in 2003



Source: Smetana a Cipra (2005)

Single premium for deferred life annuities with annual payments of 12 000 for the Czech Republic in 2003 (2.4 % p. a.)

Age	*Defer-ment*	*Unisex (U)*	*Males (M)*	*Females (F)*	*Ratio U/M (%)*	*Ratio U/F (%)*	*Ratio M/F (%)*
20	***40***	58 629	50 595	71 864	115.88 %	81.58 %	70.40 %
30	***30***	74 821	64 791	91 382	115.48 %	81.88 %	70.90 %
40	***20***	95 880	83 397	116 564	114.97 %	82.26 %	71.55 %
50	***10***	125 114	110 017	150 445	113.72 %	83.16 %	73.13 %

Source:Smetana a Cipra (2005)

- ***longevity risk must be modified if respecting the gender effect***
- e.g. ***single premium*** in the ***age of 20 with deferment of 40*** (i.e. retirement age of 60): the ***ratio M/F*** is ***70.40 %***

  ratio U/M is ***115.88 %***

  ratio U/F is ***81.58 %***

Level annual premium for deferred life annuities with annual payments of 12 000 CZK for the Czech Republic in 2003 (2.4 % p. a.)

Age	*Defer-ment*	*Unisex (U)*	*Males (M)*	*Females (F)*	*Ratio U/M (%)*	*Ratio U/F (%)*	*Ratio M/F (%)*
20	***40***	2 293	1 997	2 784	114.81 %	82.36 %	71.74 %
30	***30***	3 534	3 091	4 272	114.33 %	82.72 %	72.35 %
40	***20***	6 134	5 398	7 370	113.62 %	83.23 %	73.25 %
50	***10***	14 287	12 712	16 986	112.40 %	84.12 %	74.84 %

Source:Smetana a Cipra (2005)

- ***longevity risk must be modified if respecting the gender effect***
- e.g. ***level premium*** in the ***age of 20 with deferment of 40*** (i.e. retirement age of 60): the ***ratio M/F*** is ***71.74 %***

  ratio U/M is ***114.81 %***

  ratio U/F is ***82.36 %***

5. CONCLUSIONS

Three important aspects that must be taken into account when construc-ting products burdened by ***longevity risk***:

(1) ***sustainability of pensions***;

(2) ***the generation longevity***;

(3) ***the gender longevity***.

↑

It has been shown numerically that these aspects really play a very important role for the ***pensions in the Czech Republic***.

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REFERENCES

Blake, D., & Orszag, J. M. (1999). Annual estimates of personal wealth holdings in the United Kingdom since 1948. *Applied Financial Economics*, 9, 397-421.

Cannon, E., Tonks, I. (2008). *Annuity Market*. Oxford University Press.

Cipra, T. (1998): Cohort life tables for pension insurance and pension funds in Czech Republic. *Pojistné rozpravy*, 3, 31-57 (in Czech).

Cipra, T. (2010): *Financial and Insurance Formulas*. Heidelberg, Dordrecht, London, New York: Physica-Verlag / Springer.

Clark, G. L., Munnell A. H., Orszag, J. M. (Eds.) (2006). *Pensions and Retirement Income*. Oxford University Press.

Dufresne, D. (1990): The distribution of a perpetuity with applications to risk theory and pension funding. *Scandinavian Actuarial Journal*, 9, 39-79.

Milevsky, M. A. (1997). The present value of a stochastic perpetuity and the Gamma distribution. *Insurance: Mathematics and Economics*, 20, 243-250.

Milevsky, M. A. (2006). *The Calculus of Retirement Income*. Cambridge University Press.

OECD (2011). *Pensions at a Glance 2011: Retirement-income Systems in OECD and G20 Countries*. OECD Publishing.

Sheshinski, E. (2005). *The economic theory of annuities*. Princeton University Press.

Smetana, P., Cipra, T. (2005): Unisex life-tables for the Czech Republic and consequences for insurance calculations. *Pojistný obzor*, 82, 10-17 (in Czech).

Uebelmesser, S. (2004): *Unfunded Pension Systems: Ageing and Migration*. Elsevier.