Rethinking Age-Period-Cohort Mortality Trend Models

Daniel Alai Michael Sherris

Centre of Excellence in Population Ageing Research

Actuarial Studies, Australian School of Business
University of New South Wales

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Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions

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Motivation

- Models where age parameters interact with period parameters are well studied in the literature (e.g. Lee-Carter models).
- Rather than building upon such a structure, with its accepted pitfalls, we propose to take a step back.
- By rethinking model fundamentals, our approach is to clearly separate the influence of factors that drive mortality improvement.
- Inspired by models found in non-life insurance, we
  - identify trends with reliable estimation procedures (maximum likelihood), and
  - propose a framework that has the potential to improve forecasting performance.

The Mortality Triangle

- Traditionally, mortality data has been presented with an emphasis on the calendar year of death (period tables).
- By transforming the data as shown below, we
  - shift the emphasis to year of birth, and
  - format the data in a non-life setting (triangle data).

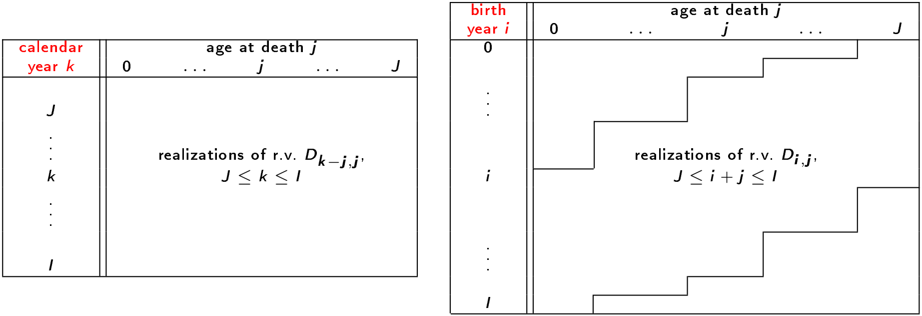

Figure: Transforming life insurance data, the mortality triangle.

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The Non-Life Insurance Model

Model Assumptions

- *Increments $X_{i,j}$ are independent (overdispersed) Poisson distributed.*
- *The regression formula is given by*

$$\eta_{i,j} = \beta_0 + \beta_{1,i} + \beta_{2,j}, \qquad i \in \{0, \ldots, I\},\ j \in \{0, \ldots, J\},$$

  where $\beta_{1,0} = \beta_{2,0} = 0$.
- *The link function is given by $g(\mu) = \ln(\mu)$.*

- This model replicates the expected future liabilities produced by the classic deterministic chain ladder method!

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Age-Period-Cohort Models in the GLM Framework

Model Assumptions

- *Deaths $D_{i,j}$ are independent (overdispersed) Poisson distributed.*
- *The regression formula is given by*
  - *Age-Period:* $\eta_{i,j} = \beta_0 + \beta_{2,j} + \beta_{3,i+j}$, *(AP)*
  - *Age-Cohort:* $\eta_{i,j} = \beta_0 + \beta_{1,i} + \beta_{2,j}$, *(AC)*
  - *Age-Period-Cohort:* $\eta_{i,j} = \beta_0 + \beta_{1,i} + \beta_{2,j} + \beta_{3,i+j}$, *(APC)*

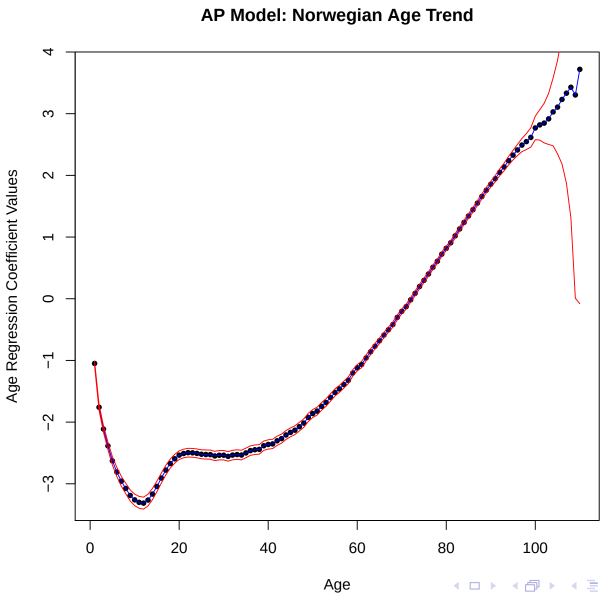
  where $\beta_{1,0} = \beta_{2,0} = \beta_{3,0} = 0$.
- *The link function is given by* $g(\mu) = \ln(\mu)$.

- Age, period, and cohort effects are modelled with distinct parameters for each age, calendar year of death, and year of birth.
- Note that the age-cohort model is exactly the non-life insurance model.

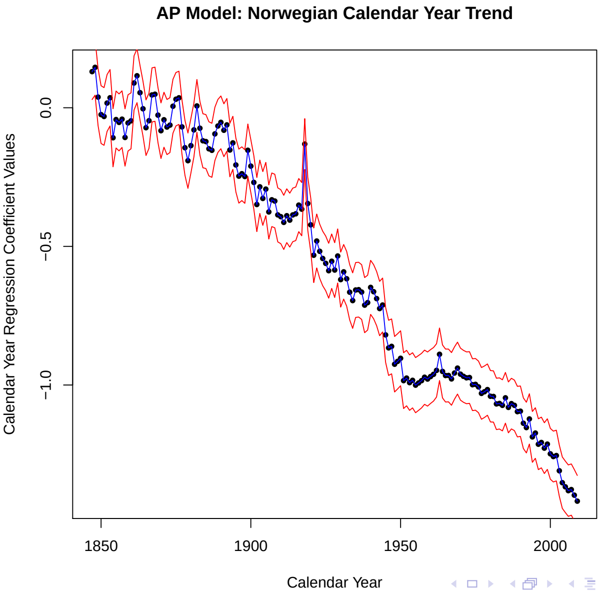
Fitting the Models to Norwegian Mortality Data

- Norwegian period mortality data dating back from 1846 to 2008 *(source: the Human Mortality Database)*.
- In the paper, we contrast this with Australian data. The models can be fit to any country data with relative ease.
- We study the regression parameter trends:
  - age trend,
  - calendar year (period) trend,
  - birth year (cohort) trend,
- To gauge the fit, we study the errors with respect to the omitted trend (either period or cohort).

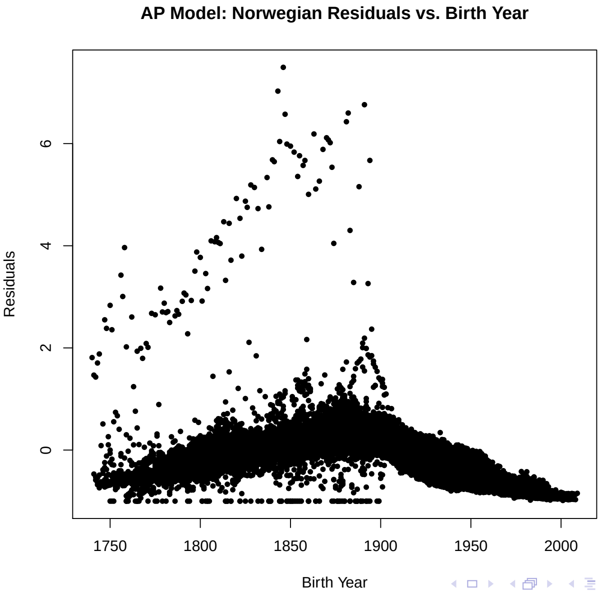
The Age-Period Model: Age Trend



The Age-Period Model: Period Trend



Residuals of the AP Model Plotted Against Birth Year



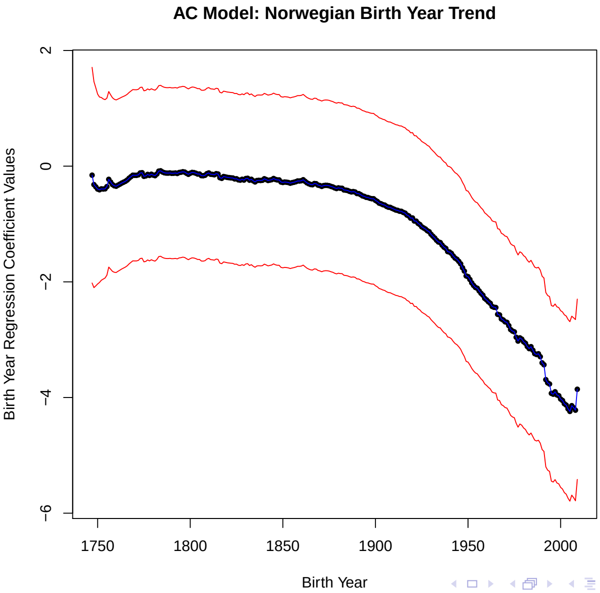
The Age-Period Model

- The age trend follows the typical shape of average log-mortality rates.
  - Relatively high infant mortality.
  - Accident hump present in early adulthood.
  - Decaying mortality for the older ages (typically modelled as linear).
- The age trend retains this pattern for the age-cohort and age-period-cohort models, where it exhibits slightly more decay for the older ages.
- The model has difficulty fitting the older ages (centenarians).

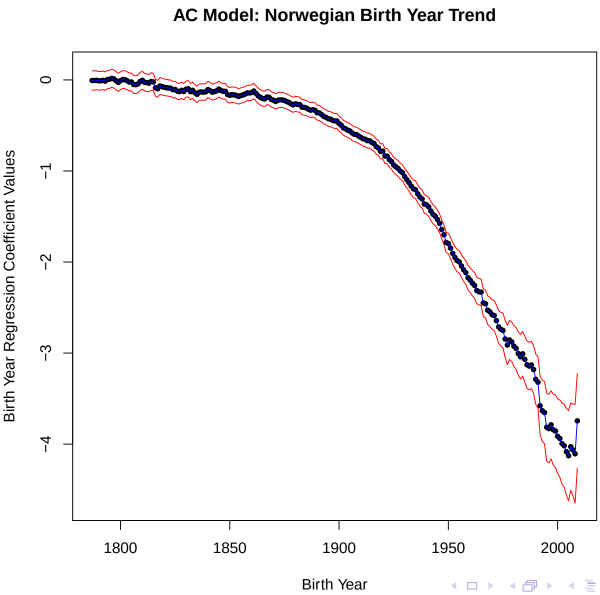
- The period trend is downward (indicating mortality improvement) and roughly linear.
  - Similar to the mortality index found in the Lee-Carter model.

- The residuals are not well behaved. It appears birth year is a significant covariate and aught to be included in the model!
  - Note: large residuals are result of the model's difficulties fitting centenarians.

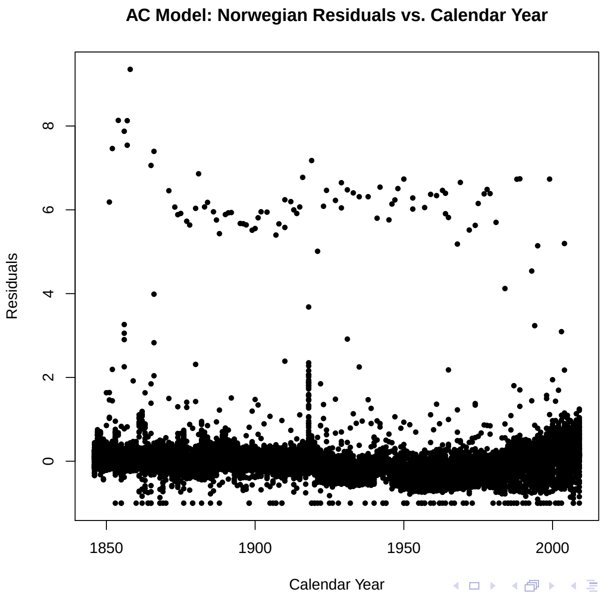
The Age-Cohort Model: Cohort Trend



The Cohort Trend - Omitting the First 50 Cohorts



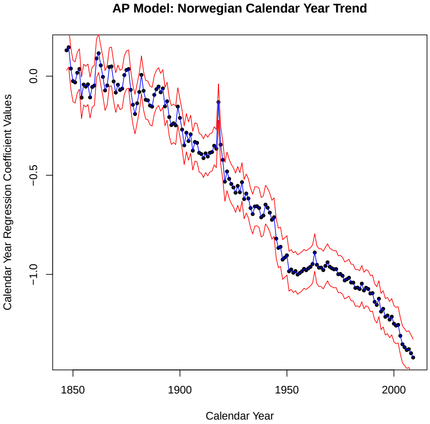
Residuals of the AC Model Plotted Against Calendar Year

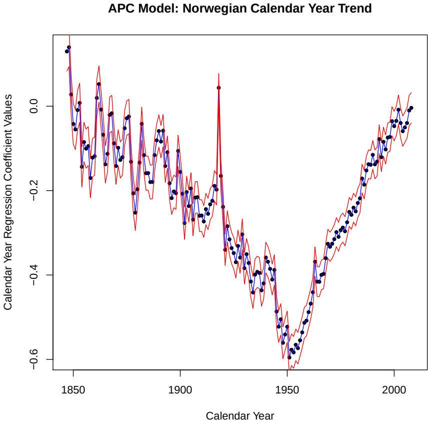


The Age-Cohort Model

- The cohort trend is downward (indicating mortality improvement) and roughly quadratic.
  - It is a smooth trend compared with the period trend found in the AP model.
- It *appears* to have high uncertainty!
  - This uncertainty originates from the earliest cohorts. If we remove, say, the first 50 cohorts, our confidence intervals become much tighter.
- The residuals are much better behaved. With the exception of 1918, the period covariates do not appear needed in the model.
  - Note: large residuals are result of the model's difficulties fitting centenarians.

The Age-Period-Cohort Model





- In the full age-period-cohort model (right), the period trend changes dramatically compared to the age-period model (left).
- The cohort trend remains unchanged when moving from the age-cohort to the age-period-cohort model.

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The Lee-Carter Model

Mortality improvements vary considerably among different age groups.

Lee and Carter introduce:

- κ_t, an index representative of the mortality improvement over time, $t \in \{1, 2, \ldots, T\}$.
- b_x, a measure of the share of that general improvement by age, $x \in \{0, 1, \ldots, \omega\}$.

They model the log mortality rate as

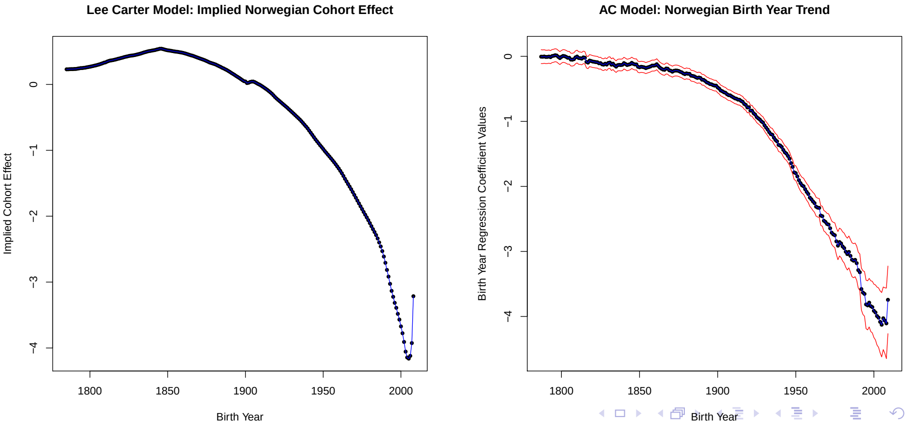
$$\ln(m_{x,t}) = a_x + b_x \kappa_t + \varepsilon_{x,t},$$

where $\varepsilon_{x,t}$ has mean zero and variance σ_ε.

The Implied Lee-Carter Cohort Effect

An age-period interaction term is really just a stand-alone cohort term.

- Let $\gamma_i = \text{average}\{b_x \kappa_t\}$, where the average is over the available combinations of x and t suitable for cohort i.
- We call γ_i the implied cohort effect and compare it with the stand-alone cohort effect in our model framework.



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The Moral of the Story

We fit mortality rates using three available indices, age, calendar year, and year of birth.

- Age certainly plays an important role.
- We show that year of birth dominates over calendar year of death and should be the second index included in the model!
- If necessary, one off period effects can be included, such as 1918 for the Norwegian data.

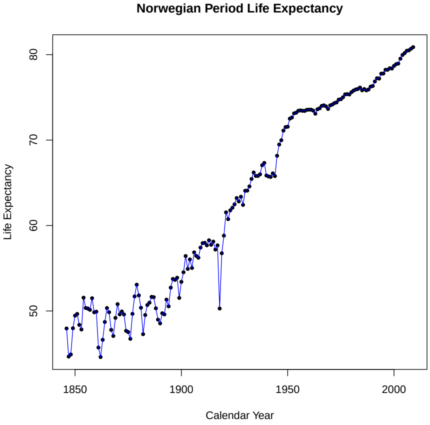
Comparison with the Lee-Carter model

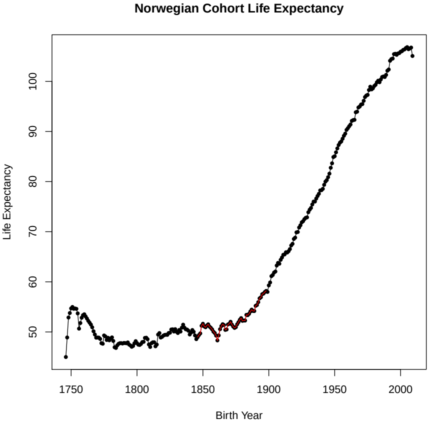
- The popular Lee-Carter model uses a bilinear period effect that compares to a stand alone cohort effect.
- It is well known that the Lee-Carter model has difficulties forecasting due to the fact that the share of overall mortality improvement for age x is constant over time.
- A stand alone cohort effect does not have this problem.

On the Horizon

Problems to consider in the future

- Cohort effects need to be updated after realizing a new year of data.
- Life expectancy, forecasting, and uncertainty.





Thank you!