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Individual Tontine Accounts

Richard K. Fullmer & Michael J. Sabin

Motivation

We desire to explore the boundaries of fair tontine design

Why?

- Shift from defined-benefit (DB) to defined-contribution (DC) pension systems
- Tendency of individuals to rollover DC balances to individual accounts after retirement

- Need for efficient, practical retirement income solutions
- Need for solutions to the “annuity puzzle”

Tontines

A tontine is an investment combined with a payout scheme

- Generally irrevocable
- Upon death, balances are transferred to surviving tontine members

The total return is a function of:

- Investment gains/losses
- Mortality gains/losses
  - Mortality losses occur at death when a member's balance is forfeited
  - Mortality gains ("tontine gains") occur when forfeited balances are redistributed to surviving members

Collective pooling of longevity risk… *without insurance*

- Surviving investors collect a "tontine yield"
- No guarantees whatsoever
- No risk reserves, no mortality risk charges, no counterparty risk

Fair Tontines

A tontine is fair when the expected value of each member's tontine gain/loss is zero

Accomplished by redistributing forfeited balances in an actuarially neutral way

- The expected amount received from dying members matches the expected amount forfeited by dying
- This is true for all members, at all times

Previous work has shown this can be achieved using a common investment pool and payout scheme…

…but what about individual accounts?

Fair Tontine Theorem

Survivor j's tontine gain is a random amount C_j. For fairness:

$$\underbrace{-q_j s_j}_{\text{Expected forfeiture}} + \underbrace{(1-q_j)\mathrm{E}\big(C_j \mid j \text{ survives}\big)}_{\text{Expected gain}} = 0.$$

Solving:

$$\mathrm{E}\big(C_j \mid j \text{ survives}\big) = \underbrace{\frac{q_j s_j}{1-q_j}}_{\text{nominal gain } c_j} = \underbrace{\left(\frac{q_j}{1-q_j}\right)}_{\text{nominal gain rate } r_j} \times s_j.$$

Theorem. In a fair tontine,

$$\mathrm{E}\big(C_j \mid j \text{ survives}\big) = c_j = r_j s_j = \left(\frac{q_j}{1-q_j}\right) s_j$$

for every member j.

Surprise! A member's expected gain depends only on his or her own age and gender (for q) and balance s.
The parameters of other members do not matter.

The Actual Values Will Not Exactly Match

What we have: Amount forfeited by decedents is:

$$\sum_{j \in D} s_j,$$

where D is an index set that identifies who died.

What we need: Total nominal gains of survivors is:

$$\sum_{j \notin D} c_j.$$

In general, the two are not equal so we cannot give survivors their exact nominal gains.

But They Do Match on Average (Ex Ante)

Expected value forfeited by decedents is:

$$\mathrm{E}\left(\sum_{j\in D} s_j\right) = \sum_j q_j s_j.$$

Expected value of nominal gains of survivors is:

$$\mathrm{E}\left(\sum_{j\notin D} c_j\right) = \sum_j (1-q_j)c_j = \sum_j (1-q_j)\frac{s_j q_j}{1-q_j} = \sum_j q_j s_j\,.$$

The two expected values match.

If the pool is large, survivors can expect to receive an amount that is close to their nominal gains.

Nominal-Gain Method

Distribute forfeited amounts in proportion to survivors' nominal gains.

Surviving member j's actual gain is:

$$C_j = c_j \times \underbrace{\left(\frac{\sum_{j \in D} s_j}{\sum_{j \notin D} c_j} \right)}_{\text{group gain } G} = c_j G = r_j s_j G.$$

Verify match:

$$\underbrace{\sum_{j \notin D} C_j}_{\text{amount received by survivors}} = G \sum_{j \notin D} c_j = \underbrace{\sum_{j \in D} s_j}_{\text{amount forfeited by decedents}} .$$

Note that $G \geq 0$ holds at all times.

Is the Nominal-Gain Method Fair?

Recall that fairness requires

$$\mathrm{E}\big(C_j \mid j \text{ survives}\big) = c_j = r_j s_j.$$

Since $C_j = Gc_j = Gr_j s_j$ in our tontine, fairness requires

$$\mathrm{E}(G \mid j \text{ survives}) = 1$$

for each member j.

In practice, $\mathrm{E}(G \mid j \text{ survives})$ is very nearly, but not exactly, equal to 1. The difference is negligible [see Sabin and Forman (2016)].

Moral: For practical purposes, it is fair.

We could have used a precisely fair method as in Sabin (2010), but it is complex and much harder for the typical retiree to understand. We like the practical simplicity of the Nominal-Gain Method.

Exploring the Boundaries of Fair Tontine Design

Fairness de-correlates my tontine gain from the investment returns of other members

- Expected value of my tontine gain depends only on my account balance and my probability of dying
- Same is true for every other member

By simulating individual tontine brokerage accounts, we demonstrate that:*

- The demographic characteristics of other members does not matter
- The contribution amounts of other members does not matter
- The contribution timing of other members does not matter
- The investment choices of the other members does not matter
- The payout choices of the other members does not matter

* We show that under the nominal-gain method, the effect of these variables on member outcomes is negligible.

Simulation Methodology

We simulate 1,000 new members enrolled at the beginning of each year from 2019 to 2100 with randomly selected:

- Age: 65 to 85, inclusive
- Gender: male or female
- Contribution: log-uniform value from \$1,000 to \$1,000,000
- Portfolio: all stock, all bond, or 50/50 blend
- Payout schedule: life annuity or lump sum after 10 years

Life annuity payout is computed as s/a, where $a = \ddot{a}_x = 1 + \sum_{t=1}^{\infty} {}_tp_x\,(1+i)^{-t}$ with the assumed annual interest rate i set to 4%

Simulation Methodology

For this set of 82,000 members, we evaluate 10,000 simulation runs

- Each run has randomly selected investment returns for stocks and bonds
- Each run assigns a randomly selected year of death (82,000 x 10,000 = 820,000,000 random years of death)
- Thus, a total of 82 x 10,000 = 820,000 group gain values were calculated

We use the 2012 Individual Annuitant Mortality (IAM) basic table projected forward using scale G2:

- For determining nominal tontine gain yields
- In randomly selecting the death year [more about this later]

	Arithmetic Mean	Standard Deviation	Correlation	
Stock	9.0%	18.0%	1.0	
Bond	5.5%	6.5%	0.3	1.0

Operation

At the start of each year:

- Determine the nominal gain rates r for each age/gender cohort for the year
- Enroll new members
- Rebalance portfolios

At the end of each year:

- Compute each member's investment return and new balance
- Compute the Group Gain G
- Credit each surviving member with a tontine gain equal to $Gr_j s_j$
- Compute each surviving member's payout and deduct it from the member's balance
- Set each decedent's balance to zero
- Delete members with a zero balance (decedents as well as survivors that received a lump-sum payout)

Assessing Fairness

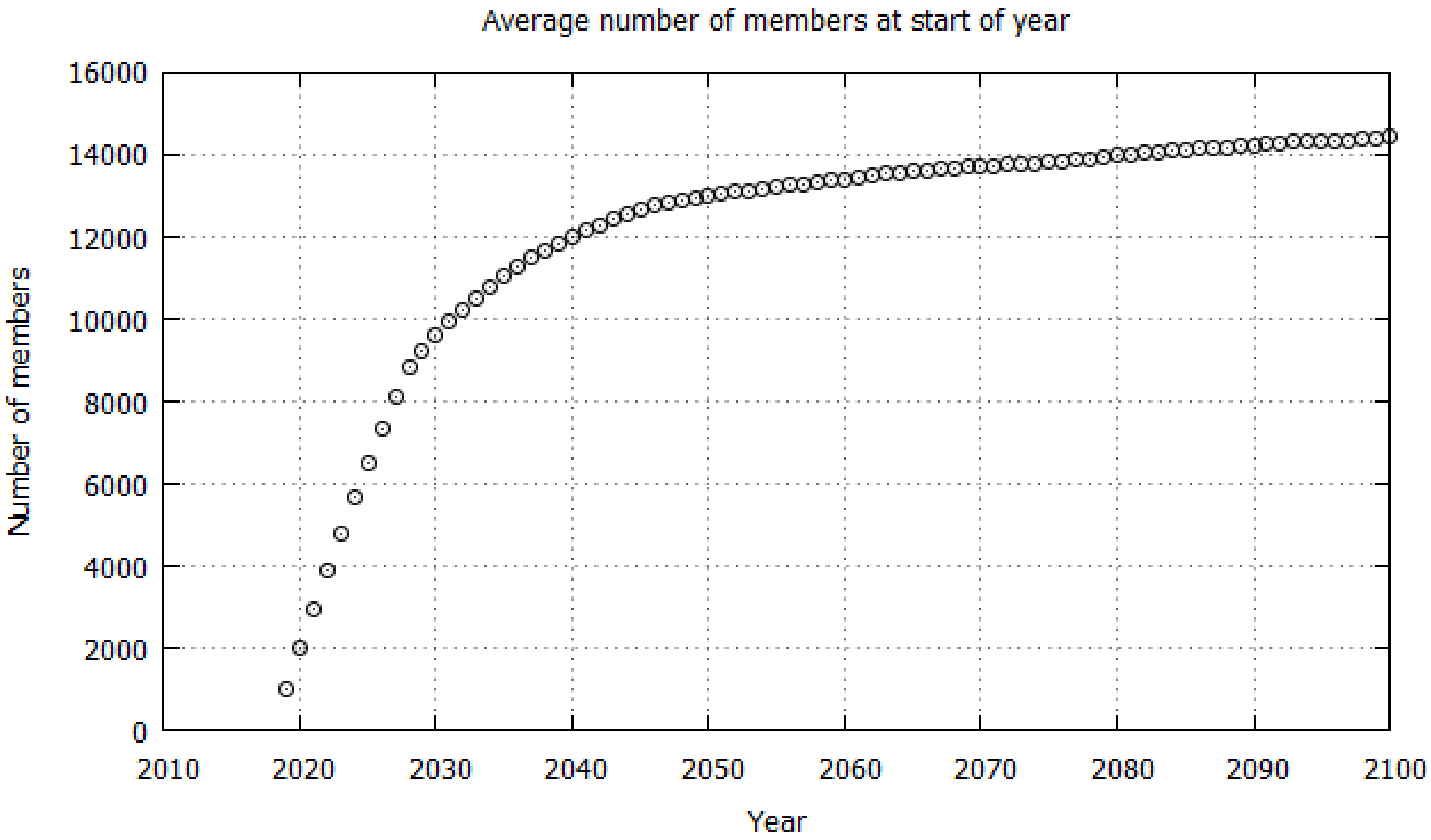
Members hope to achieve the same return as if they had invested outside of a tontine, plus a nominal tontine yield

But actual tontine gains will naturally vary from their nominal values to some degree because mortality pooling does not eliminate mortality risk completely

However, if the group gain G is close to 1 then members will receive actual tontine yields of rG that are close to their nominal values of r

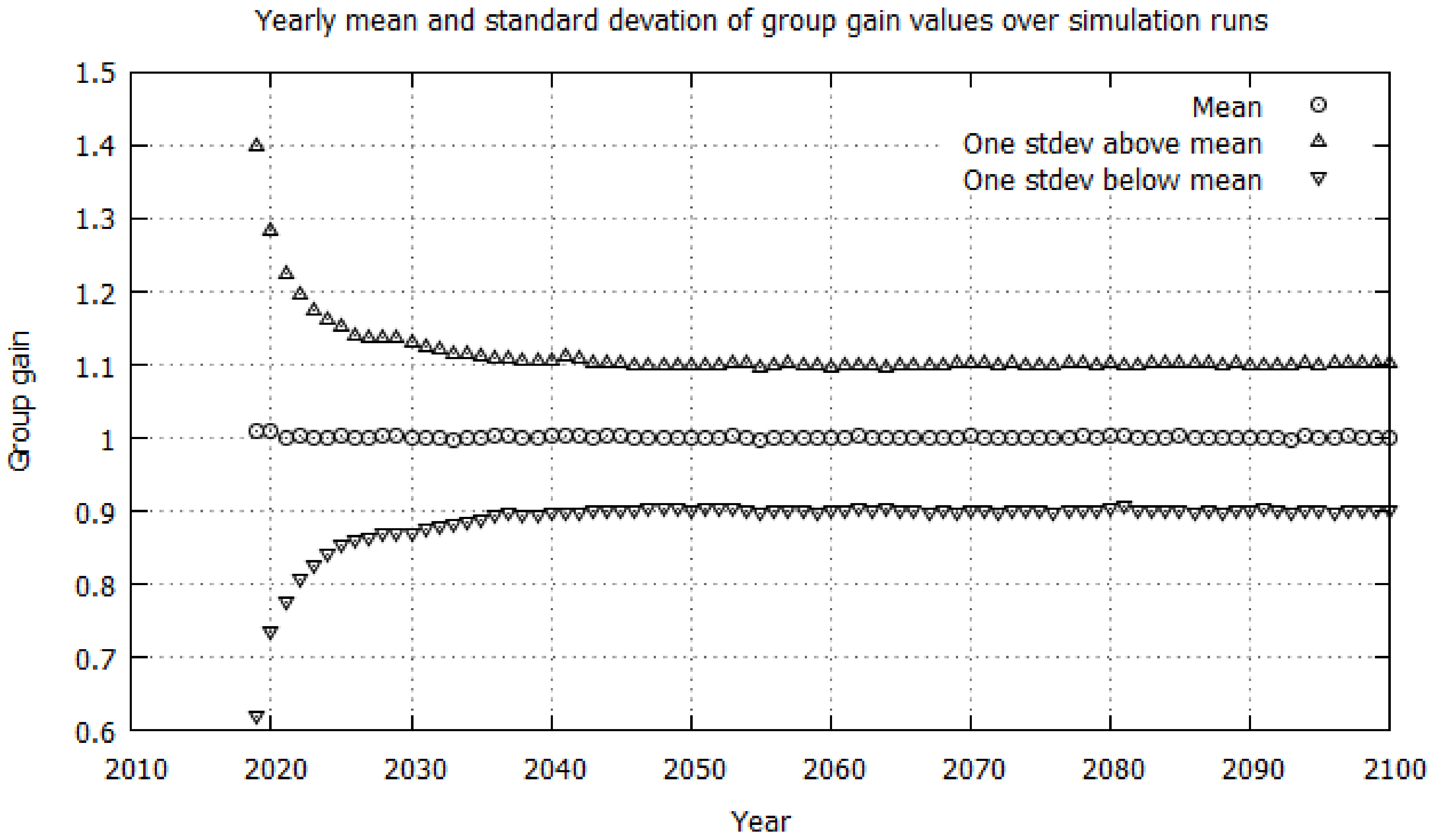
Thus, an assessment of fairness boils down to the characteristics of the group gain G

Pool Size



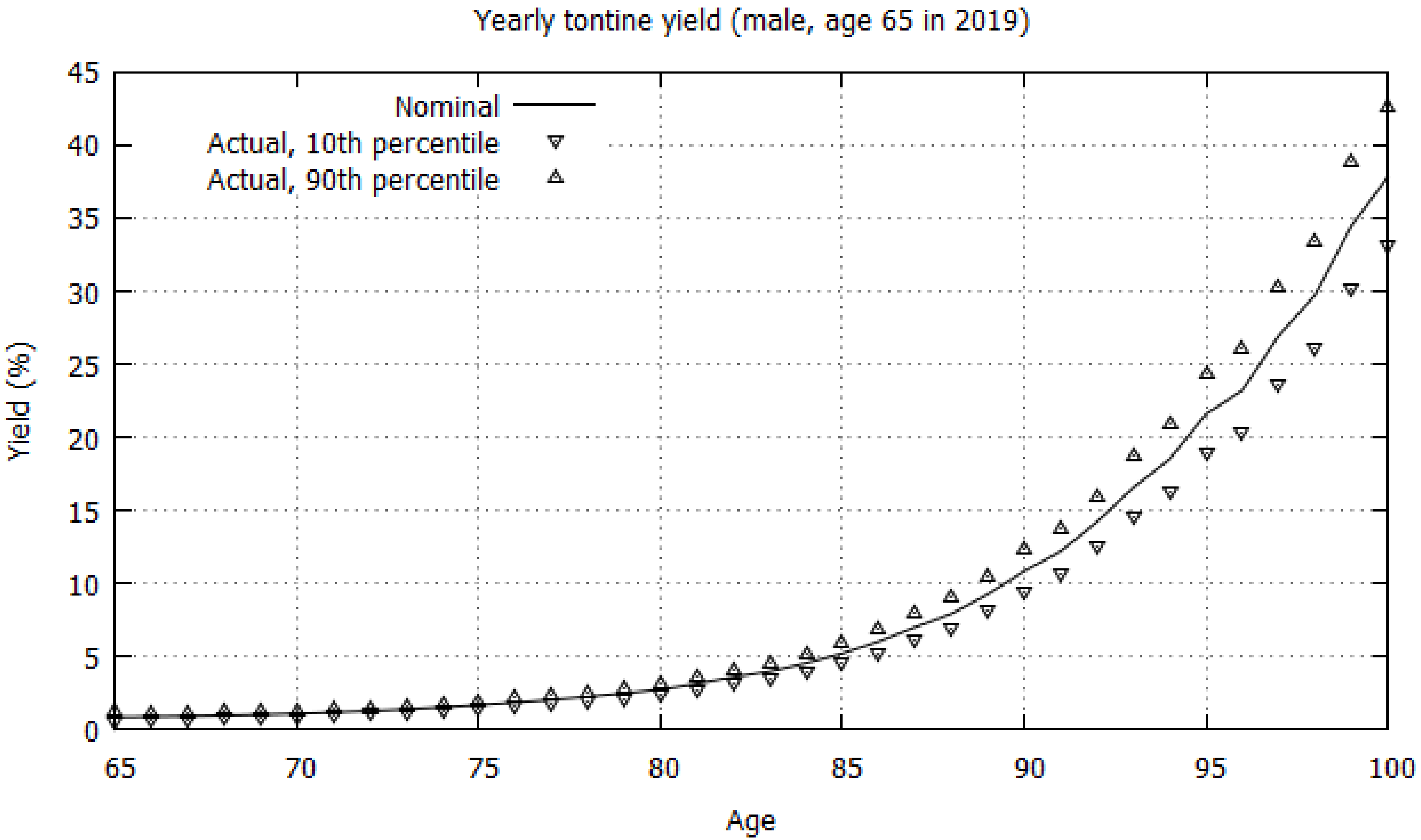
Average number of members at start of year
Number of members
16000
14000
12000
10000
8000
6000
4000
2000
0
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
Year

Group Gain

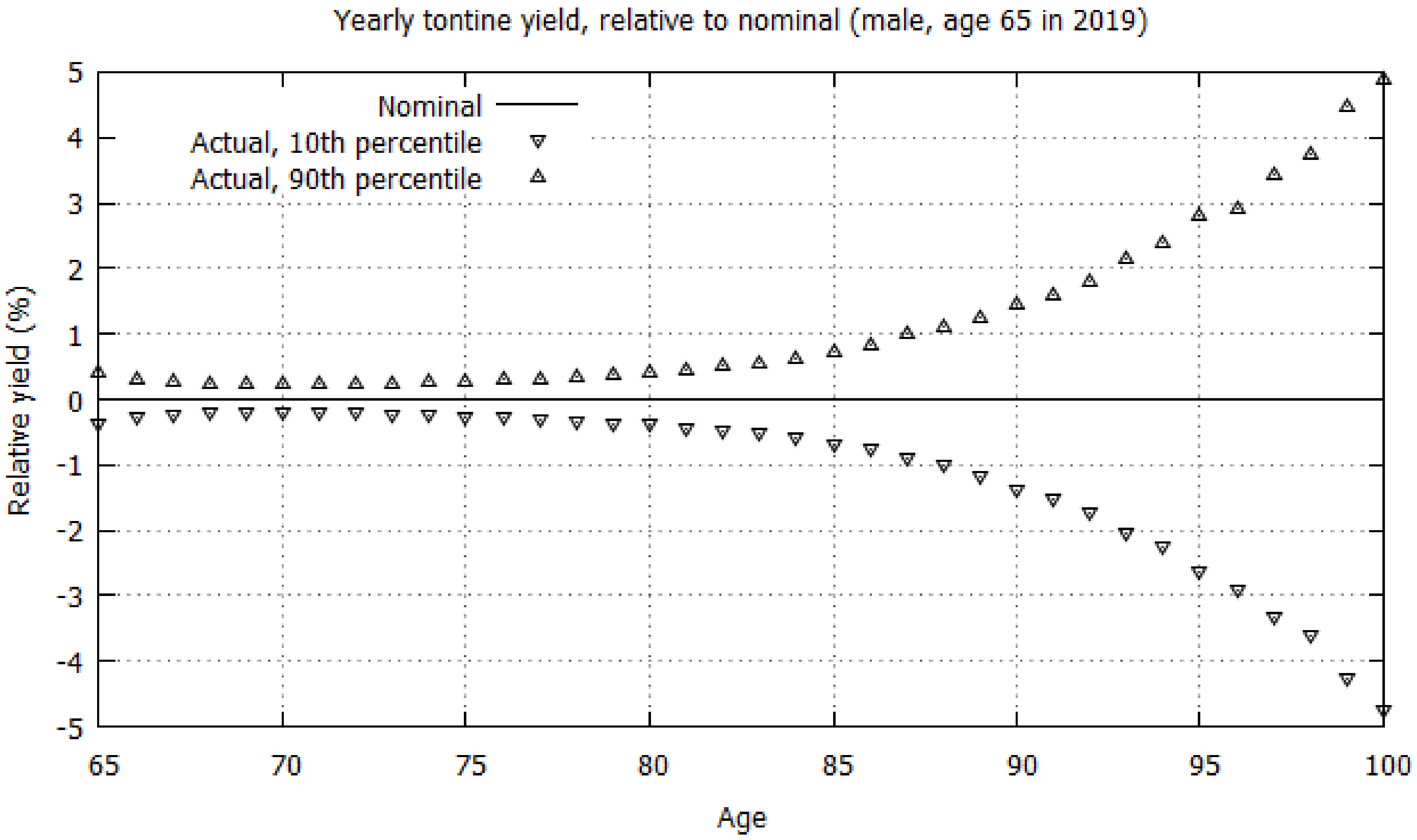


Yearly mean and standard devation of group gain values over simulation runs
Mean
One stdev above mean
One stdev below mean
Group gain
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
Year

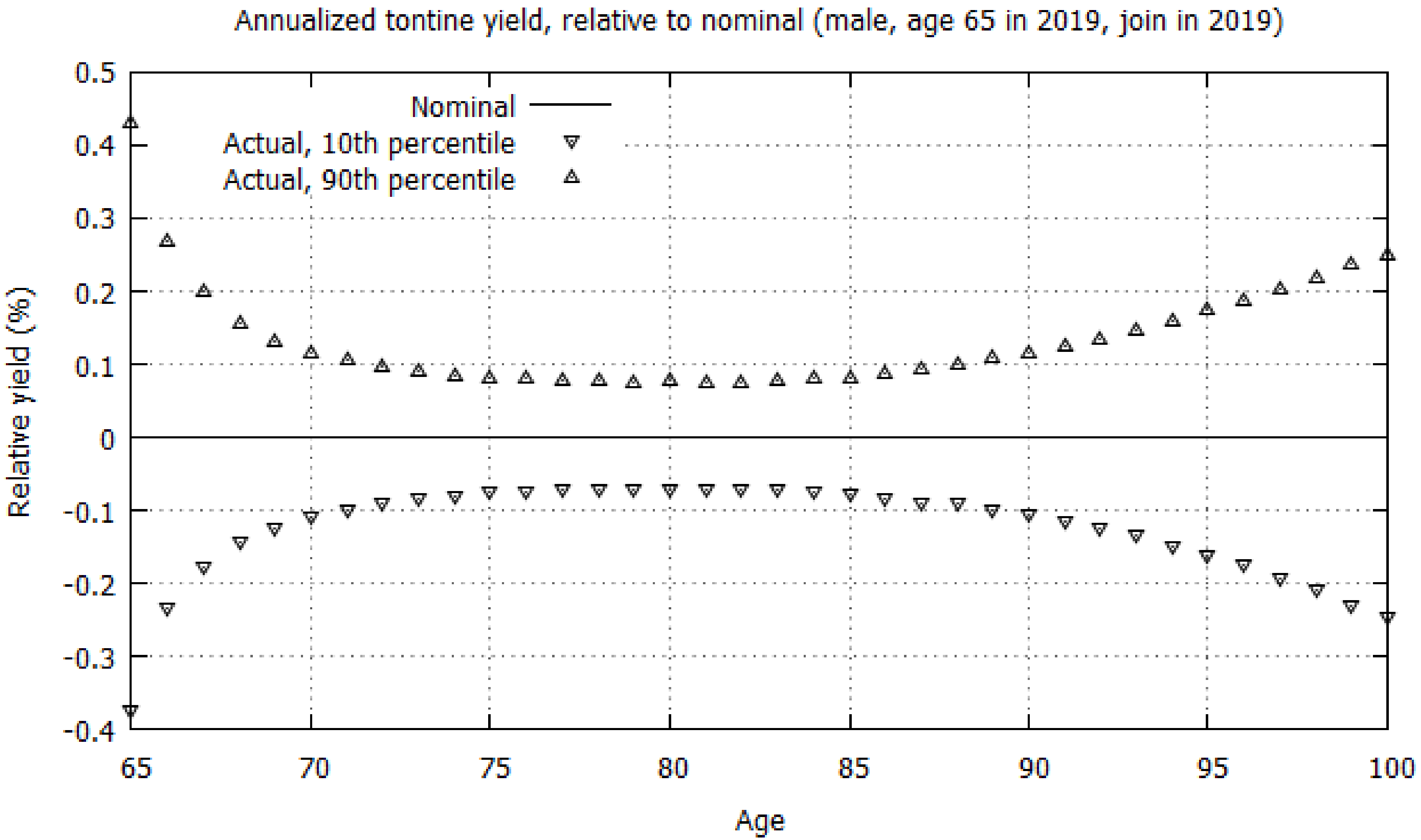
Yearly Tontine Yield



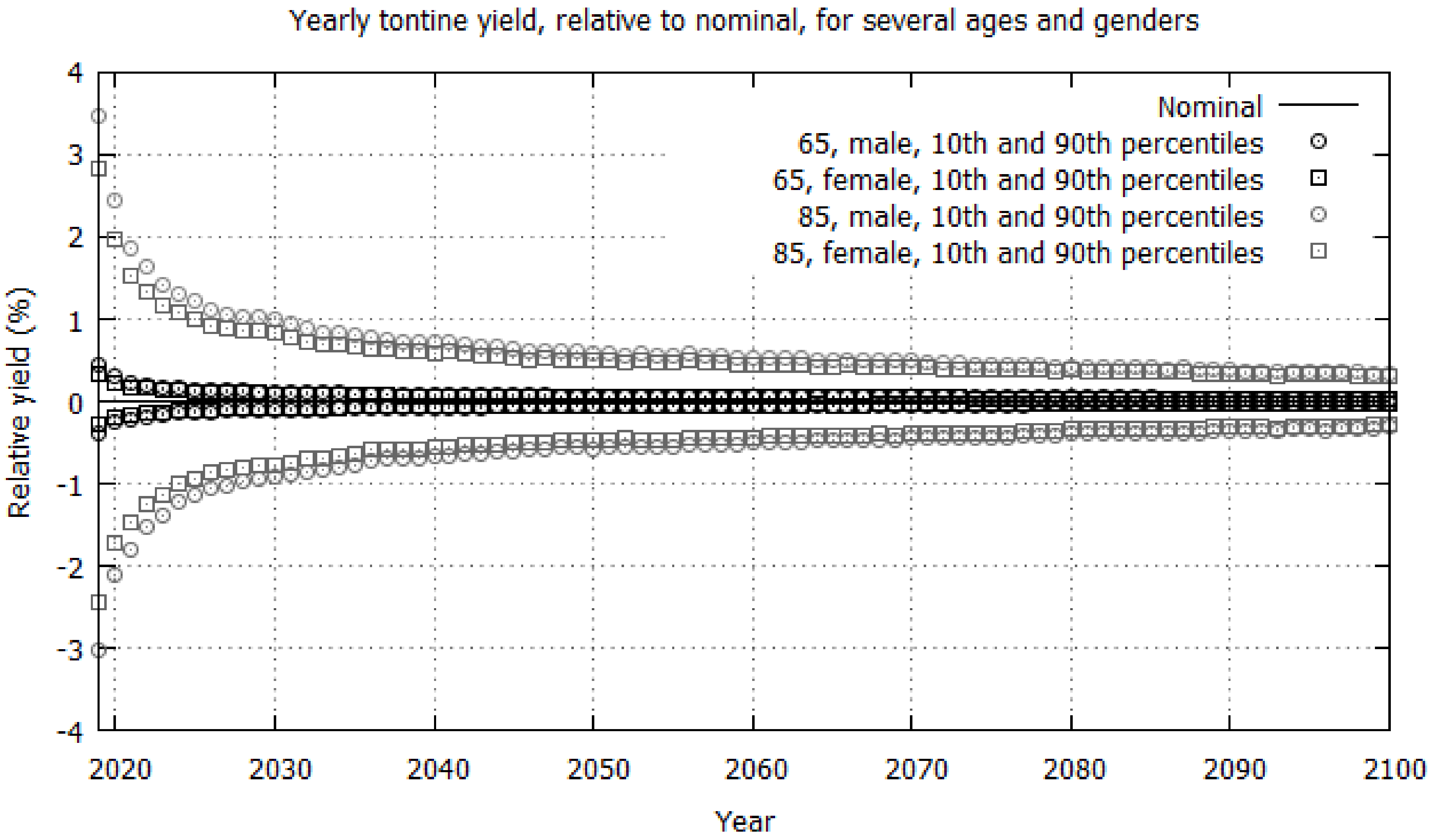
Yearly Tontine Yield, Relative to Nominal



Annualized Tontine Yield, Relative to Nominal



Yearly Tontine Yield, Relative to Nominal, by Calendar Year



Effect of the Investment Decisions of Others

Q) Why isn't an aggressive investor disadvantaged when in a pool dominated by conservative investors who are likely to die with lower balances?

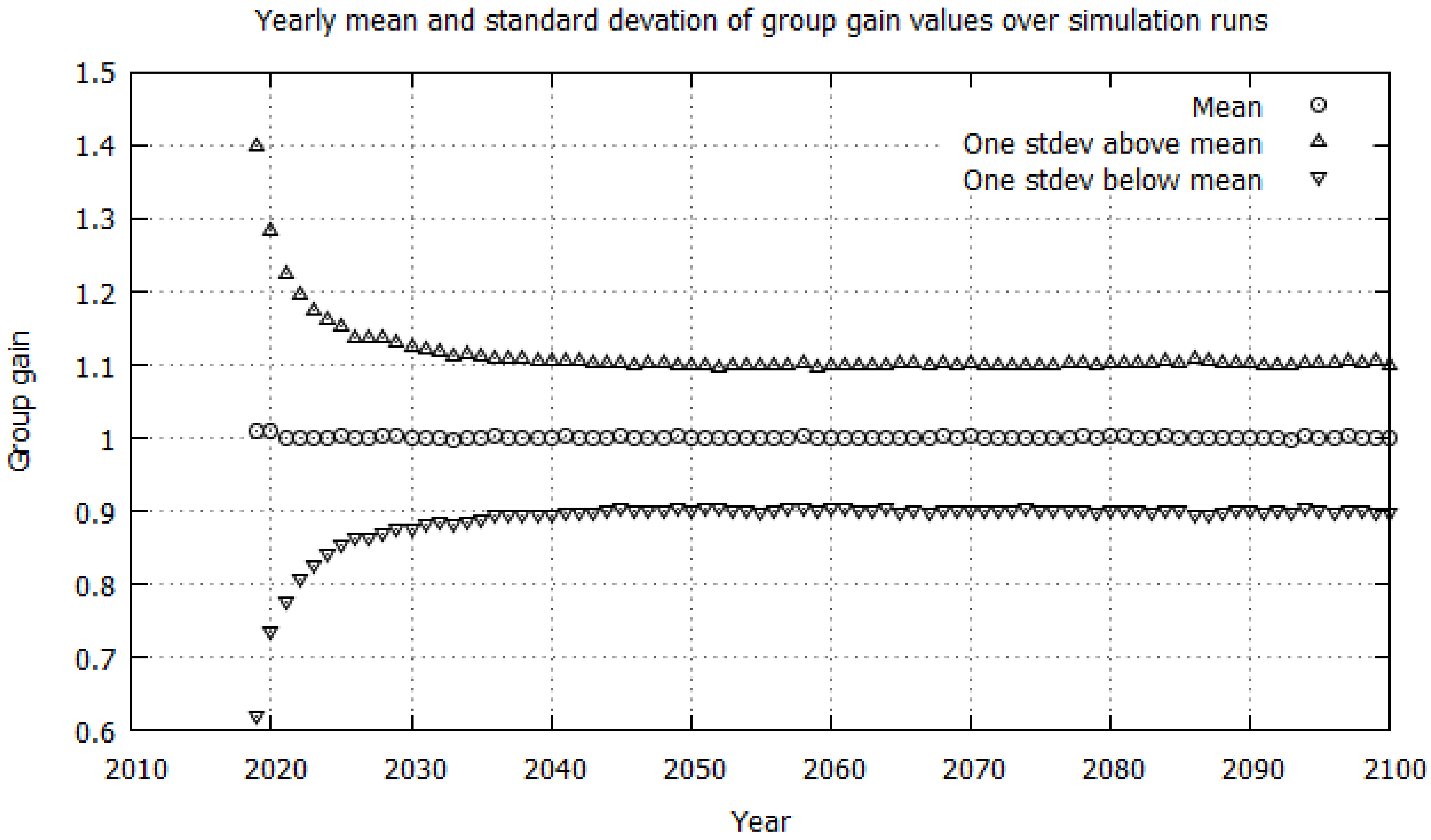
A) By ensuring that reallocations are fair, each member will have an expected nominal tontine gain of $rs = sq/(1-q)$ regardless of how anyone else invests

- Higher returns lead to a higher balance, s
- A higher value of s results in a higher value of rs and thus a greater share of forfeiture allocations
- Member investment choices affect both the numerator and the denominator of the group gain formula such that the expected value of G remains very close to 1
- Thus the actual tontine gain rsG received by each member remains very close to its fair value of rs

To demonstrate this, we reran our base analysis with the following change:

- 95% probability of being assigned an all-bond portfolio
- 5% probability of being assigned an all-stock portfolio

Group Gain when Portfolio Selection is Skewed Conservatively



Comparing Expected Mortality Yields: ITAs vs. Annuities

Our ITA tontine annuity is similar to a variable-income annuity (VIA)

The main difference lies in who bears systematic mortality risk

- In the ITA, the members bear it
- In the VIA, the insurer bears it… and charges a risk premium to do so, which lowers the payout

We compare the "mortality yields" of ITAs versus VIAs

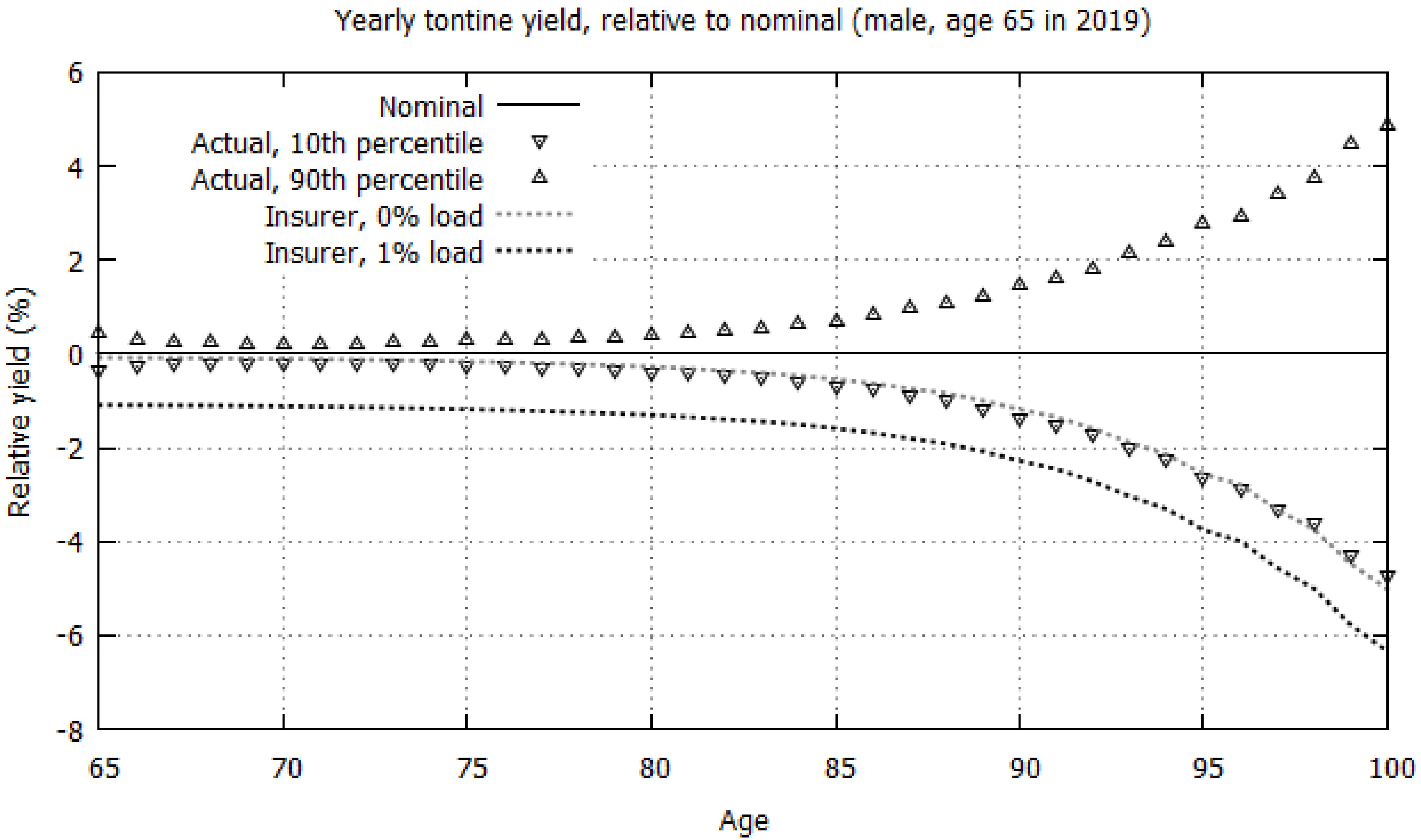
- ITA mortality yield is computed as rG as before, recalling that $r = q/(1-q)$
- VIA mortality yield is computed as $\hat{r} = \hat{q}/(1-\hat{q})$

The ITA's q's are taken from the projected Individual Annuitant Mortality (IAM) table (no risk load)

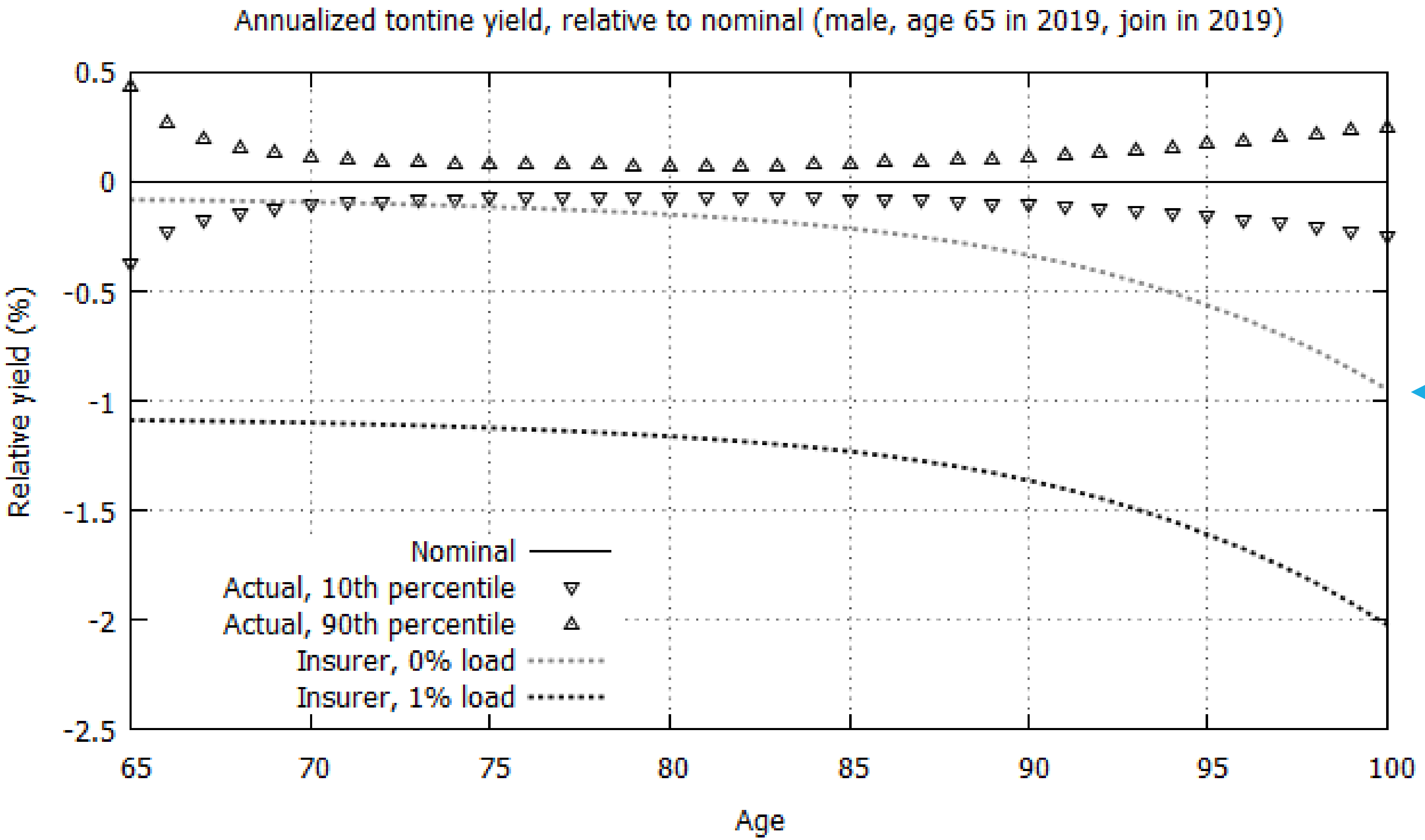
The VIA's $\hat{q}$'s are taken from the projected Individual Annuitant Reserving (IAR) table (≈10% risk load)

- "No load" VIA yield is $\hat{r} = \hat{q}/(1-\hat{q})$
- "1% load" VIA yield is $\hat{r}_{1\%} = \hat{r} - 0.01(1+\hat{r})$

Yearly Mortality Yields of ITA vs. VIA, Relative to Nominal



Annualized Mortality Yields of ITA vs. VIA, Relative to Nominal



Note the no load VIA curve. We will reference it later.

The Effect of Systematic Mortality Risk

Our research focused on idiosyncratic mortality risk

We did not stochastically model systematic mortality risk (future research project)

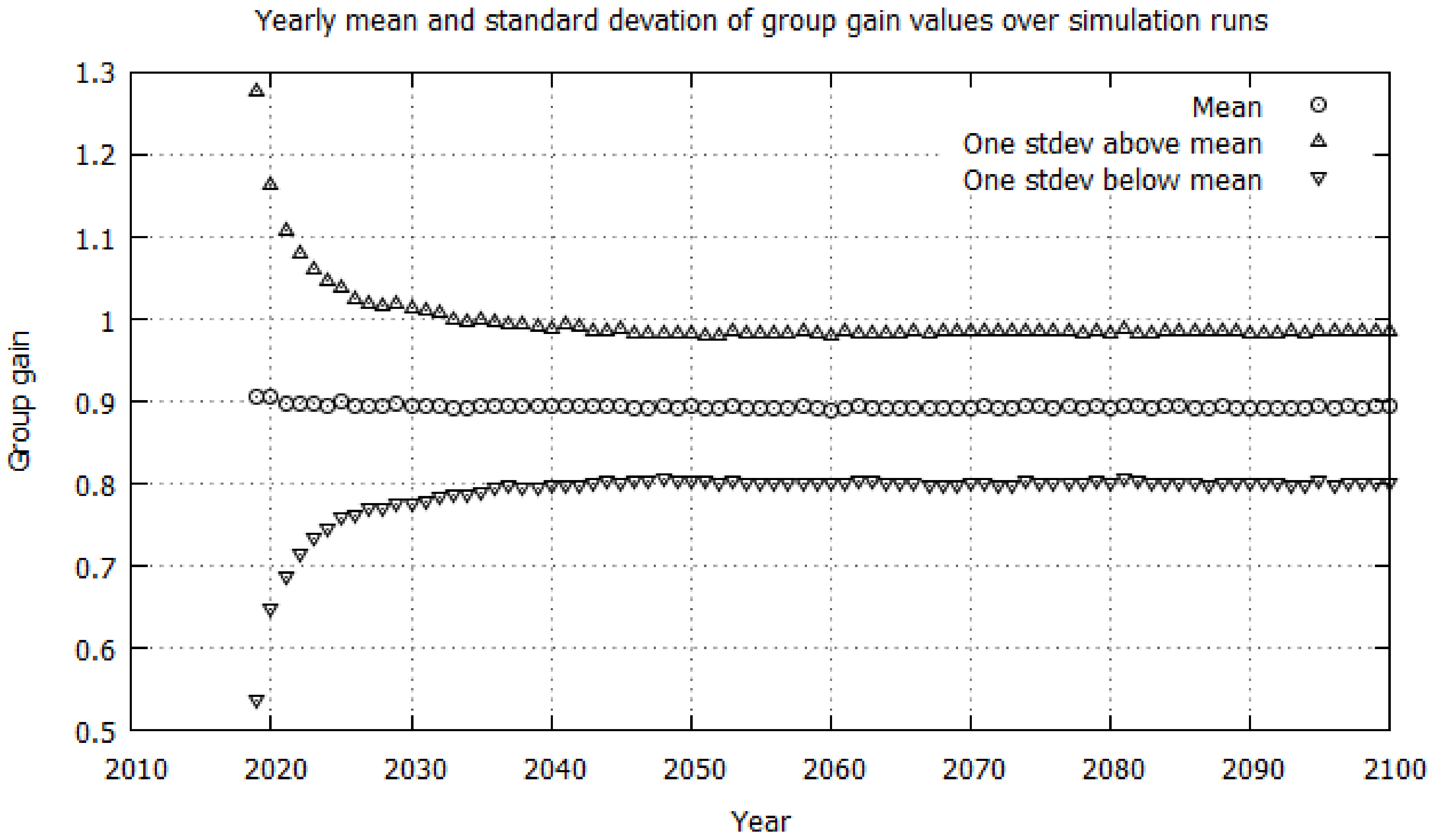
However, we did consider the potential effect of systematic mortality risk by asking the question:

What would happen if an ITA is designed using IAM mortality rates,
but actual mortality evolves according to the significantly lower IAR rates?

To illustrate, we reran the base analysis as follows:

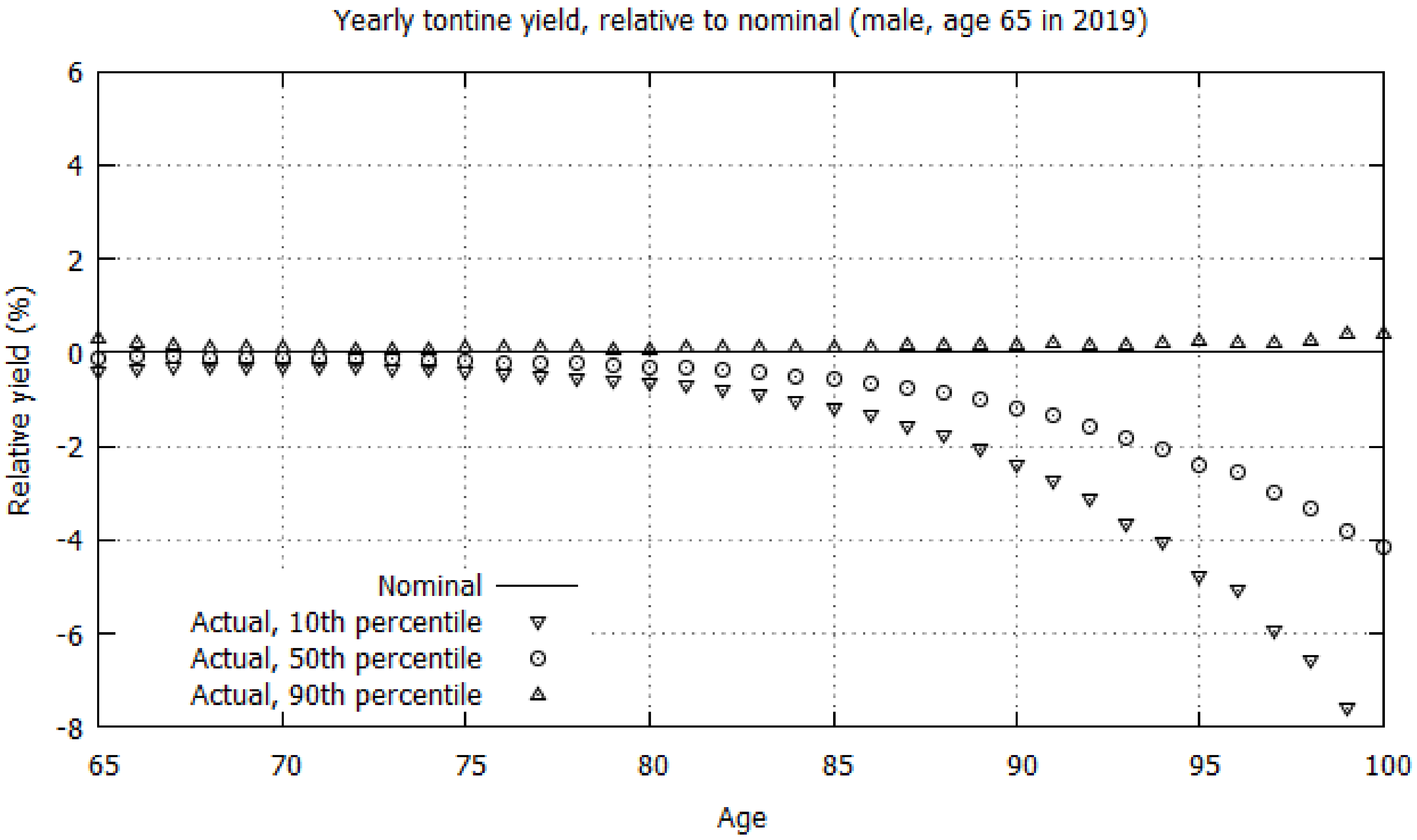
- The ITA nominal yields r are computed according to the IAM table, as before (no reserve loading)
- But the year of death is determined by the *IAR* table (≈10% reserve loading)

Group Gain when Mortality Evolves according to IAR Rates



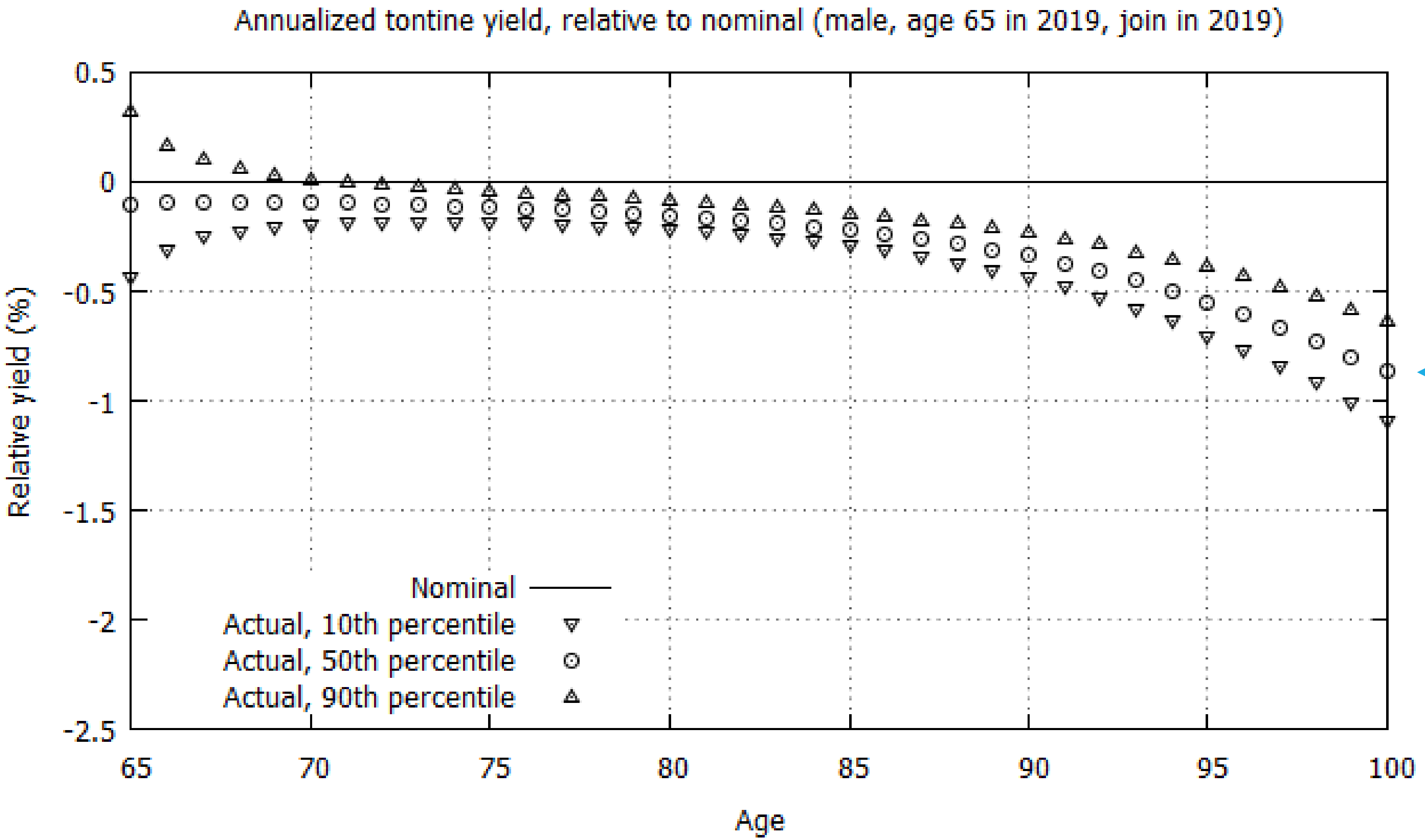
Yearly Tontine Yield, Relative to Nominal

When mortality evolves according to IAR rates



Annualized Tontine Yield, Relative to Nominal

When mortality evolves according to IAR rates



Note: The median curve is similar to the no load VIA curve shown earlier.

Even if the ITA is designed using incorrect mortality rates, it largely *behaves* as if designed using true mortality rates.

Conclusion

The fair tontine principle does not require a common portfolio or common payout option

Fair ITAs offer:

- A nonnegative and uncorrelated tontine yield on top of a member's underlying investment return
- A low-cost way to derive extra income from one's savings without taking on additional investment risk
- The freedom to invest and trade when and as desired
- The freedom to choose from a wide array of payout contracts based on preferences and needs
- A partial remedy to the annuity puzzle
  - Lower cost
  - Greater transparency
  - Continually self-adjusts to behave according to true mortality experience

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Sample Member Statement

Tontine Subaccount Statement As Of:	**March 31, 2020**
<u>**Account Overview**</u>	
Value on March 31, 2019	$ 102,613.86
Market appreciation/depreciation	$962.17
Dividends, interest, and capital gains	$ 1,600.00
Balance before tontine gain	$ 105,176.03
Tontine gain	$ 2,015.07
Balance before payout	$ 107,191.10
Tontine payout (to regular subaccount)	$ (10,017.44)
Value on March 31, 2020	$ 97,173.66

	Tontine Gain (if alive as of December 31, 2019)	
	Your Nominal Tontine Yield for 2019	0.019166
x	Common Group Gain Factor for 2019	0.999644
=	Your Actual Tontine Yield	0.019159
x	Your Balance Before Tontine Gain	$ 105,176.03
=	Your Tontine Gain	$ 2,015.07

Payout Option: Single Life Annuity 9.3454%