Embedding the Natural Hedging of Mortality/Longevity Risks into Product Design

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Outline

- Introduction
- Theoretical Development
- Numerical illustrations
- Cases of Annuities
- Conclusions

Introduction 1/3：How to Manage Mortality Risk

- Transferring longevity/mortality risk externally
  - Mitigation solutions through capital market
    - including survivor bonds (Denuit, Devolder, and Goderniaux, 2007), survivor swaps (e.g. Dowd and Blake, 2006; Dowd, Blake, Cairns, and Dawson, 2006), mortality swaps (Lin and Cox, 2007), mortality securitization (e.g. Dowd, 2003; Lin and Cox, 2005; Cairns, Blake, and Dowd, 2006a; Blake, Cairns, and Dowd, 2006; Cox, Lin, and Wang, 2006; Blake, Cairns, and Dowd, 2006; Blake, Cairns, Dowd, and MacMinn, 2006).

Introduction 2/3

- Mitigating the risks within the entity
  - Natural hedging strategy - through product portfolio matching
    - Hedging effect with portfolios of Life insurance and annuity products (Cox and Lin, 2007)
    - Optimal portfolios of Life insurance and annuity product including Wang et al. (2010) immunization model, Tsai and Chung (2013) and, Lin and Tsai (2013) extend the application to more than two products.
    - Tsai, Wang, and Tzeng (2010), Wang, Huang, and Hong (2013)

Introduction 3/3

- Our thinking - if the emerging risk is from product itself,…
  - why not mitigating the risk from product design
  - within a product design: "when to pay" and " how much to pay"

Theoretical Development

- Idea Scratching
  - Flat whole life insurance product

$$\int_0^\infty e^{-\delta s}\, b(s)\ {}_s p_x\ \mu_{x+s}\, ds = \mathcal{L}\{b(s)\cdot f(s)\} = \phi(\delta)$$

  - Insert a factor of "how much to pay"

$$\phi(\delta-\gamma) = \int_0^\infty e^{-\delta s} e^{\gamma s}\ {}_s p_x\ \mu_{x+s}\, ds = \mathcal{L}\{e^{\gamma s} f(s)\}$$

Theoretical Development

- Formal Development

$$ {}_tV_x = \int_0^\infty F_0 e^{\gamma t} e^{\gamma s} e^{-\delta s} {}_s p_{x+t} \, \mu_{x+t+s} \, ds $$

- without losing generality, let $F_0 = 1$.

$$ \frac{d}{dx} {}_tV_x = -e^{\gamma t} \mu_{x+t} + {}_tV_x \left(\mu_{x+t} + \delta - \gamma \right) $$

- Rearranging equation above, we get the expected reserves

$$ {}_tV_x = \frac{\frac{d}{dx} {}_tV_x + e^{\gamma t} \mu_{x+t}}{\mu_{x+t} + \delta - \gamma} $$

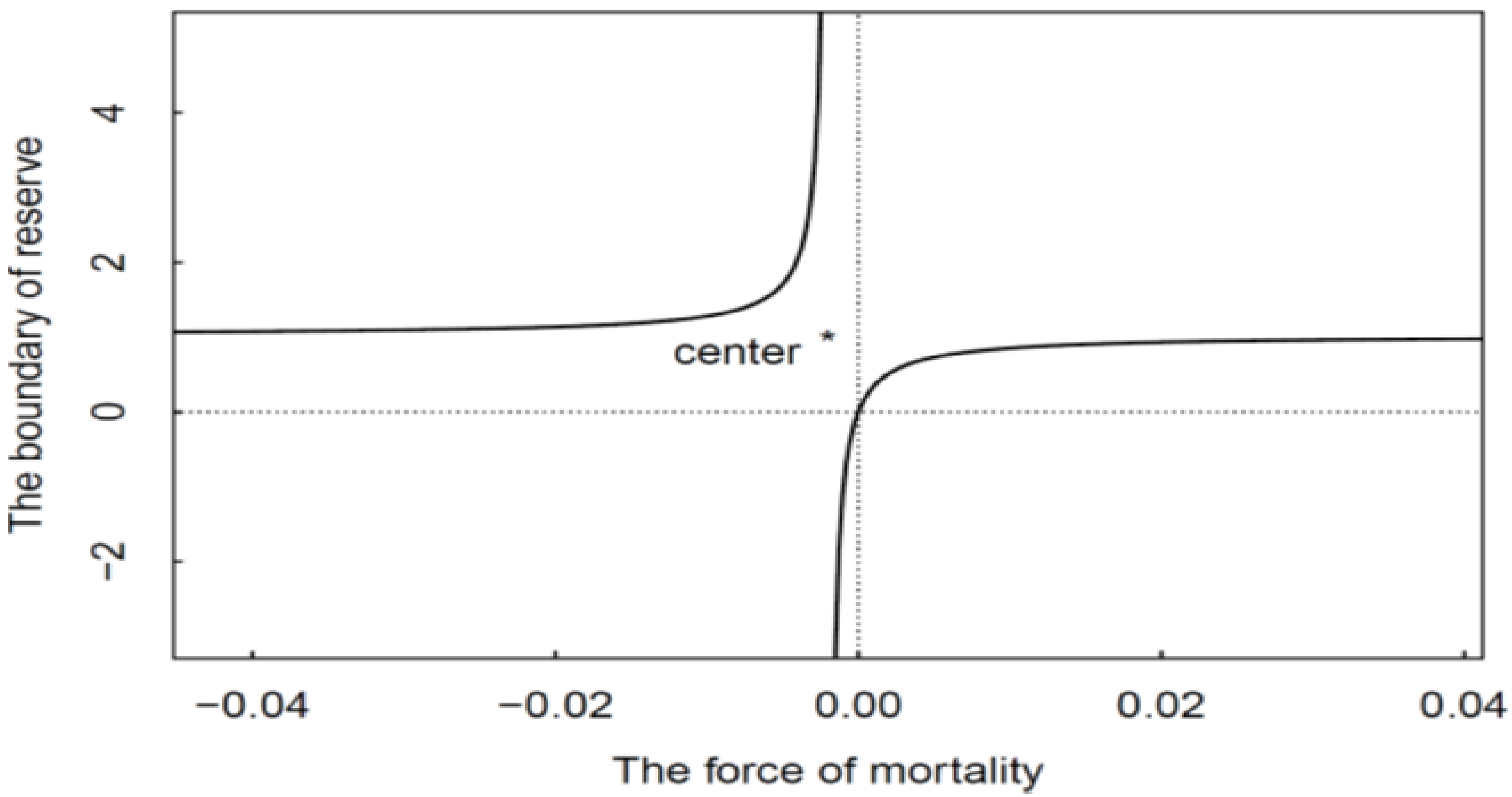
Theoretical Development

- The boundary conditions of the expected reserves:

1. when $\mu_{x+t}+\delta-\gamma > 0$ and ≥ 0, $\quad {}_tV_x \geq \frac{e^{\gamma t}\mu_{x+t}}{\mu_{x+t}+\delta-\gamma}$,
2. when $\mu_{x+t}+\delta-\gamma < 0$ and ≥ 0, $\quad {}_tV_x \leq \frac{e^{\gamma t}\mu_{x+t}}{\mu_{x+t}+\delta-\gamma}$,
3. when $\mu_{x+t}+\delta-\gamma > 0$ and ≤ 0, $\quad {}_tV_x \leq \frac{e^{\gamma t}\mu_{x+t}}{\mu_{x+t}+\delta-\gamma}$,
4. when $\mu_{x+t}+\delta-\gamma < 0$ and ≤ 0, $\quad {}_tV_x \geq \frac{e^{\gamma t}\mu_{x+t}}{\mu_{x+t}+\delta-\gamma}$

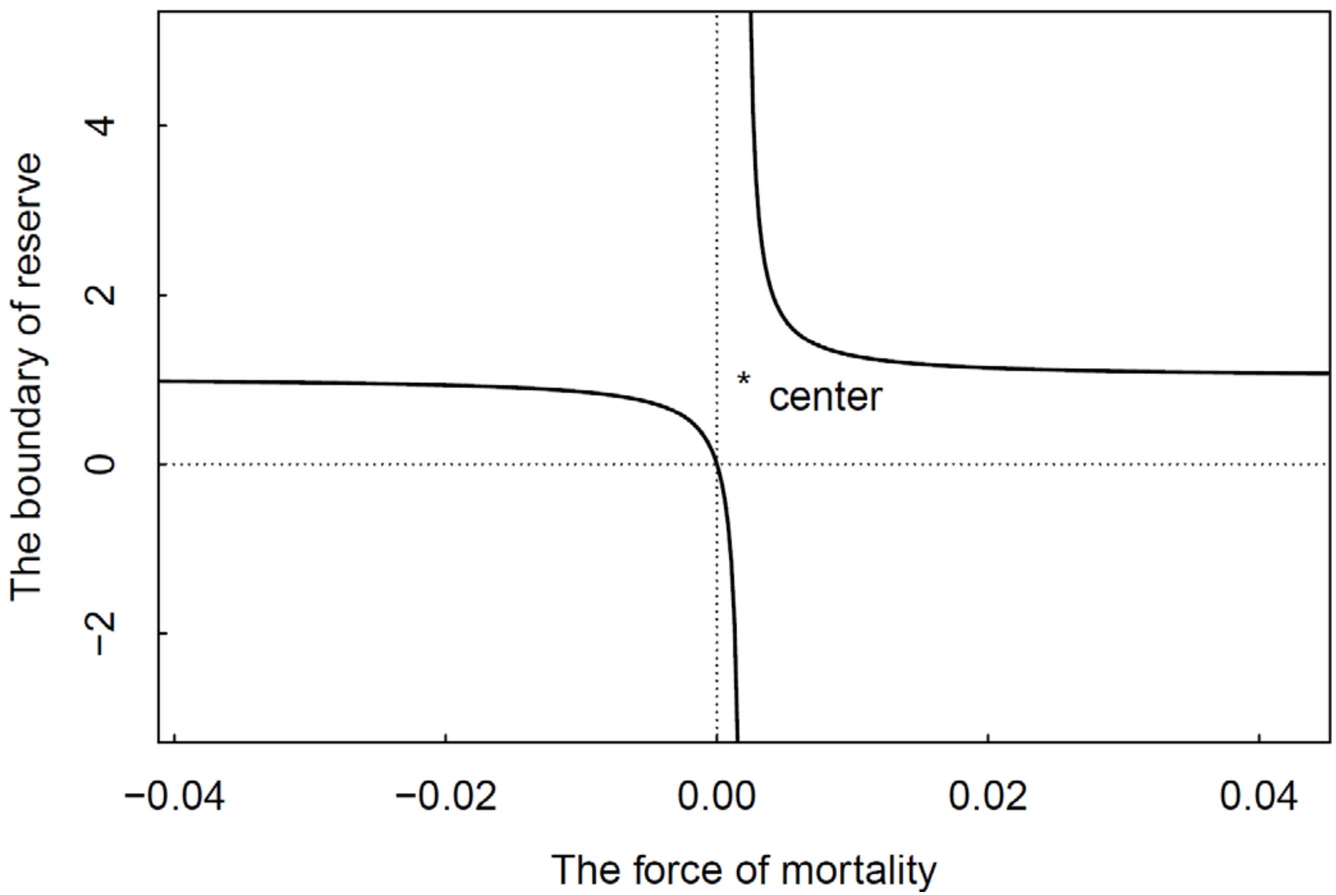
Theoretical Development

- The boundary of the expected reserve with positive $(\delta - \gamma)$ on the $(\mu_{x+t}, {}_tV_x)$ plane



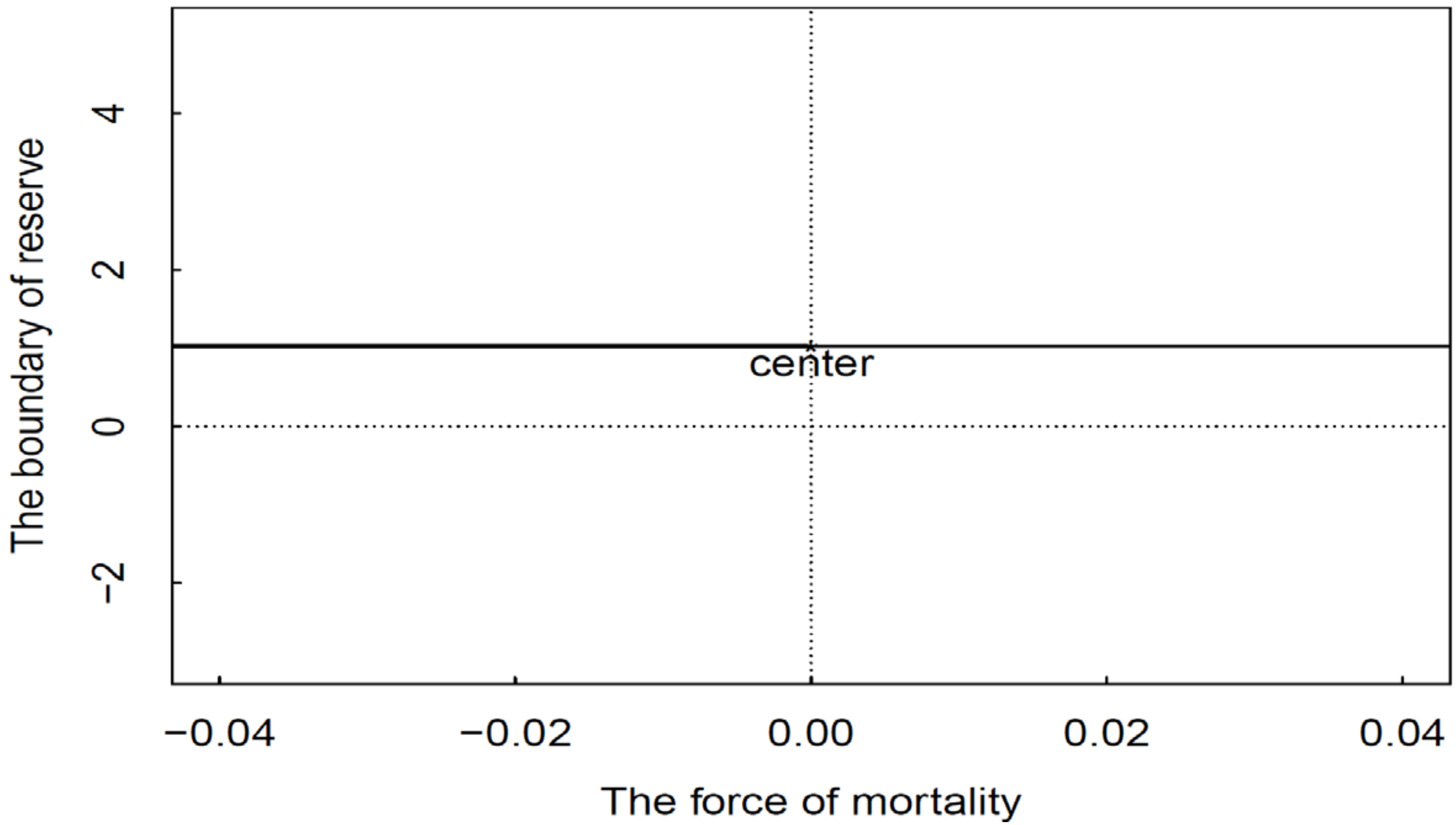
Theoretical Development

- with negative $(\delta - \gamma)$ on $(\mu_{x+t}, {}_tV_x)$ plane



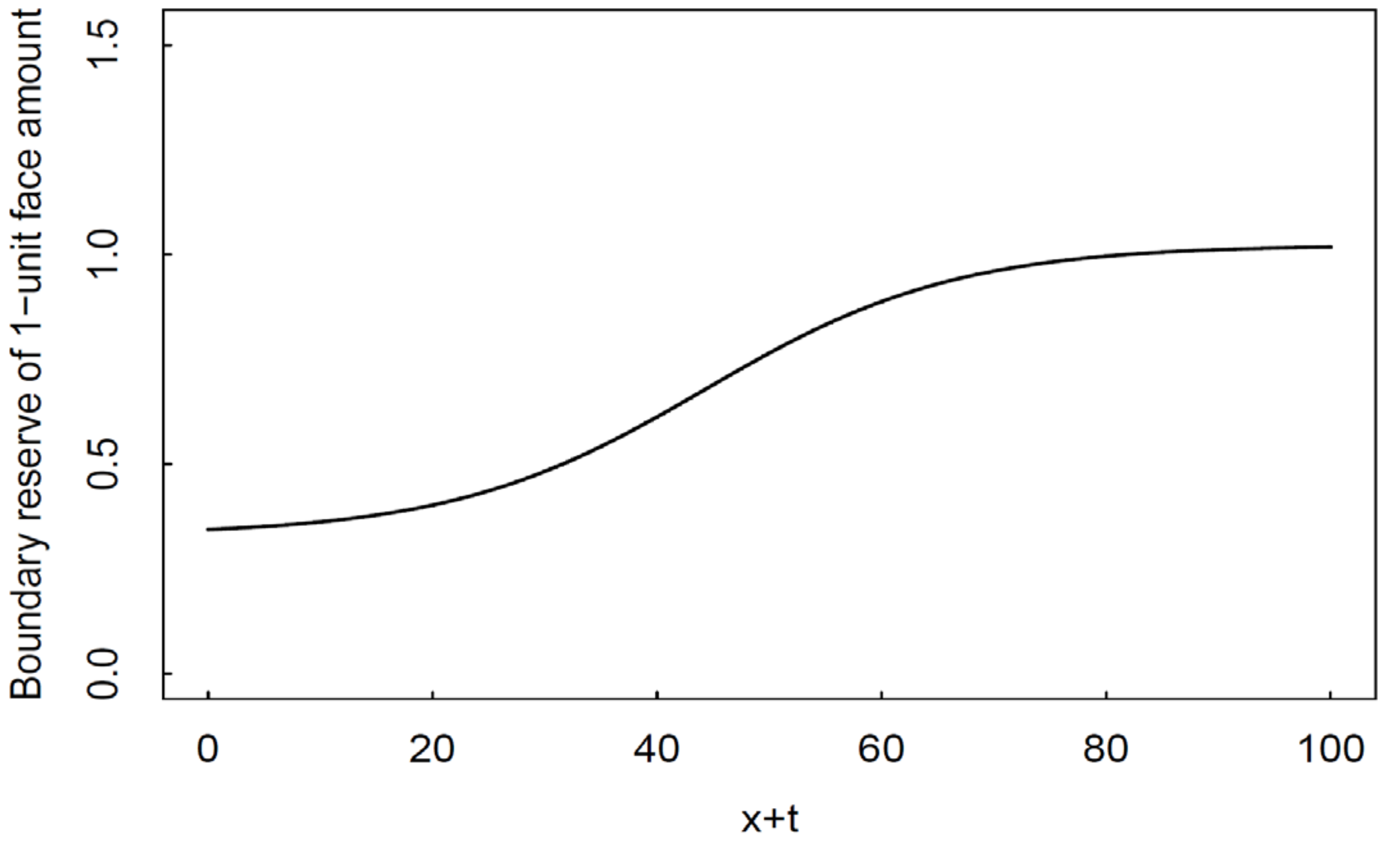
Theoretical Development

- with $(\delta - \gamma) = 0$ on the $(\mu_{x+t}, {}_tV_x)$ plane



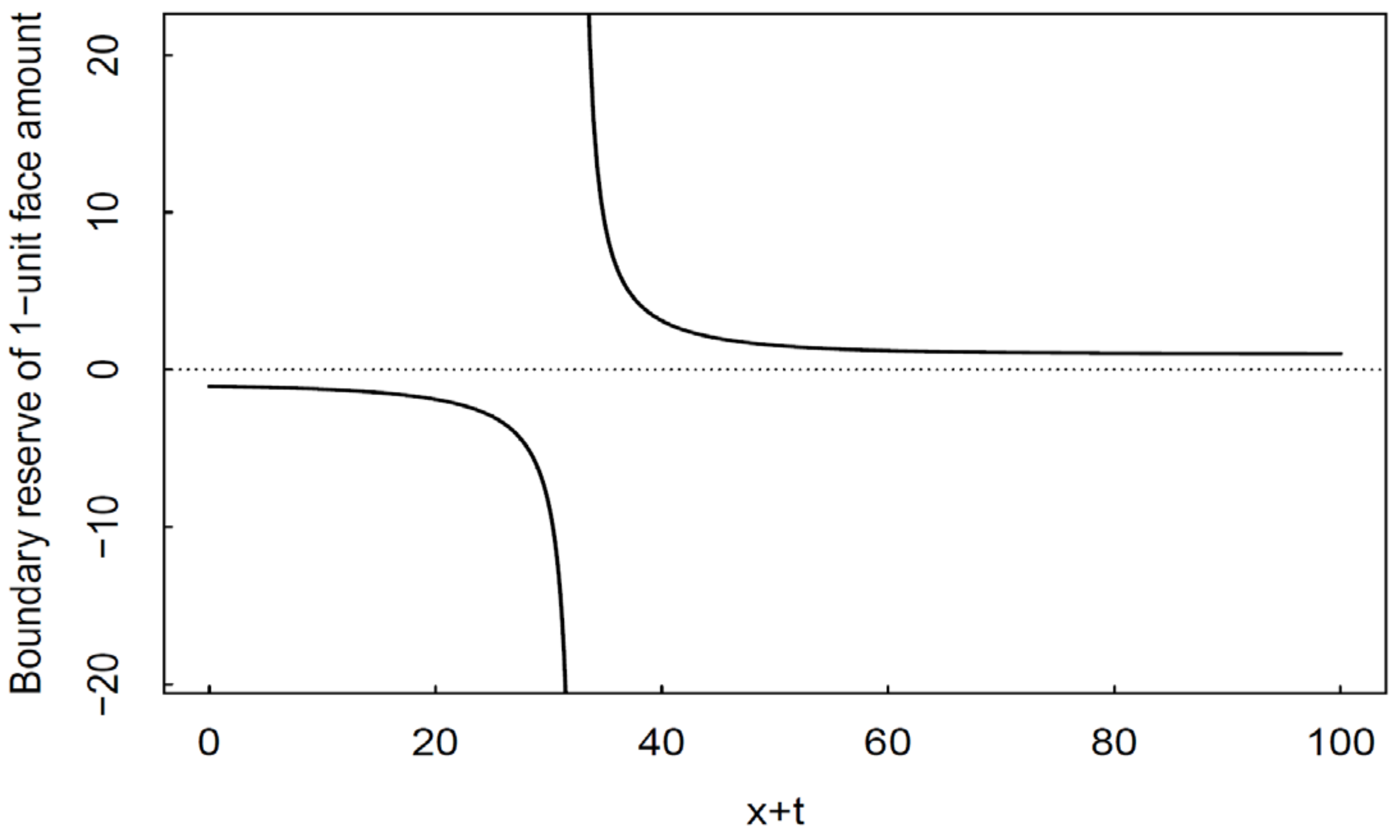
Theoretical Development

- Further Analyses on the $(x+t,\ {}_tV_x)$ plane, with positive $(\delta - \gamma)$



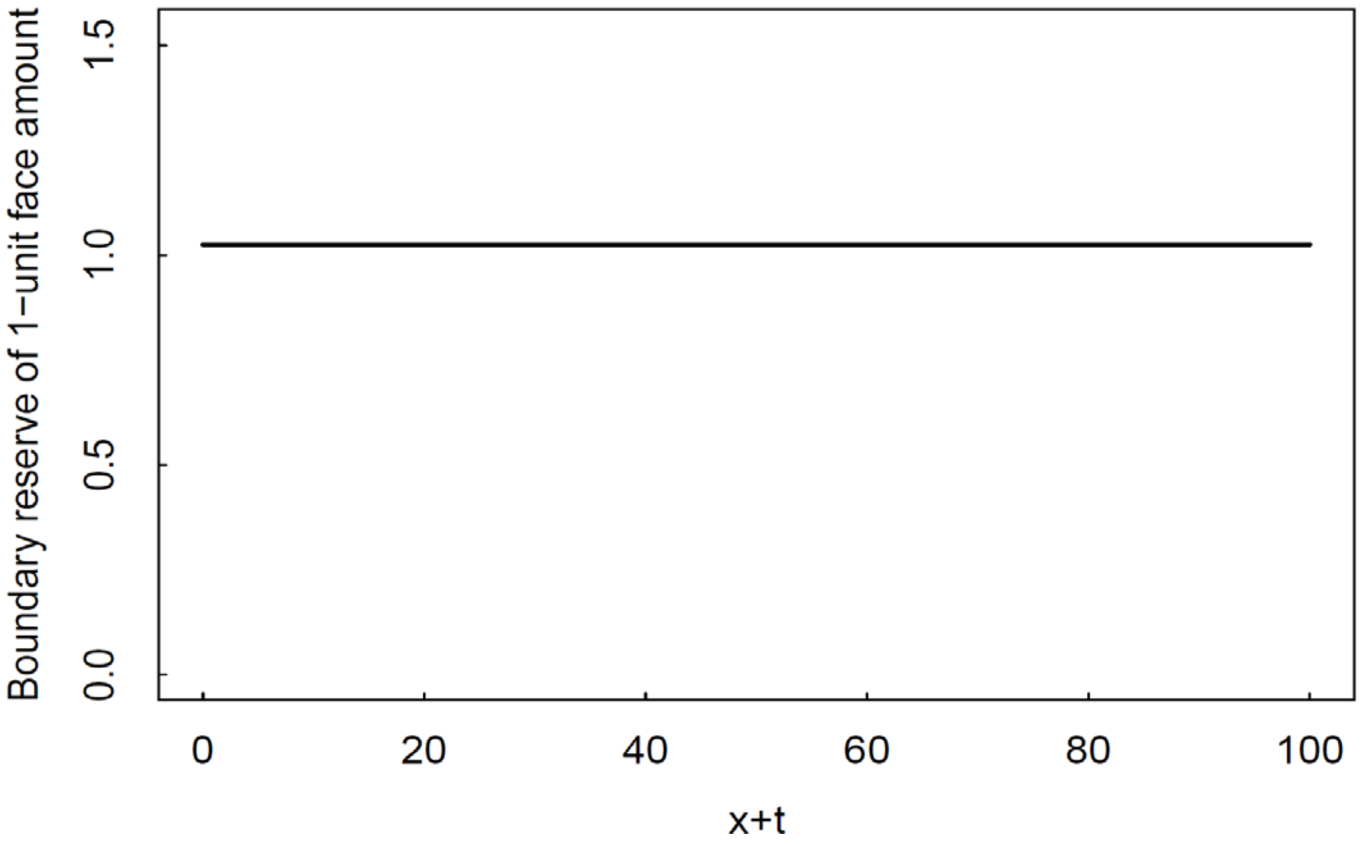
Theoretical Development

- with negative $(\delta - \gamma)$ on $(x+t,\ {}_tV_x)$ plane



Theoretical Development

- with $\delta - \gamma = 0$ on $(x+t\ ,\ {}_tV_x)$ plane



Numerical illustrations

- Basic assumption for the new form of the whole life insurance product

Age of insured	25, 45
Gender	Male
Face amount	100,000
The initial value of force of interest rate (δ)	4%
Death benefit	100,000 compounded by $\gamma(t)$
Benefit period	Whole life
Method of paying premium	Single premium

Numerical illustrations

- The optimal strategy $\gamma = \delta$
  - if δ = constant = 4% through the whole policy year.
  - Apply $\gamma = \delta$ in product design, $F_t = F_0 \exp(\gamma t)$,
  - The outcome is

Numerical illustrations

- The Liability at the End of the 5th Policy Year of Illustrated Insurance Product for Different Mortality Bases

$\gamma(=4\%) = \delta(=4\%)$					
	(1)	(2)	(3)	(4)=[(2)-(1)]/(1)	(5)=[(3)-(1)]/(1)
age	Basis	20% Up Shock	20% Down Shock	Reserve Changed	Reserve Changed
25	122,140	122,140	122,140	0%	0%
45	122,140	122,140	122,140	0%	0%

Numerical illustrations

- δ obtained by CIR model with a constant long term mean 6.5%
- and mortality rate model, LC (Lee & Carter, 1992) model
- We display the results of $\gamma = 6.5\%$ and the other comparison panel of $\gamma = 0\%$

Numerical illustrations

- The Liability at the End of the 5th Policy Year of Illustrated Insurance Product with CIR Model and LC Model for Different Mortality Bases

Panel A: γ (=6.5%) = Long Term Mean of Interest Rate Model

age	(1) Basis	(2) Mortality Rate 20% Up Shock	(3) Mortality Rate 20% Down Shock	(4)=[(2)-(1)]/(1) Reserve Changed	(5)=[(3)-(1)]/(1) Reserve Changed
25	153,363	152,143	153,547	-0.80%	0.12%
45	146,222	145,267	146,424	-0.65%	0.14%

Panel B: γ (= 0%) ≠ Long Term Mean of Interest Rate Model

age	(1) Basis	(2) Mortality Rate 20% Up Shock	(3) Mortality Rate 20% Down Shock	(4)=[(2)-(1)]/(1) Reserve Changed	(5)=[(3)-(1)]/(1) Reserve Changed
25	6,378	7,133	5,536	11.84%	-13.20%
45	17,509	19,195	15,570	9.63%	-11.07%

Annuity

- Theoretical Development is similar to the case of life insurance.
- There is no chance we can get the situation of annuity that $(\delta + \gamma)=0$ with $\delta>0$ and $\gamma>0$.
- While annuity product with decreasing benefit design (i.e. $\gamma>0$) can obtain the outcome of reducing longevity risk comparing to the one product design without γ (that is $\gamma=0$).

Annuity

- Numerical illustrations
  - The Liability at the End of the 5th Policy Year of Illustrated Annuity Product for Different Mortality Bases

A: γ (=4%) <δ (=4%)					
	(1)	(2)	(3)	(4)=[(2)-(1)]/(1)	(5)=[(3)-(1)]/(1)
age	Basis	20% Up Shock	20%Down Shock	Reserve Changed	Reserve Changed
25	90,472	89,608	91,389	-0.955%	1.014%
45	79,509	77,469	81,759	-2.565%	2.830%
B: γ (=0%) <δ (=4%)					
25	195,256	191,308	199,719	-2.022%	2.286%
45	152,721	146,562	159,903	-4.032%	4.703%

Conclusions

- Mitigating risks ov "when to pay" and "how much to pay".
- The optimal strategy of life insurance product design, to keep with the criteria of settings on γ as to let $\gamma = \delta$.
- The strategy for annuity product design with $\gamma > 0$ can obtain reducing on the longevity risk.
- Following the optimal strategy in the product design, the insurance product has a lot more possibility in engaging to financial instrument.

Thank You