Mortality Risk Management under the Factor Copula Framework

- with Applications to Insurance Policy Pools

MING-HUA HSIEH, JASON C. TSAI AND JENNIFER L. WANG

RISK AND INSURANCE RESEARCH CENTER
COLLEGE OF COMMERCE, NATIONAL CHENGCHI UNIVERSITY

Major ways to mitigate the mortality risks

- Capital market solutions including mortality securitization (see, for example, Cowley and Cummins, 2005; Lin and Cox, 2005; Blake et al., 2006a and 2006b; Cox et al., 2006), survivor bonds (e.g., Blake and Burrows, 2001; Dowd, 2003; Denuit et al., 2007), and survivor swaps (e.g., Dowd et al., 2006).
- Through internal self-insurance that takes place within an insurer such as the natural hedging suggested by Cox and Lin (2007) and Wang et al. (2010) and/or within the industry using the reinsurance swap (Lin and Cox, 2005).
- Building mortality projection models to provide accurate estimations of future mortality rates (e.g., Lee and Carter, 1992; Renshaw and Haberman, 2003, and Cairns, Blake, and Dowd, 2006a; 2006b).

Natural hedging

- life insurance may be used to hedge against the longevity risk of annuity products.
- Life insurers have difficulties in implementing the intended hedging because new sales may be insufficient and/or because life insurers do not have full control over the composition of new sales.
- Hedging the mortality risks from the asset side of the insurer’s balance sheet may be more flexible and cost effective than through the liability side.

Factor copulas

- Managing the mortality risks of its liabilities demands finding a counter portfolio to the chosen policy pool and modeling the future lifetimes of the associated insureds.
- The to-be-hedged pool is always composed of hundreds of, thousands of, or more policies.
- The dependence structure among these policies as well as the structure between the policy pool and the hedging portfolio is too complex to be modeled without finding a way to reduce the dimension of the structure and finding a way that better describes the dependence than using correlation coefficients.
- We propose to use factor copulas to model the dependence structure.

Valuation

$$V_{wl}(j) = \frac{-A_j}{(1+r_{wl}(j))^{\tau_j}} + \sum_{t=premium\ payment\ times\ before\ \tau_j} \frac{Q_j(t)}{(1+r_{wl}(j))^t},$$

$$V_{ls}(i) = \frac{B_i}{(1+r_{ls})^{T_i}} - \sum_{t=\ premium\ payment\ times\ before\ T_i} \frac{P_i(t)}{(1+r_{ls})^t}$$

Dependence Structures

Let the marginal distribution function of T_i and τ_j be denoted by $F_i(.)$ and $G_j(.)$, respectively.

Assume that the joint dependence structure of T_i and τ_j can be described by Gaussian factor copulas that include normal copulas and *t* copulas.

Under the framework of normal factor copulas, we first express T_i by a corresponding latent variable X_i as:

$$T_i = F_i^{-1}\big(N(X_i)\big),$$

We impose a two-factor structure by expressing the dependence among X_i through common factors M and M_{ls} as follows:

$$X_i = aM + bM_{ls} + \sqrt{1 - a^2 - b^2} Z_i, \quad (4)$$

where M, M_{ls}, Z_1, ..., Z_n are independent standard normal random variables and a and b denote constant factor loadings.

The common factor M represents the global trend in the mortality improvements of life settlements and life insurance policies while M_{ls} reflects the improvement trend of the life settlement fund only.

On the other hand, Z_1,…., Z_n are specific factors pertaining to each life settlement.

Y_j are the latent variables used to model the joint distributions of τ_j.

The dependence among Y_j is structured by common factors M *and* M_{wl} as follows:

$$Y_j = cM + dM_{wl} + \sqrt{1 - c^2 - d^2} W_j,$$

where M_{wl}, $W_1, \ldots, W_m$ are independent standard normal random variables and constants c and d denote factor loadings.

M_{wl} represents the factor affecting the mortality improvements of the insurance pool and $W_1, \ldots, W_m$ are specific factors pertaining to each insurance policy.

We also tried t_ν copulas to better capture tail dependence

Risk Mitigation Arrangement

The total value of the positions including the value of the insurance policy pool and the investments in the life settlement fund, V_h, can be expressed as:

$V_h = V_{wl} + h\, V_{ls}$,

in which $V_{wl} = \sum_{j=1}^{m} V_{wl}(j)$,

h represents the so-called hedge ratio,

and $V_{ls} = \sum_{i=1}^{n} V_{ls}(i)$.

Optimal Mitigation

When σ is chosen to be the risk measure, the objective function will be:

$$\min_{h} \sigma\,(V_{wl} + h\,V_{ls}).$$

When *ES* is chosen to be the risk measure, the objective function will be:

$$\min_{h} ES\,(V_{wl} + h\,V_{ls}).$$

Case study

- We analyze a case in which an insurer intends to mitigate the mortality risk of a life insurance policy pool.
- The pool consists of about 400 policies from a medium-size insurer.
- For the risk mitigation tool, we choose a portfolio of life settlements originated by Coventry, a major market maker in US.

Summary Statistics of the Life Insurance Policy Pool

The samples consist of 418 whole life insurance policies, and their summary statistics are

	Mean	Median	Standard Deviation	Maximum	Minimum
Insured's Age	63.6	64.1	4.6	75.8	55.1
Gender (Male = 1)	0.46	1	0.5	1	0
Death Benefit ($million)	0.47	0.20	0.96	15.0	0.1

Summary Statistics of the Life Settlement Portfolio

	Mean	Median	Standard Deviation	Maximum	Minimum
Insured's Age	75.8	75.7	4.5	86.8	63.4
Life Expectancy	12.9	13.0	2.8	19.7	5.9
Gender (Male = 1)	0.73	1	0.45	1	0
Policy Year	3.1	2.3	3.6	23.7	0.1
Death Benefit ($million)	4.1	3.0	3.8	20.0	0.2
Acquisition Cost ($million)	0.46	0.24	0.69	6.80	0.02

Numerical Results ($a = 0.5$, $c = 0.8$, $b = 0.3$, $d = 0.3$, $r_{ls} = 4.85\%$)

	Normal Factor Copula		
	Standard Deviation	Value at Risk	Expected Shortfall
Risk of V_{wl} (1)	34,143,483	69,133,688	86,103,577
Risk of $V_{wl} + h\ V_{ls}$ (2)	22,121,620	38,578,204	49,255,396
Risk Reduced $\frac{(1)-(2)}{(1)}$	35.2%	44.2%	42.8%
Optimal Hedge Ratio h^*	8.2%	10.1%	10.3%

T factor copula

	t_5 Factor Copula		
	Standard Deviation	Value at Risk	Expected Shortfall
Risk of V_{wl} (1)	34,757,525	72,183,048	88,677,591
Risk of $(V_{wl} + h\ V_{ls})$ (2)	22,737,005	40,008,071	54,508,115
Risk Reduced $\frac{(1)-(2)}{(1)}$	34.6%	44.6%	38.5%
Optimal Hedge Ratio h^*	8.2%	9.4%	9.6%

Robustness Checks

Case	a	c	b	d	r_{ls} (%)	h^* (%)	$VaR(V_{wl})$	$VaR(V_{wl}+hV_{ls})$	Risk Reduced
Benchmark	0.5	0.8	0.3	0.3	4.85	9.4	72,183,048	40,008,071	44.6%
Smaller b, d	0.5	0.8	0.1	0.1	4.85	10.9	64,172,539	24,007,962	62.6%
Larger a	0.8	0.8	0.3	0.3	4.85	7.2	71,127,485	33,655,689	52.7%
Smaller c	0.5	0.5	0.3	0.3	4.85	5.9	44,326,413	30,706,049	30.7%
Higher r_{ls}	0.5	0.8	0.3	0.3	8.6	10	70,417,613	37,436,261	46.8%

Concluding Remarks

This article sets up a factor copula framework to deal with the mortality risk management of a life insurance pool consisting of many policies.

The future lifetime of each insured underlying the pool is a risk factor, and the dependence structure among these insureds' future lifetimes determines the mortality risk of the pool.

We contribute to the literature by introducing the factor copula approach to tackle the (dimension) issue of modeling the dependence structure.

The literatures on managing/hedging mortality risks seldom dealt with many risk factors.

The framework can be applied to natural hedging that employs the insurer's another policy pool to mitigate the concerned pool.

It is also useful in constructing the hedging strategy involving the securities linked to dozens of age and country specific mortality rates such as the Kortis longevity bond.

The factor copula models can be extended to incorporate time-varying dependence as well.