Value Hedging with an Uncertain Market Price of Longevity Risk

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Motivation

Presentation: Focus on results; Maths in paper.

Large exposure to longevity risk (pension funds & insurers).

No hedging products available.

Cash flow matching:

2004: EIB/BNP longevity bond withdrawn prior issue:

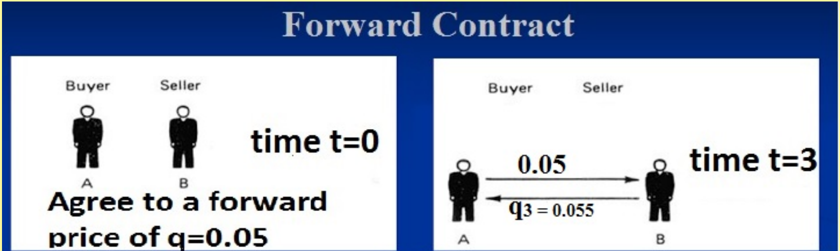
- Duration: 25 years;
- High capital relative to the risk exposure;
- Parameter and model risk;
- Not flexible.

Capital markets:

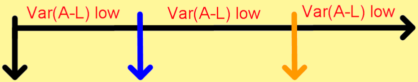
Overcome issues of longevity bond: q-forwards (Coughlan et al, 2007).

Liquid market & flexible products; q-forwards building blocks:

- Maturity;
- Gender;
- Age group.



Value hedging



t=0
Buy
contract
A-L given

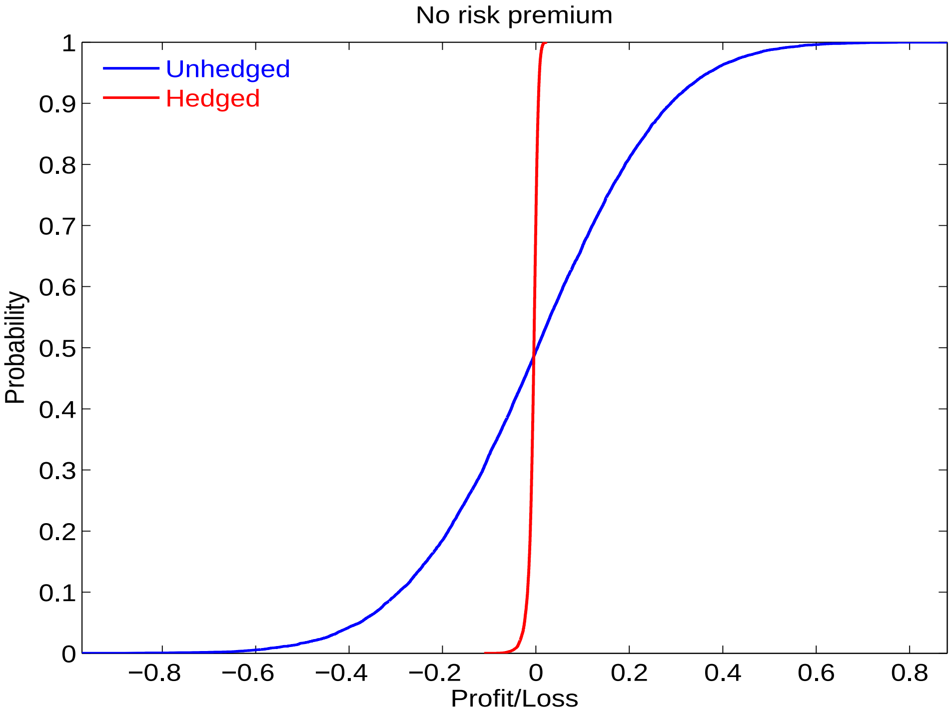
t=3
q-forward
payments;
->Var(A-L) low
Buy new
contract

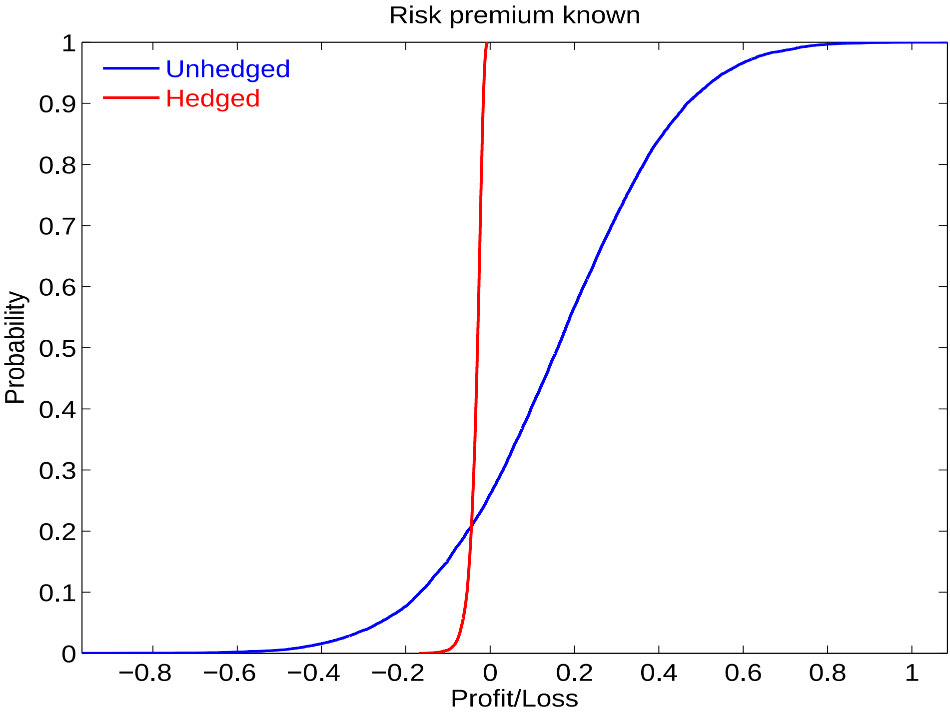
t=6
q-forward
payments
->Var(A-L) low
Buy new
contract

Typical assumption: constant price longevity risk over time?

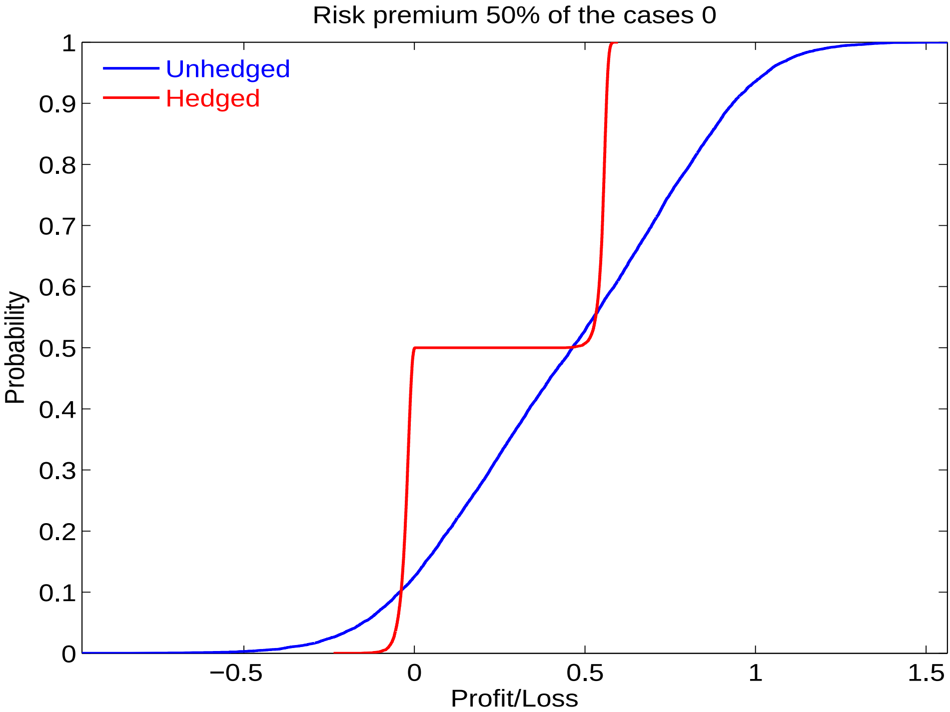
Stochastic volatility risk premium (Bollerslev, Gibson, and Zhou, 2012).

Q-forwards shown to be effective hedge.


No risk premium
Unhedged
Hedged
Probability
Profit/Loss
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8



Risk premium known
Unhedged
Hedged
Probability
Profit/Loss
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1


Risk premium 50% of the cases 0
Unhedged
Hedged
Probability
Profit/Loss
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
−0.5
0
0.5
1
1.5

Value Hedging with an Uncertain Market Price of Longevity Risk

GGIW & Bayesian

The parameters in the random walk with drift are estimates.

Limited data $\Rightarrow$ estimates are uncertain.

New information $\Rightarrow$ update estimates.

- Wishart distribution: multivariate χ^2 distribution;
- Inverse Wishart distribution: ensures positive definite Σ;
- Gaussian Inverse Wishart distribution: uncertain mean & variance;
- Generalized Gaussian Inverse Wishart distribution: different number of observations.

Cairns Blake Dowd Model

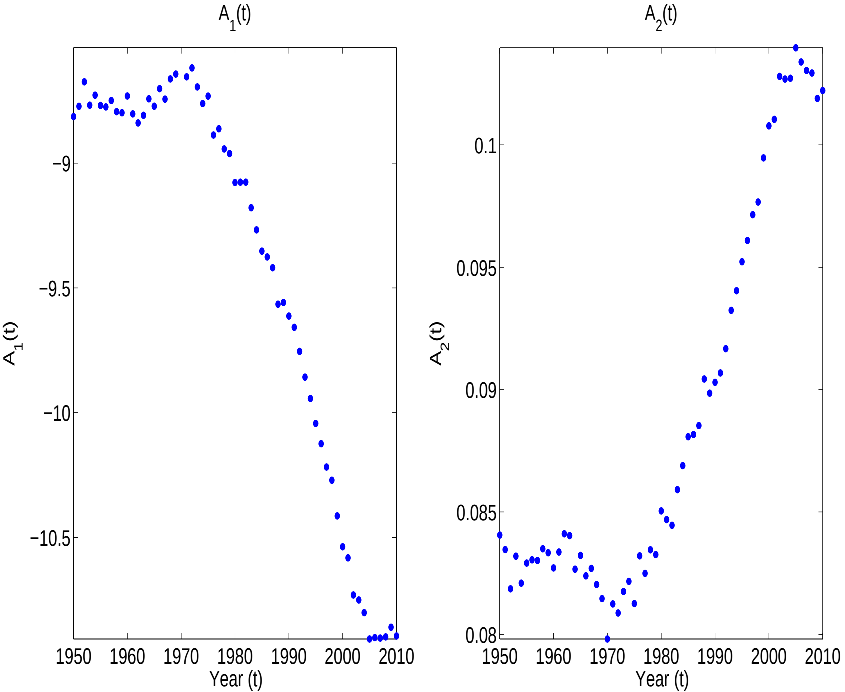
CBD model:

$$\begin{aligned} p(x,t) =& 1 - q(x,t) \\ q(x,t) =& \frac{\exp\left(A_1(t) + x \cdot A_2(t)\right)}{1 + \exp\left(A_1(t) + x \cdot A_2(t)\right)}. \end{aligned}$$

Two stochastic processes (random walk with drift):

$$\begin{aligned} A(t) =& [A_1(t)\ A_2(t)]^\top \\ D(t) =& A(t) - A(t-1) \\ =& \mu + CZ(t). \end{aligned} \tag{1}$$

US male mortality age 65-95 from 1950-2010.


A1(t)
A1(t)
−9
−9.5
−10
−10.5
1950
1960
1970
1980
1990
2000
2010
Year (t)
A2(t)
A2(t)
0.1
0.095
0.09
0.085
0.08
1950
1960
1970
1980
1990
2000
2010
Year (t)

Longevity risk premium

Mortality dynamics (including parameter risk):

$$V|D \sim \text{Inverse Wishart}_2(T, \widehat{V}) \quad (2)$$

$$\mu|V, D \sim N_2(\widehat{\mu}, T^{-1}V). \quad (3)$$

Change of measure:

$$\begin{aligned} A(t+1) - A(t) &= \mu + C(\widetilde{Z}(t+1) - \lambda) \\ &= \widetilde{\mu} + C\widetilde{Z}(t+1), \end{aligned}$$

where $\widetilde{\mu} = \mu - C\lambda$.

Dynamics for λ:

- Similar GIW as mortality dynamics;
- Allow for parameter risk in covariance.

Value Hedging with an Uncertain Market Price of Longevity Risk

Insurer

Issuing an annuity to a 65 year old male.

(Real) payment of annuity is 1 if insured is alive and 0 otherwise.

(Real) interest rate is set at 2%.

No idiosyncratic longevity risk.

No financial risk.

Insurer

Value hedging for 3 years.

Two q-forwards:

- 75 years, maturity 3 years;
- 85 years, maturity 3 years.

Insurer minimizes portfolio variance by optimally selecting # q-forwards.

Net asset value includes:

- risk premium (q-forward);
- payments (via assets);
- liability value at time 3 (surviving, mortality dynamic & risk premium).

Value Hedging with an Uncertain Market Price of Longevity Risk

Scenarios

Risk adjusted process is unknown.

In base case we set:

$\widehat{\lambda} = [0.1167\ 0.1167] \Rightarrow$ risk premium 5%.

$\tau = 5$ (not much information)

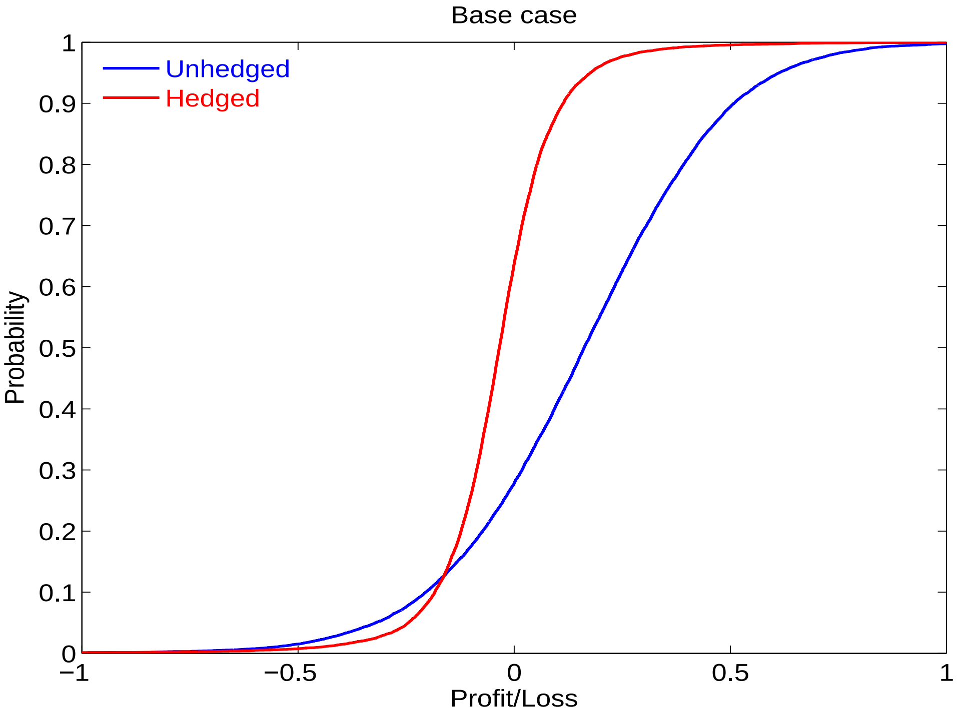
$\widehat{V}$ such that at $t = 3$ the effect on the risk premium of an annuity:

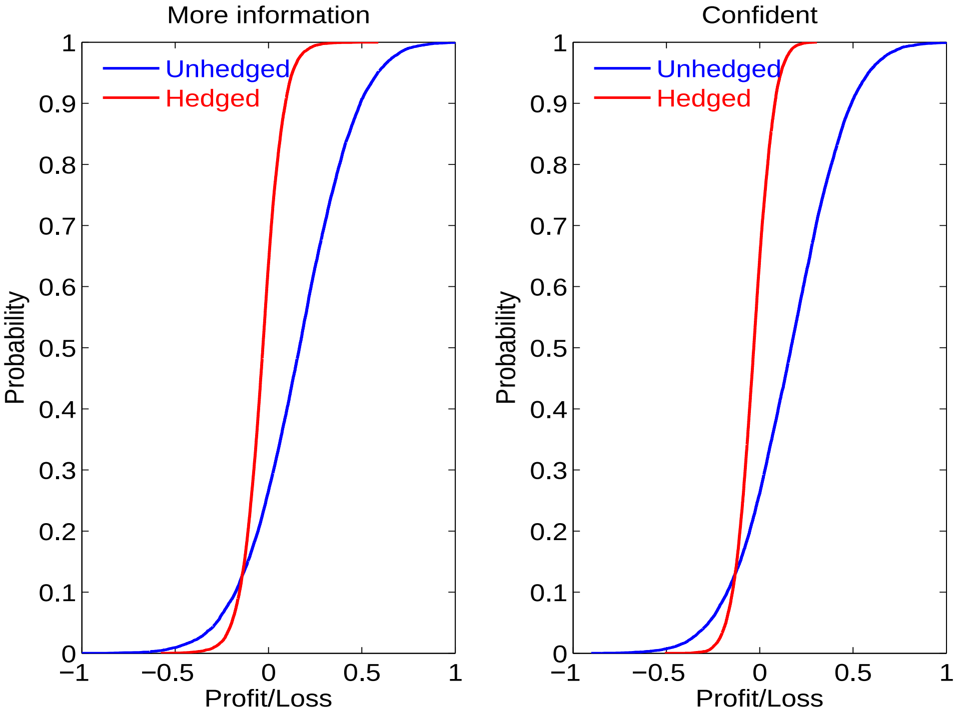
- 25% – 75%CI: -0.5% – 0.5%;
- 10% – 90%CI: -1.1% – 1.2%.

25% – 75%CI of λ at year 3: 0.102-0.131.

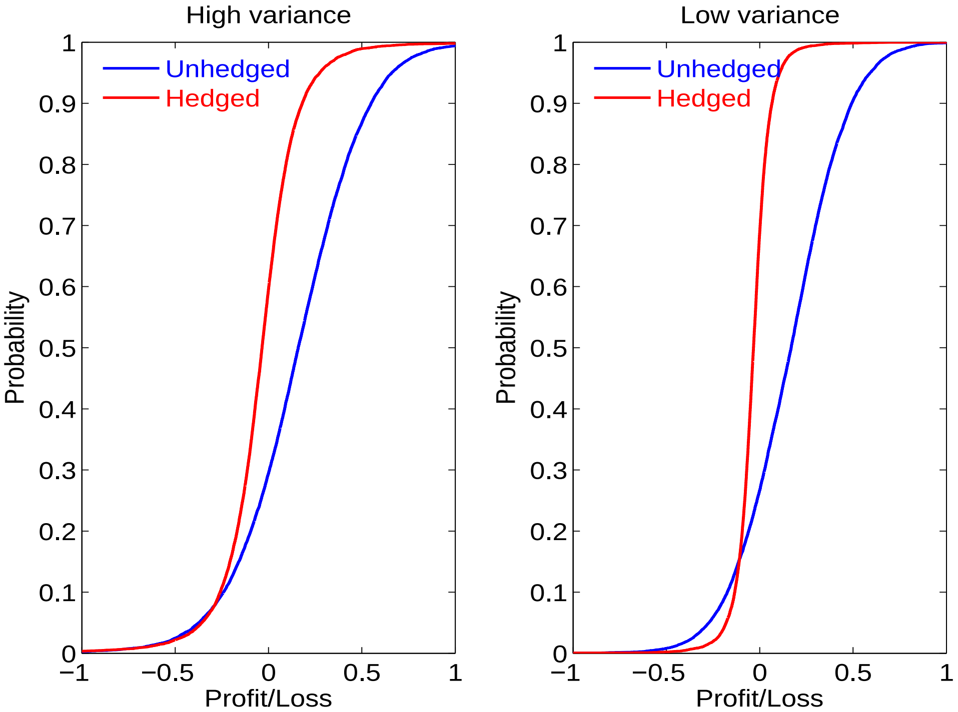
Robustness checks:

- Use τ of 10 & 30;
- Increase standard deviations by 50%, decrease by 33%.


Base case
Unhedged
Hedged
Probability
Profit/Loss
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
−1
−0.5
0
0.5
1



More information
Confident
Unhedged
Hedged
Probability
Profit/Loss
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
−1
−0.5
0
0.5
1



High variance
Low variance
Unhedged
Hedged
Probability
Profit/Loss
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
−1
−0.5
0
0.5
1


RISK
AHEAD

Portfolios and risk reduction

Scenario	β_{75}	β_{85}	std	std hedged	reduction
No risk premium	101	30	0.228	0.010	95%
Known risk premium	97	36	0.249	0.017	93%
Risk premium 50%	93	35	0.375	0.290	23%
Base case	99	35	0.290	0.152	48%
More information $\tau = 10$	92	37	0.264	0.101	62%
Confident $\tau = 30$	99	35	0.259	0.090	65%
High variance	95	36	0.340	0.236	30%
Low variance	100	35	0.266	0.103	61%

Optimal hedging portfolio robust to longevity risk premium;

Knowing uncertainty in variance more important than level of the variance.

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Conclusions

Longevity risk can effectively be hedged using q-forwards if market price of longevity risk is known.

Optimal hedging portfolio robust to longevity risk premium.

Value hedging less effective is market price of longevity risk becomes uncertain in the future. Risk-Reward tradeoff?

Knowing uncertainty in variance more important than level of the variance.

Value hedging: Could have potential, but there are risks!

Academics need information on market price!