Reverse Mortgage Pricing and Risk Analysis Allowing for Idiosyncratic House Price Risk and Longevity Risk

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Topic Coverage

1. Introduction
2. Pricing Framework
3. Idiosyncratic House Price Risk
4. Termination Model
5. Pricing and Risk Analysis
6. Conclusions

Topic coverage

Research Motivation

- Population ageing: how to finance health care in retirement?
  - large component of household wealth in home equity
  - role for reverse mortgages and other equity release products

- Growing literature on reverse mortgage pricing and risk management
  - impact of longevity risk (Wang et al., 2008; Li et al., 2010; Yang, 2011)
  - new pricing framework (Alai et al., 2013; Cho et al., 2013)
  - house price risk typically analysed using time series models based on one market-wide index (e.g., Chen et al., 2010; Yang, 2011; Lee et al., 2012)

- This study: idiosyncratic house price risk and longevity risk
  - portfolio of options (not: option on portfolio)
  - idiosyncratic house price risk substantial (Hanewald and Sherris, 2013)
  - disaggregated house price indices: more volatility and different trends (Shao et al., 2013)

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Accumulated loan amount

Consider a reverse mortgage loan with:

- variable interest rate
- lump-sum payment
- single borrower

The accumulated loan amount at termination:

$$L_{T_x} = L_0 \exp\left(\sum_{t=1}^{K_x+1} \left(y_t^1 + \varphi + \pi\right)\right)$$

- T_x: random termination time for age (x), $K_x = [T_x]$
- L_0: initial loan amount
- y_t^1: quarterly risk-free rate
- φ: quarterly lending margin
- π: mortgage insurance premium

No-Negative Equity Guarantee (NNEG)

At termination, the repayment amount is $\min\{L_{T_x},\ H_{T_x}\}$
Borrower's net equity is:

$$\begin{aligned} \text{Net Equity}_{T_x} &= H_{T_x} - \min\{L_{T_x},\ H_{T_x}\} \\ &= \max\{H_{T_x} - L_{T_x},\ 0\} \end{aligned}$$

Present value of lender's loss:

$$\text{Loss}_{T_x} = \max\{L_{T_x} - (1-c)H_{T_x},\ 0\} \prod_{s=1}^{K_x+1} m_s$$

- c: transaction cost
- m_s: risk-adjusted stochastic discount factor

The value of NNEG is:

$$\text{NNEG} = \sum_{t=0}^{\omega-x-1} E\left[{}_{t|}q_x^c \ \text{Loss}_{t+1}\right] \tag{1}$$

- ${}_{t|}q_x^c$: probability of terminating between t and $t+1$

Mortgage insurance premium

Premium accumulation:

$$dP_t = \pi L_t dt$$

EPV of charged premium is:

$$\text{MIP} = \pi \sum_{t=0}^{w-x-1} E\left[{}_t p_x^c \; L_t \prod_{s=0}^{t} m_s \right] \tag{2}$$

- ${}_t p_x^c$: in-force probability

Solve for π by equating expectations of MIP and NNEG: Equations (1) and (2).

$$\text{SF} = \sum_{t=0}^{\omega-x-1} \left\{ {}_{t|}q_x^c \; E\left[\text{Loss}_{t+1}\right] - {}_t p_x^c \; \pi E\left[L_t \prod_{s=0}^{t} m_s \right] \right\} \tag{3}$$

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Hybrid house price model (Shao et al., 2013)

$$V_{it} = \alpha + T'\beta + X'\gamma + X'\Delta T' + \eta_i + \xi_{it} \tag{4}$$

- β: coefficients for time dummy variables T
- γ: coefficients for house characteristic variables X
- Δ: coefficients for the interactions between time dummy and house characteristic variables
- η_i: individual house specific error, uncorrelated with ξ

Differencing Equation (4):

$$V_{jt} - V_{js} = D'\beta + X'\Delta D + \xi_{jt} - \xi_{js} \tag{5}$$

- D: differenced time dummy variables
- No data on changes of X

Economic scenario generation

- Model: VAR(2)

$$Y_t = \kappa + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \Sigma^{1/2} Z_t$$

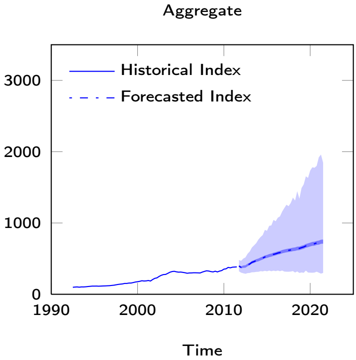
- State variables included in Y_t:
  1. aggregate house price index growth rates
  2. rental yields
  3. GDP growth rates
  4. 1-quarter zero-coupon bond yield rates
  5. spread of 5-year over 1-quarter bond yield rates
- Z_t: vector of independent standard normal variables
- Derive bond yield curve based on the development of the state variables under the VAR(2) model.
- Derive risk-adjusted stochastic discount factors (m_t) assuming no arbitrage (Cochrane and Piazzesi, 2002; Alai et al., 2013).

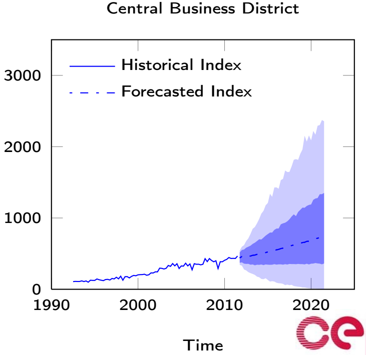
Projection of individual house prices

- Model: VARX(1,0)

$$IHG_t = \tilde{\kappa} + \tilde{\Phi} IHG_{t-1} + \tilde{\beta} HG_t + \tilde{\Sigma}^{1/2} \tilde{Z}_t,$$

- IHG_t: disaggregated house price growth rate
- HG_t: aggregate house price growth rate





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Termination triggers (Ji et al., 2012)

Four major triggers of repayment:

- λ: age-specific factor to scale down at-home **mortality**
- κ: age-specific factor that relates **LTC incidence** to mortality
- q_i^{pre}: duration-dependent annual **prepayment** probability
- q_i^{ref}: duration-dependent annual **refinancing** probability

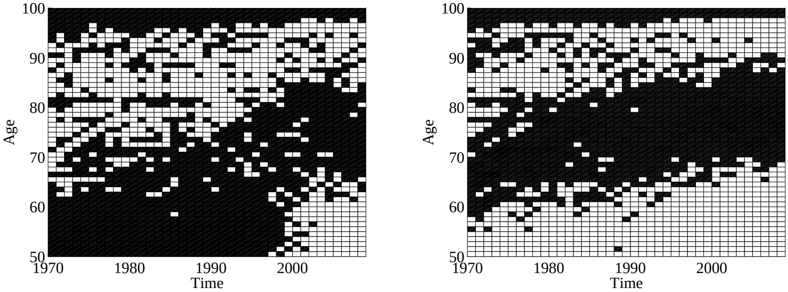
In-force probability of the reverse mortgage contract is:

$$ {}_tp_x^c = \exp\left\{\int_0^t (\lambda_{x+s} + \kappa_{x+s})\hat{\mu}_{x+s} ds\right\} \prod_{i=1}^{t} \left[(1 - q_i^{pre})(1 - q_i^{ref})\right]^{1/4} \quad (6) $$

Stochastic mortality: two factor CBD model

$$\text{logit } q(t,x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) \tag{7}$$

- $q(t,x)$: death probability
- $\bar{x}$: average age



Males (left) and females (right), + (white) and - (black)

Stochastic mortality: Wills-Sherris model

$$\begin{aligned} \Delta_c \ln(\mu(x,t)) &= \ln(\mu(x,t)) - \ln(\mu(x-1,t-1)) \\ &= ax + b + \sigma\varepsilon(x,t) \end{aligned} \quad (8)$$

- $\mu(x,t)$: force of mortality for a cohort aged x at time t
- a, b and σ: parameters to be estimated
- $\varepsilon(x,t)$: a standard normal variable

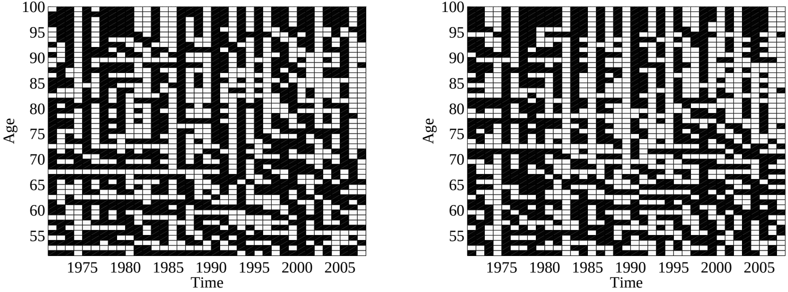
Age dependence:

$$[\varepsilon(x_1,t), \varepsilon(x_2,t), \cdots, \varepsilon(x_N,t)]' = \Omega^{\frac{1}{2}} W_t$$

Stochastic mortality: Wills-Sherris model (cont'd)

Parameter estimation:

Parameter	Male	Female
$\hat{a}$ ($\times 10^{-4}$)	-2.36	4.02**
$\hat{b}$ ($\times 10^{-2}$)	8.38***	4.59***
$\hat{\sigma}$ ($\times 10^{-2}$)	5.76***	6.52***



Males (left) and females (right), + (white) and - (black)

Projected survival curve: three models

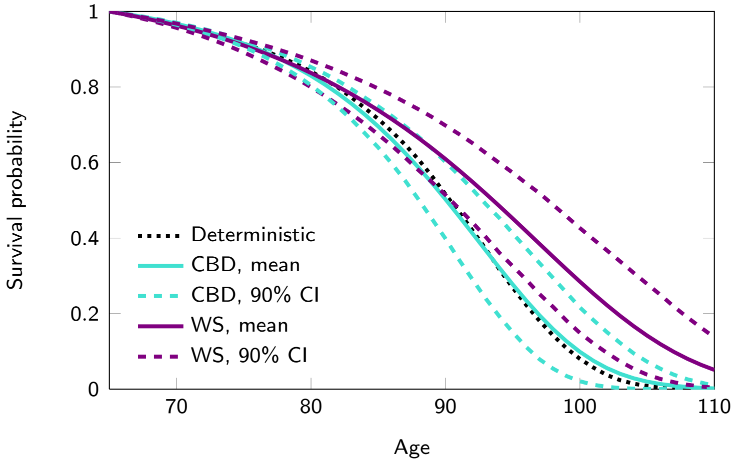


Figure: Simulated survival probabilities of 65-year-old females.

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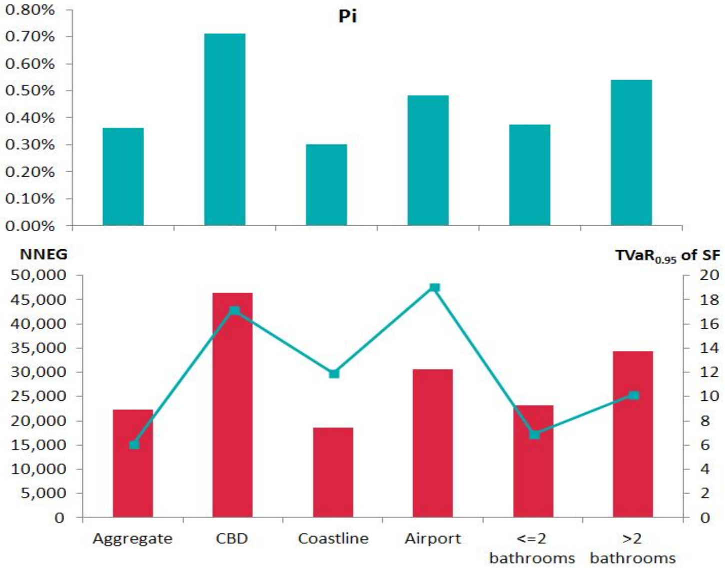
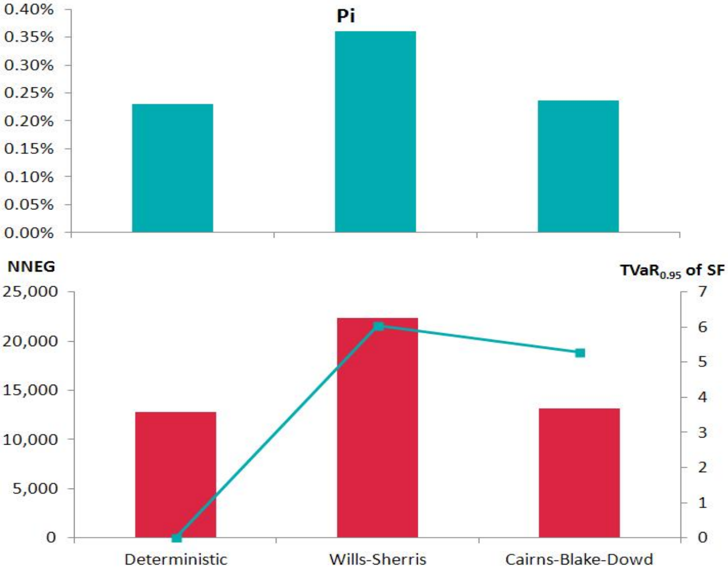
NNEG and mortgage insurance premium (Age 65, LTV 0.4): house price risk



NNEG and mortgage insurance premium (Age 65, LTV 0.4): longevity risk



Topic coverage

Conclusions

- Pricing and risk analysis of reverse mortgages
  - idiosyncratic house price risk
  - longevity risk
- Considerable house price risk
  - using an aggregate index underestimates the risk
  - risk factors associated with house characteristics should be used
- Longevity risk
  - mortality improvement modelling has a large impact
  - effect smaller than that of house price risk
- Improved insights into product pricing and risk analysis

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Appendix A: Australian market for reverse mortgages

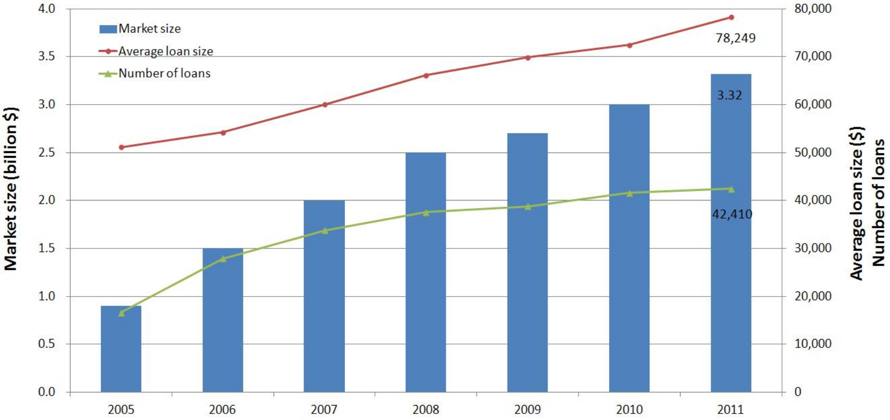

Figure: Australian Market for Reverse Mortgages, data source: Deloitte and SEQUAL media release 2012

Appendix B: NNEG and mortgage insurance premium

Model	Deterministic			Wills-Sherris			Cairns-Blake-Dowd		
LTV	0.2	0.4	0.6	0.2	0.4	0.6	0.2	0.4	0.6
A. Overall Sydney house price index									
π (p.a.)	0.003%	0.230%	3.246%	0.009%	0.360%	2.583%	0.003%	0.237%	3.126%
NNEG	71	12,794	400,017	279	22,393	335,952	90	13,147	379,366
S.E.	17	498	2,131	36	639	2,038	18	491	2,094
TVaR	0.000	0.000	0.000	0.467	6.048	12.913	0.179	5.278	13.487
B. Houses near the central business district									
π (p.a.)	0.218%	0.720%	1.829%	0.239%	0.711%	1.621%	0.218%	0.716%	1.819%
NNEG	6,043	42,421	186,092	7,298	46,370	181,302	6,036	42,138	184,776
S.E.	470	1,673	4,092	494	1,680	3,876	463	1,651	4,048
TVaR	0.000	0.000	0.000	6.654	17.148	29.594	6.424	17.779	31.168
C. Houses near to coastlines									
π (p.a.)	0.076%	0.255%	1.184%	0.088%	0.302%	1.183%	0.076%	0.257%	1.173%
NNEG	2,062	14,238	110,932	2,624	18,645	124,031	2,070	14,284	109,598
S.E.	289	879	2,399	308	939	2,402	286	866	2,359
TVaR	0.000	0.000	0.000	4.387	11.923	21.331	3.893	11.512	22.120
D. Houses with less than or equal to two bathrooms									
π (p.a.)	0.010%	0.247%	3.078%	0.019%	0.374%	2.485%	0.011%	0.253%	2.968%
NNEG	269	13,752	370,431	561	23,275	318,003	294	14,080	352,317
S.E.	86	566	2,239	99	692	2,148	87	558	2,198
TVaR	0.000	0.000	0.000	1.028	6.913	13.748	0.634	6.170	14.350
E. Houses with more than two bathrooms									
π (p.a.)	0.058%	0.418%	2.868%	0.081%	0.540%	2.376%	0.059%	0.420%	2.781%
NNEG	1,577	23,759	335,272	2,412	34,438	298,788	1,612	23,871	321,653
S.E.	209	893	2,871	232	1,005	2,717	205	874	2,807
TVaR	0.000	0.000	0.000	3.391	10.145	17.505	2.914	9.912	18.420