Case Study – Modeling Longevity Risk for Solvency II

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Background

- **SOLVENCY II** – New Minimum Capital Requirements
  - Standard Formula - Stipulated Methodology
  - Internal Model … *may produce smaller amount*

- Internal Models Current Focus on Market Risks
  - Capture Asset Volatility including Asset-Related Liability Risks

- Fifth Quantitative Impact Study (QIS5)
  - Issued by EIOPA *(European Insurance and Occupational Pensions Authority)*
  - Analysis in advance of implementation of Solvency II in 2011
  - Identified Most Material Risk Modules for Life Undertakings:

    After **Market Risk**, the next most material risk is:

    Life Underwriting Risk (Lapse and Longevity)

Solvency II Capital

- **Two Approaches:**
  - Standard Formula - Stipulated Methodology
  - Internal Model – *must satisfy certain standards:*
    - Widely used and plays important role in decision-making
    - Sufficiently sophisticated to support standards of statistical quality
    - Calibrated to external and internal trends and volatility
    - Back-tested to demonstrate sources of profit and loss
    - Validated regularly against results
    - Sufficient documentation including limits and deficiencies

Goals of this Presentation

- Demonstrate Advantages of Internal Model Using Volatile Mortality
  - Potential reduction in Required Capital
  - Better Understanding of Capital Requirements

This will be accomplished by a relatively simple case study…

The case study can be found at

http://www.milliman.com/expertise/life-financial/products-tools/reveal/pdfs/modelling-longevity-risk.pdf

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Case Study

- **Calculate Required Capital for Liabilities**
- **Ignore Market Risk**
- **Model Sample Portfolio**
- **Calculate Minimum Required Assets Under Solvency II as Sum of:**
  1. **Best Estimate Liability**
  2. **Solvency Risk Requirement (SCR)** → 1-in-200-year event
     (99.5^{th} Percentile)

  3. **Risk Margin**

- **Used proprietary modeling software, Milliman REVEAL…**

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REVEAL stands for:

Risk and Economic Volatility Evaluation of Annuitant Longevity

REVEAL is a system developed to analyze longevity risk.

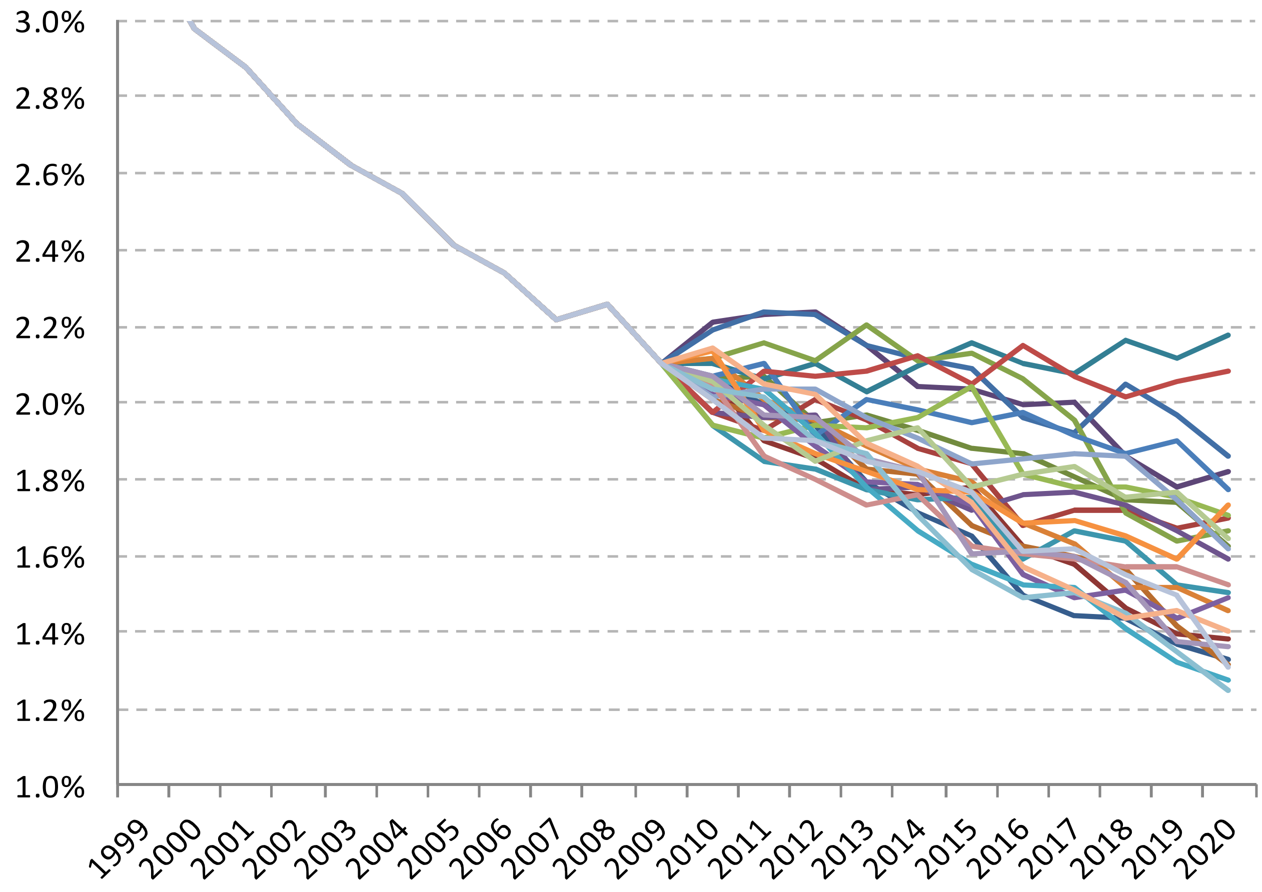
Stochastic model for pension & annuity liabilities with volatility:

- baseline mortality,
- mortality improvement,
- extreme mortality and longevity events, and
- plan participant behavior (e.g., retirement dates & benefit elections)

For more information about REVEAL, please visit:

http://www.milliman.com/expertise/life-financial/products-tools/reveal/

Historical and Projected
General Population Annual Mortality Rate
(Male 70 years old, 25 scenarios)



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Modeling Mortality Rate Volatility

1. Mortality improvement may be modeled as a combination of:
   - Long-term waves with lingering effects over multiple years, and
   - Random annual fluctuations.
2. May group ages to minimize offsetting fluctuations across model population
3. May designate random probability of the contingency of a significant long-term shift by specific cause of death related deaths (e.g., infection or cure)
4. May assign random probability of short-term mortality spike (e.g., epidemic or terrorism)

Modeling Mortality Rate Volatility

Long-Term Waves and Short-Term Fluctuations

1. Long-Term: Develop random average annualized mortality improvement factor for each T-year period.
   (e.g., T is 10 years in case study.)

2. Short-Term: Develop random annual mortality improvement factor for each year that fluctuate around the random annualized long-term improvement factor for each *T-year period.*

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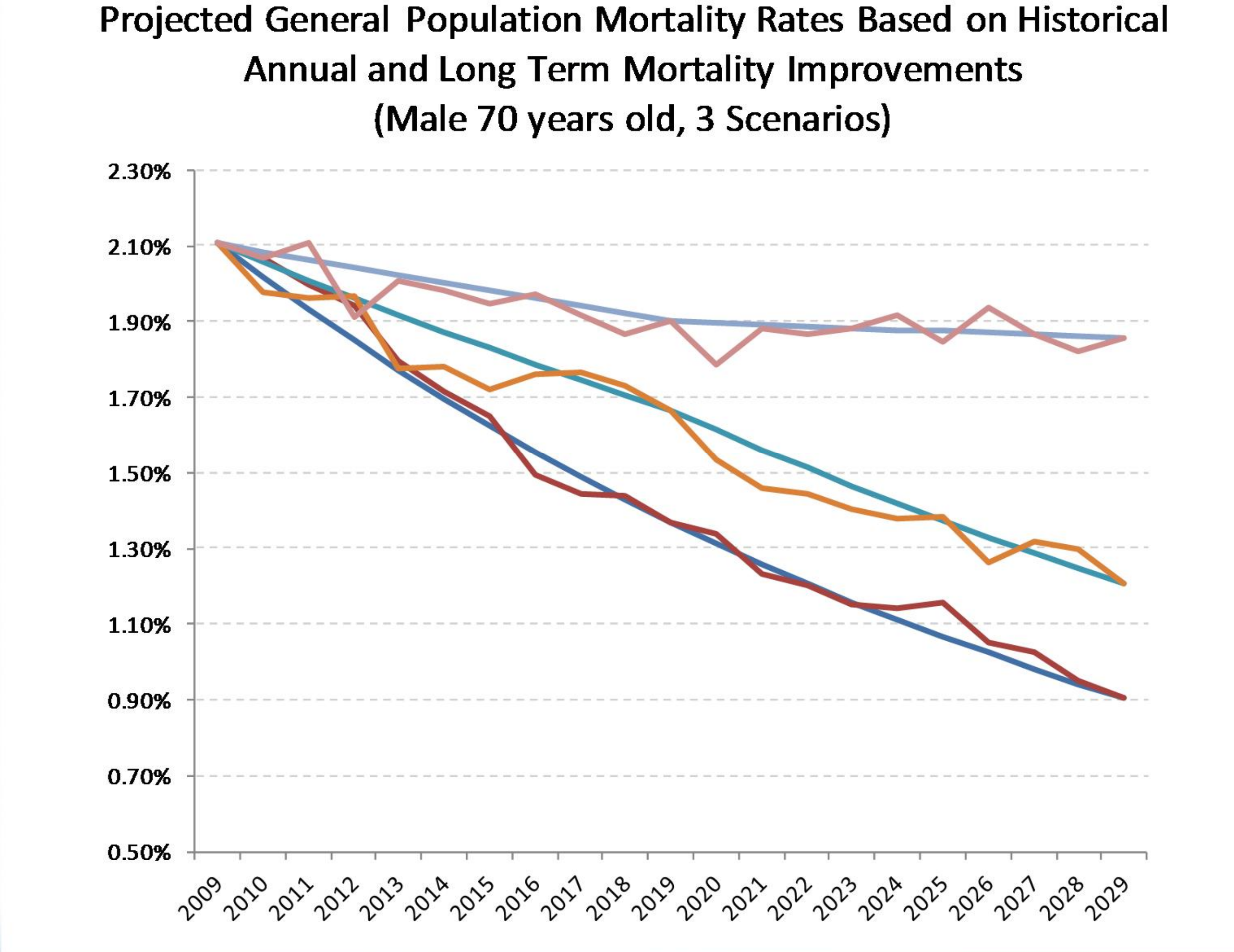
Modeling Mortality Rate Volatility

Sample Projection of Long-Term Trend and Short-Term Volatility



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Modeling Mortality Rate Volatility

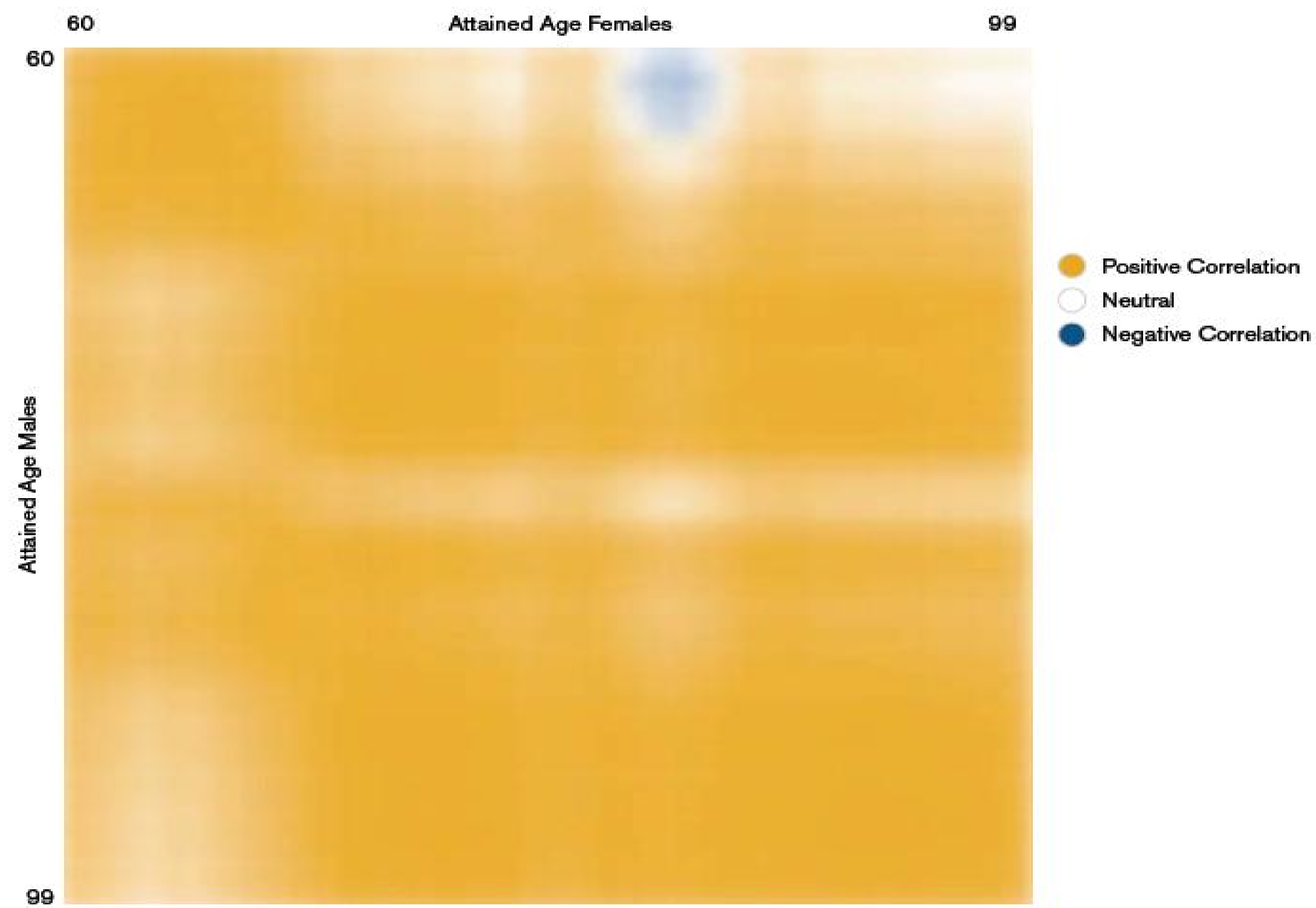
Calibration to Historical Results

- Calibrate the model, in this case, using moment fitting approach:
- The stochastic mortality model is calibrated such as:

1. The expected value of the projected improvements matches with the average historical improvements (or current expectation);
2. The volatility of the long term improvement component as well as the short term improvement component match with the historical volatility (or current expectation);
3. The long term and short term correlation match with the historical correlation (or current expectation).

Modeling Mortality Rate Volatility

Historical Long-Term Mortality Improvement Correlation, UK population



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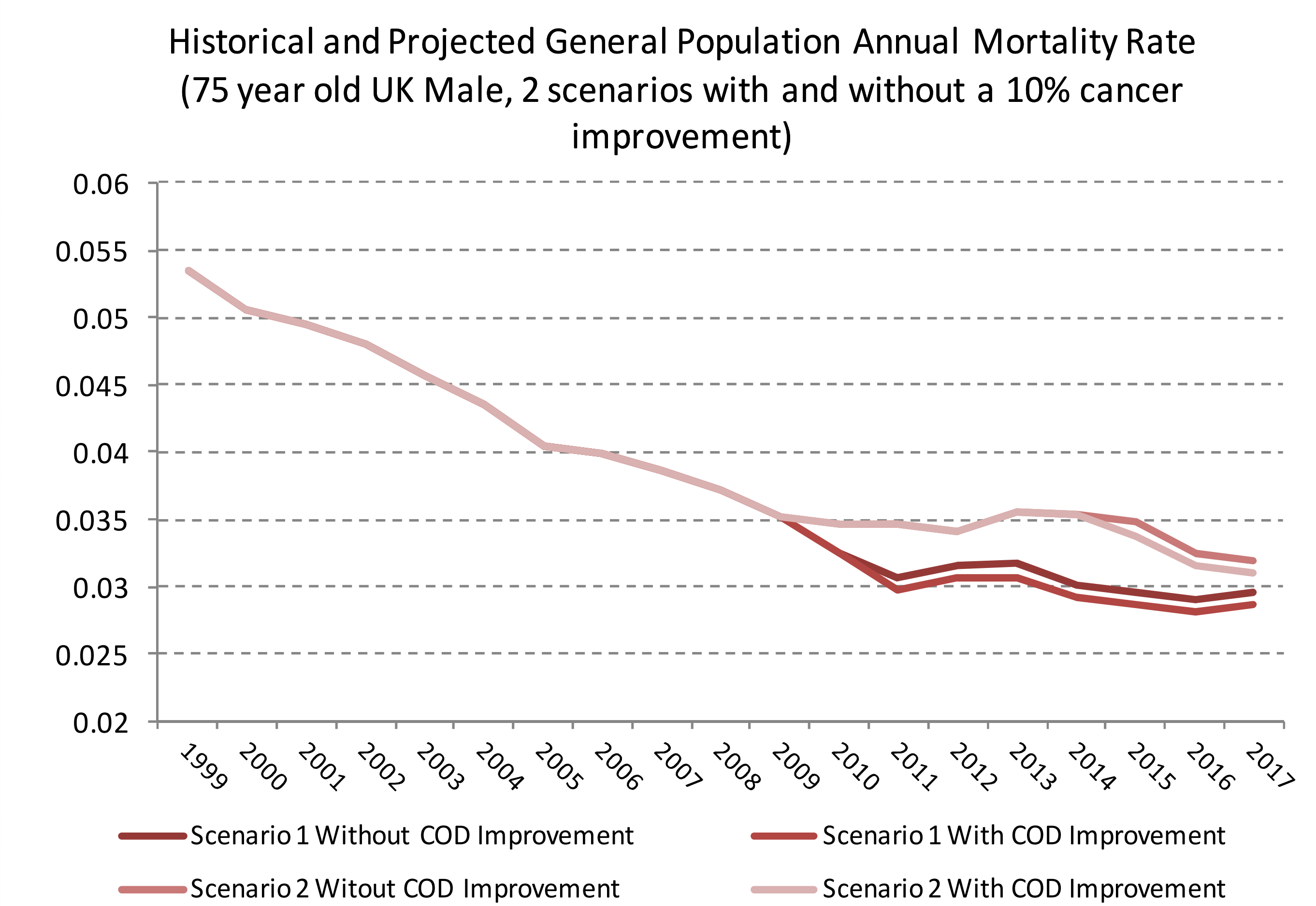
Modeling Mortality Rate Volatility

Projecting Improvement by Specified Cause of Death



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The Hypothetical Portfolio

"The Hypothetical Portfolio was designed to:

…be consistent with a typical block of in payment annuities held by an insurer.

…be sufficiently large to minimize random 'basis' volatility,

…highlight the effect of other volatility factors."

Assumptions

- Valuation Date 12/31/2010
- Hypothetical Portfolio of 50,000 in-payment annuities
- Husband Age = Wife Age + 3
- 50% Benefit to Surviving Spouse
- Annualized Benefits Increase 5% Each Year
- Quarterly Payments
- Discount Interest at the 12/31/2009 Spot Rates

Assumptions

- Best Estimate Mortality (before improvement):
  - 90% of the PCMA00 and PCFA00

    PCMA00 & PCFA00 tables in the CMI Library are described:

    “Life Office Pensioners, Combined, amounts – ultimate”
- Best Estimate Mortality Improvement:
  - Male: CMI 2010 projection model,

    with a long term rate of 1.2% p.a., applied from 2000 onwards.
  - Female: CMI 2010 projection model,

    with a long term rate of 0.9% p.a., applied from 2000 onwards.

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Distribution of Hypothetical Portfolio 1

by Annualized Benefit Amount

Measuring Life	
Primary Annuitant	55%
Spouse (Widow/Widower)	45%

Gender of Measuring Life	
Male	55%
Female	45%

Annual Benefits	
Indexed to CPI	84%
Fixed	16%

Annualized Benefit Amount	
< 1,000	13%
1,000 – 4,999	22%
5,000 – 9,999	21%
10,000 – 19,999	7%
20,000 – 29,999	1%
30,000 +	2%

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Distribution of Hypothetical Portfolio 2

by Annualized Benefit Amount

Age of Measuring Life	
Measuring Life Age Group	
Ages 60-64	13%
Ages 65-69	22%
Ages 70-74	21%
Ages 75-79	19%
Ages 80-84	14%
Ages 85-89	7%
Ages 90-94	2%
Ages 95-99	1%

Benefits Currently Paid to Joint-Life or Surviving Spouse	
Measuring Life Age Group	
Ages 60-64	92%
Ages 65-69	89%
Ages 70-74	84%
Ages 75-79	79%
Ages 80-84	74%
Ages 85-89	61%
Ages 90-94	47%
Ages 95-99	29%

Plan of Action – Values to Calculate

Best Estimate Liability

- Present Value of Expected Annuity Cash Flows,
  (Estimated using average of stochastic projections on expected mortality)

- Discounted using Risk-Free Spot Rates
  with 100% allowance for illiquidity premium.

Best Estimate Liability

ia_t = Annual Spot Rate from Risk-Free Curve with 100% allowance for illiquidity premium.

$BECF_t$ = Average annual annuity payments projected to be paid in year (e.g., best estimate cash flow)

BEL_0 = Best Estimate Liability at time zero

$$= \sum_{t=0,1,2,\ldots} \frac{BECF_t}{(1 + ia_t)^t}$$

= 1,725.5 million

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BEL_0 = 1,725.5 million

Standard Formula

Use Immediate & permanent 20% improvement in mortality rates.
(i.e., Best Estimate Mortality multiplied by 80% in all years)

SCR = excess of

(a) Standard Formula Liability
(Present value of cash flows reflecting the 20% margin),

over

(b) the Best Estimate Liability,
(*discounted to Valuation Date using the spot curves with 100% allowance for the illiquidity premium.)*

BEL_0 = 1,725.5 million

Standard Formula: Solvency Capital Requirement

$SFCF_t$ = Avg annual annuity payments projected (Using the Standard Formula Mortality Assumption) to be paid in year.

Standard Formula Liability = $\sum_{t=0,1,2,\ldots} \frac{SFCF_t}{(1+ia_t)^t}$

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BEL_0 = 1,725.5 million

Standard Formula: Solvency Capital Requirement

$SCR_0^{StdForm}$ = Solvency Capital Requirement

= Standard Formula Liability less BEL_0

= 1,884.1 million - 1,725.5 million

= 158.6 million

BEL_0 = 1,725.5 million

$SCR_0^{StdForm}$ = 158.6 million

Standard Formula: Risk Margin

Range of Formulations under QIS5, including:

Amortize SCR proportional to Best Estimate Liability annuity cash flows:

$SCR_t^{StdForm}$ = Amortized Solvency Capital Requirement

$$= SCR_{t-1}^{StdForm} * \frac{BECF_t}{BECF_{t-1}}$$

iz_t = Annual Spot Rate from Risk-Free Curve with 0% allowance for illiquidity premium

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BEL_0 = 1,725.5 million

$SCR_0^{StdForm}$ = 158.6 million

Standard Formula:

Risk Margin

$Risk\ Margin_0^{StdForm}$ = 6% *(i.e., proxy for cost of capital)* of PV of future amortized SCR_t rates, discounted at risk-free rates

$$= 6\% * \sum_{t=0,1,2,\dots} \frac{SCR_t^{StdForm}}{(1 + iz_t)^t}$$

= 181.0 million

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BEL_0 = 1,725.5 million

$SCR_0^{StdForm}$ = 158.6 million

$Risk\ Margin_0^{StdForm}$ = 181.0 million

Standard Formula: Excess Over Best Estimate

$ExBEL_0^{StdForm}$ = Standard Formula SCR
+ Standard Formula Risk Margin

= 339.6 million

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Standard Formula: Summary

$SCR_0^{StdForm}$	=	158.6 million
$Risk\ Margin_0^{StdForm}$	=	181.0 million
$ExBEL_0^{StdForm}$	=	339.6 million
BEL_0	=	1,725.5 million

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Standard Formula Summary

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Modeling Stochastic Mortality

The stochastic projections reflect three sources of volatility:

1. Randomized Dates of Death

   Monte Carlo Simulation applied to Scenario-Specific Mortality

2. Future Mortality Improvement Trend Volatility

   Analysis of historical population mortality (e.g., UK 1979-2009)
   to create stochastic mortality improvement scenarios reproducing
   historic mean, standard deviation, and correlation
   over annual and adjacent longer-term (e.g., 10-year) periods

3. Potential Extreme Longevity Occurrences

   Risk of immediate long-term change for specific cause of death
   (e.g., a new highly effective treatment for cancer)

$BEL_0 = 1{,}725.5$ million

Principle-Based Economic Calculation

The Economic Capital Approach was performed two ways:

- Volatility Assumptions A

  volatility in the mortality curve based solely on historical mortality improvement trends, and

- Volatility Assumptions B

  volatility in the mortality curve based on **both** historical mortality improvement trends and the possibility of a significant reduction in cancer related deaths.

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BEL_0 = 1,725.5 million

Principle-Based Economic Calculation

10,000 Scenarios for 50,000-Life Portfolio

The 99.5th Percentile of PV Future Annuity Cash Flows:

		$ExBEL_0$
Standard Formula	=	339.6 million
Econ Model Vol A	=	183.5 million
Econ Model Vol B	=	190.8 million

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BEL_0 = 1,725.5 million

Internal Model Approach

For Each Scenario:

1. Generate stochastic mortality improvement for first duration, and
2. Set the mortality improvement in years 2+ to:
   - reflecting simulated mortality improvement experience over the first year,
   - given a credibility factor of 10%.

BEL_0 = 1,725.5 million

Internal Model Approach

$\Delta q_{x,t}$ = Expected annual rate of mortality improvement at attained age, duration t

$Q_x^{scale}(0)$ = Stochastic adjustment to mortality improvement at attained age x, duration 0

$\Delta q_{x,t}^{new}$ = Expected annual rate of mortality improvement at attained age x, duration t, reflecting duration 0 stochastic improvement

$$= 1 - \left(1 - \Delta q_{x,t}\right) * Q_x^{scale}(0)$$

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BEL_0 = 1,725.5 million

Internal Model Approach

c = Credibility assigned to stochastic mortality improvement simulated in the first duration

$\Delta q_{x,t}^{Sol2}$ = Assumed annual rate of mort improvement at attained age x, duration t *(t = 1, 2, …)* adjusted to reflect credibility of duration 0 stochastic improvement

$$= (1 - c) * \Delta q_{x,t} + c * \Delta q_{x,t}^{new}$$

Note: *If credibility c equals zero, then the Assumed Improvement equals Best Estimate Expected Improvement. (The Case Study assumed c=10%)*

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BEL_0 = 1,725.5 million

Internal Model Approach

Our understanding of *regulator-approved internal model* under Solvency II :

Economic capital reflects once-in-200-years event

$$Internal\ Model\ SCR = VaR_{99.5\%}\left(\frac{BEL_1}{(1+i(1))} + \sum_{0<t\leq 1} \frac{CF_t}{(1+i(t))^t}\right) - BEL_0$$, where

$VaR_{99.5\%}(x)$ = 99.5% percentile of a random variable x

BEL_1 = Best Estimate Liability at Time t=1

Using altered mortality expectation reflecting credibility of "simulated" experience from t=0 to t=1

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BEL_0 = 1,725.5 million

Internal Model Approach

		Volatility Assumptions A	Volatility Assumptions B
$SCR_0^{IntModel}$	=	140.5 million	140.9 million
$Risk\ Margin_0^{IntModel}$	=	160.3 million	160.8 million
$ExBEL_0^{IntModel}$	=	300.8 million	301.8 million

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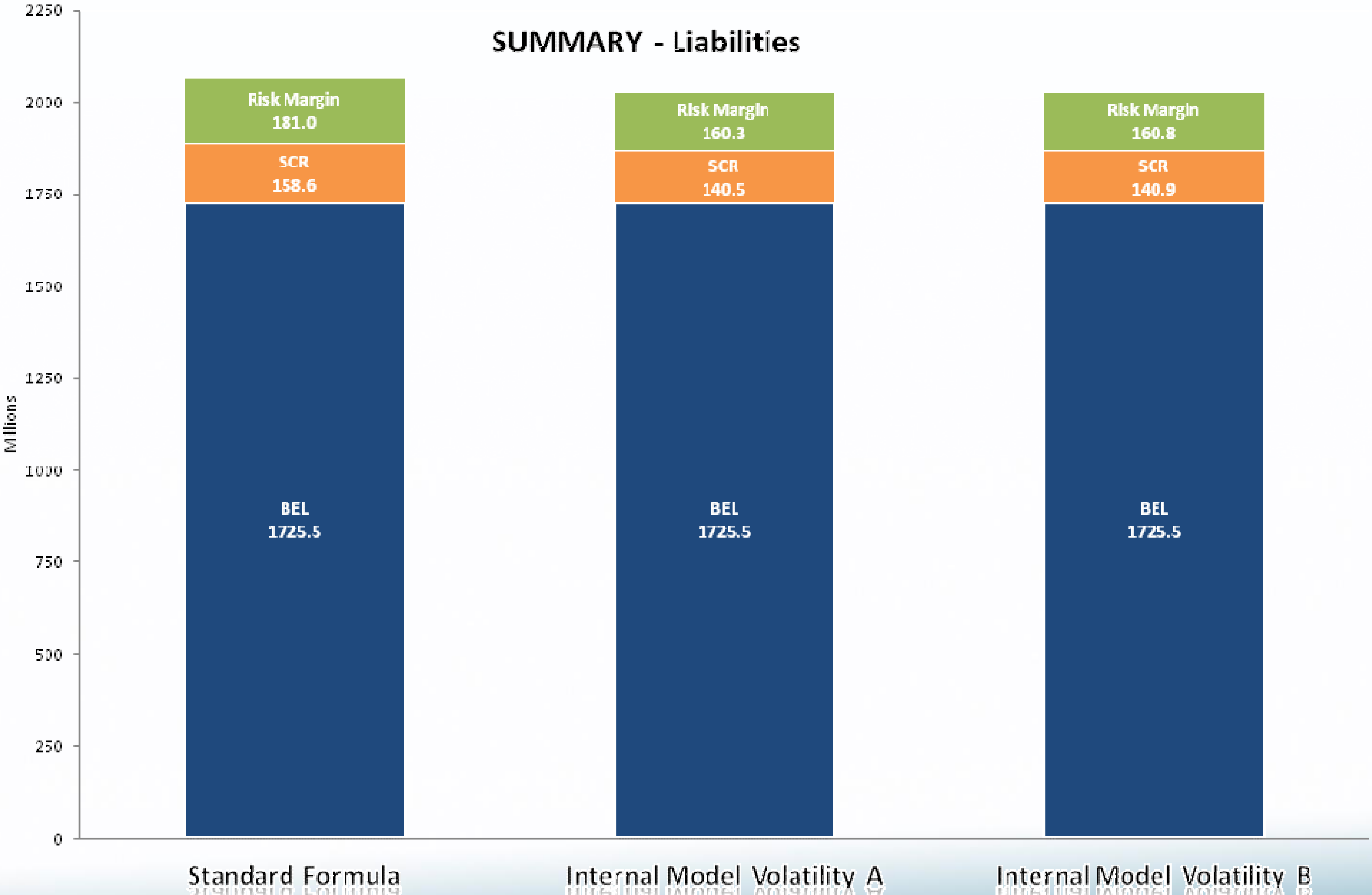
Comparison: Summary

	Standard Formula	Internal Model Volatility A	Internal Model Volatility B
SCR_0	158.6 million	140.5 million	140.9 million
$Risk\ Margin_0$	181.0 million	160.3 million	160.8 million
$ExBEL_0$	339.6 million	300.8 million	301.8 million
BEL_0	1,725.5 million	1,725.5 million	1,725.5 million


SUMMARY - Liabilities
Millions
2250
2000
1750
1500
1250
1000
750
500
250
0
Risk Margin
181.0
SCR
158.6
BEL
1725.5
Risk Margin
160.3
SCR
140.5
BEL
1725.5
Risk Margin
160.8
SCR
140.9
BEL
1725.5
Standard Formula
Internal Model Volatility A
Internal Model Volatility B

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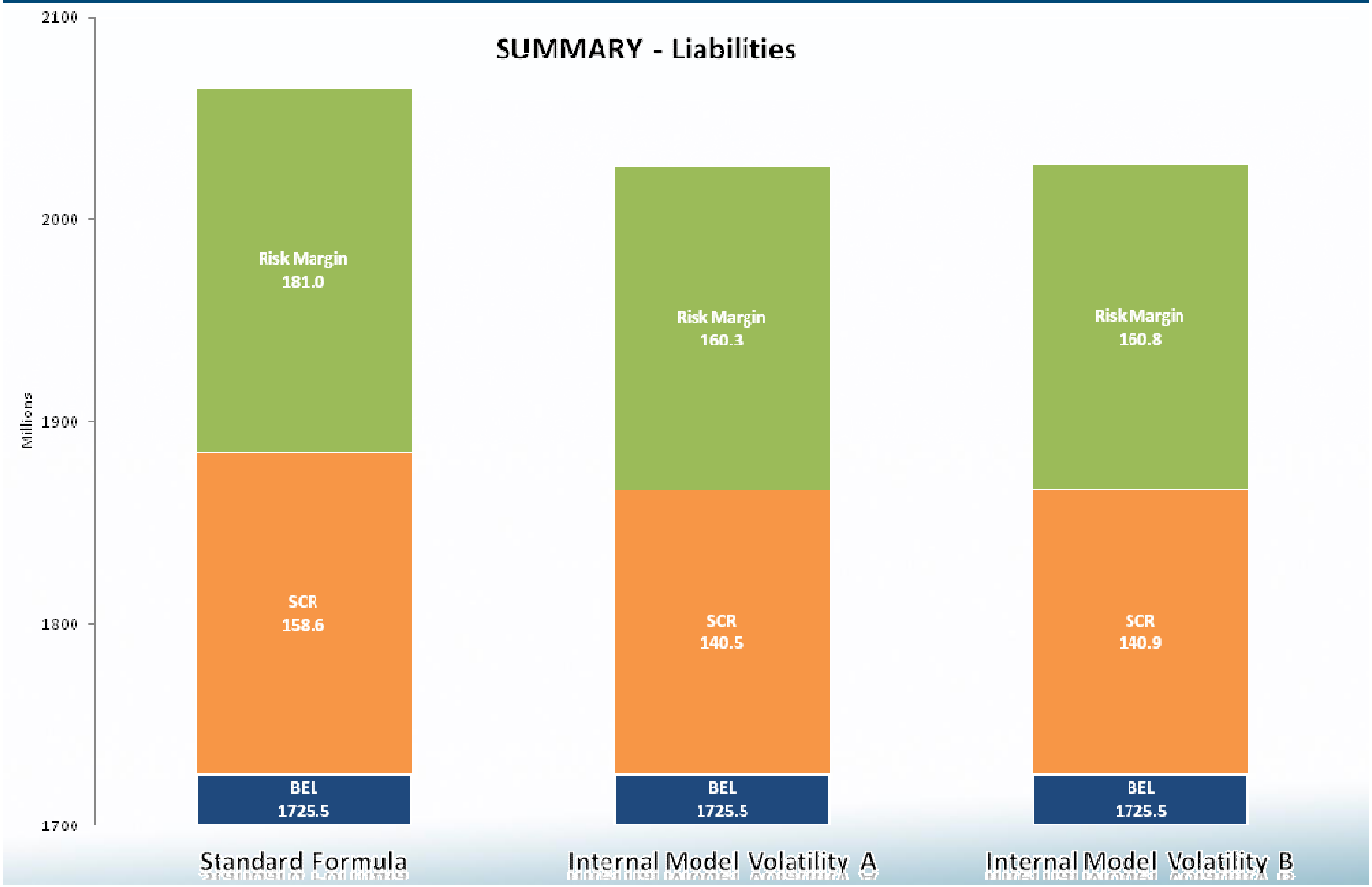
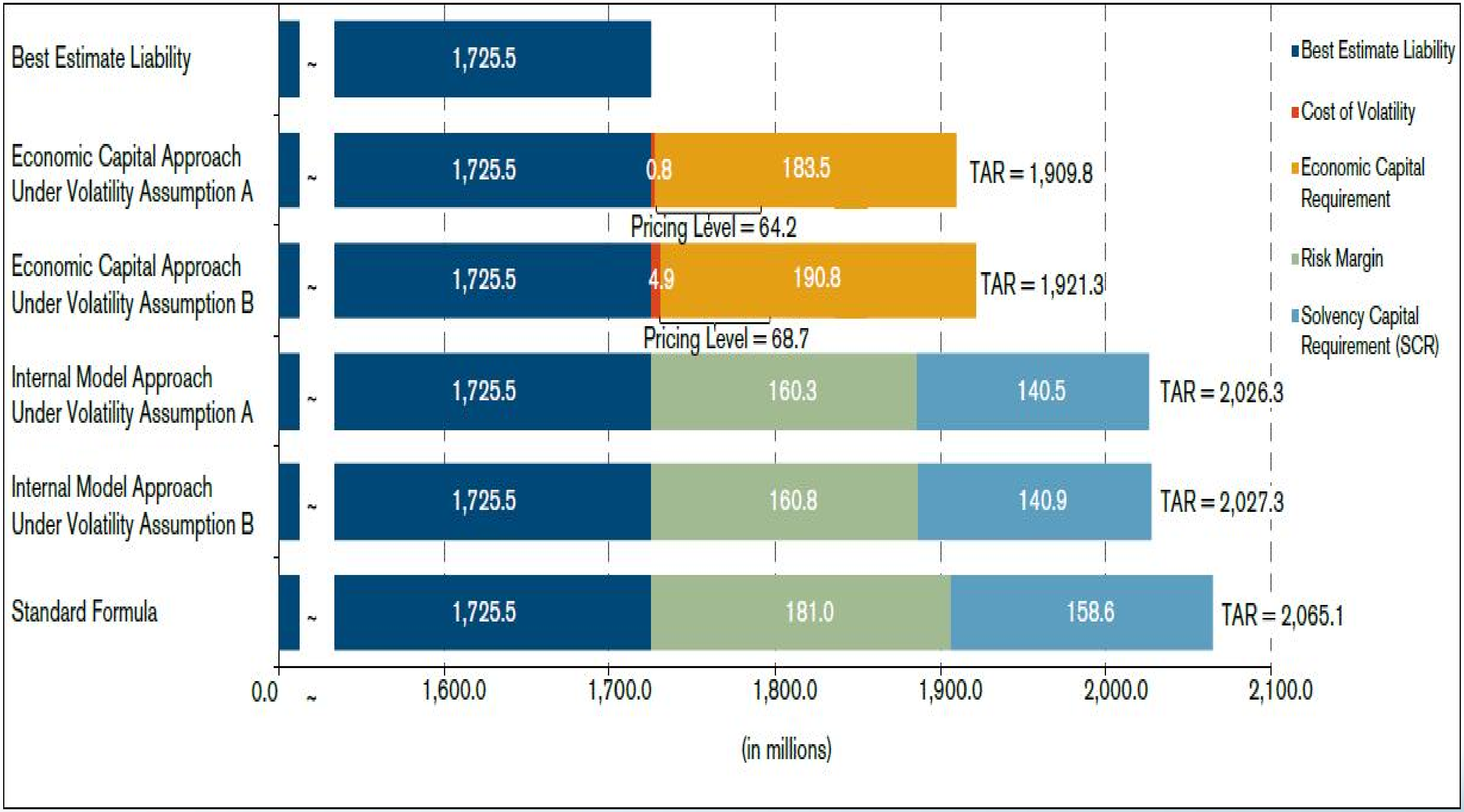

SUMMARY - Liabilities
Millions
2100
2000
1900
1800
1700
Risk Margin
181.0
SCR
158.6
BEL
1725.5
Risk Margin
160.3
SCR
140.5
BEL
1725.5
Risk Margin
160.8
SCR
140.9
BEL
1725.5
Standard Formula
Internal Model Volatility A
Internal Model Volatility B

Solvency II Capital: Internal Model Savings

	Standard Formula	Reduction Under Internal Model Volatility A	Reduction Under Internal Model Volatility B
SCR_0	158.6 million	18.1 million	17.7 million
$Risk\ Margin_0$	181.0 million	20.7 million	20.2 million
$ExBEL$	339.6 million	38.8 million	37.0 million
BEL	1,725.5 million	0 million	0 million

Components of Total Assets Requirement (TAR)



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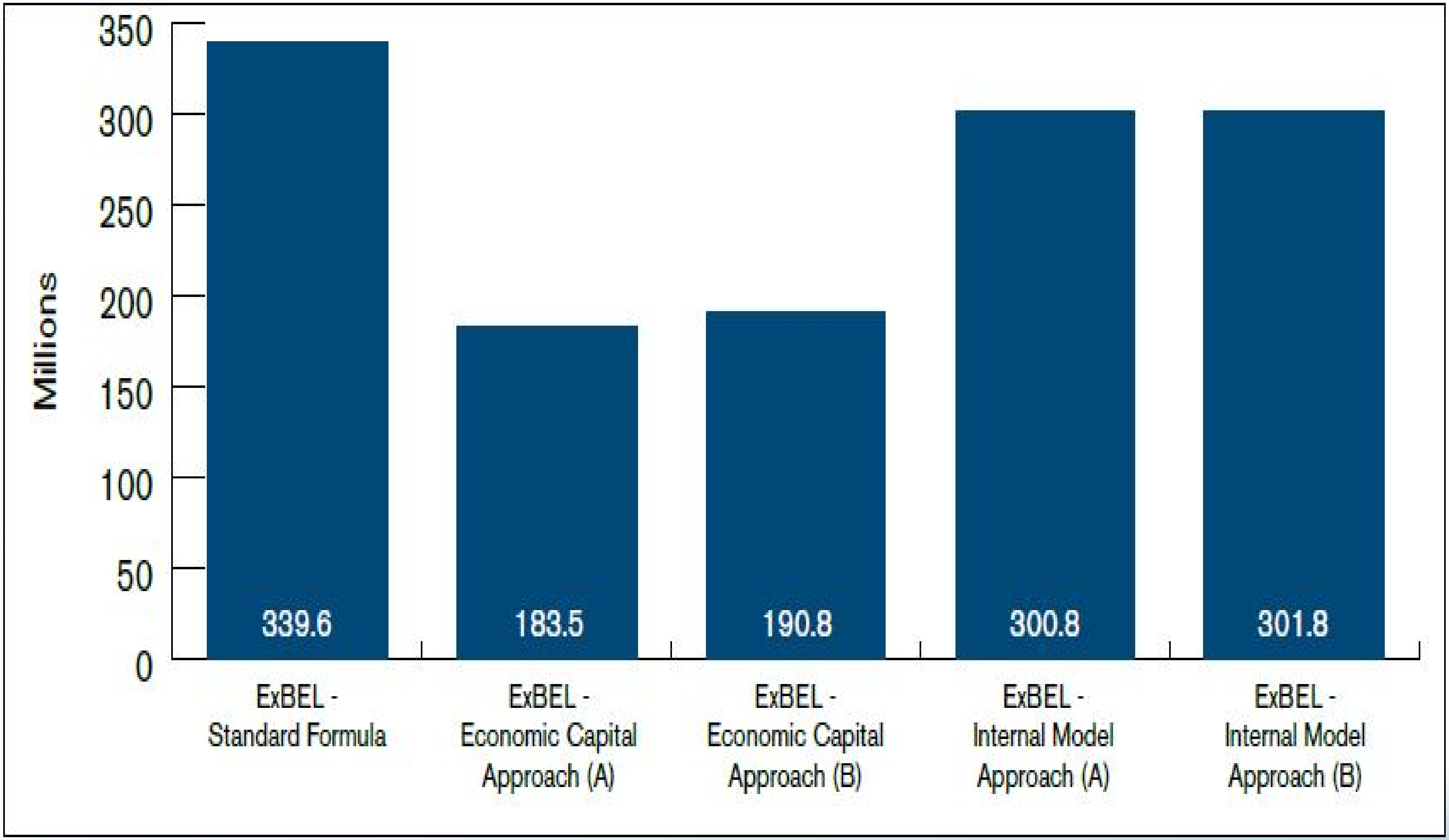
Summary – Excess Over Best Estimate Liability



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Take-Aways

1. Internal Model may produce capital savings for Longevity Risk (relative to Standard Formula)
2. Internal Model may still produce higher capital costs than a principle-based economic calculation
3. Possible advantage of financial transactions to move longevity risk to more favorable regulatory environment.

Contact Information

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