The Annuity Duration Puzzle

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- We have access to a unique dataset of over 3 million annuity prices (weekly quotes) over a period of a decade, from the major U.S. insurance companies.
- We are interested in studying how prices react to changes in interest rates, a.k.a. **annuity duration.**

Our main results are (i.) prices react to 30-year mortgage rates more than risk-free U.S. swap or Treasury rates, (ii.) duration values are much lower than expected, and (iii.) the response is asymmetric, all of which we label an **annuity duration** puzzle.

- To be clear, we are interested in the Single Premium Immediate Annuity (SPIA), a $7.6 billion U.S. market (2010).
- This is much smaller than the Variable Annuities (VA) with Guaranteed Living Income Benefits (GLIB), which is a $140 billion U.S. market (2010) in new sales.

[1.] **Portfolio Choice and Timing of Annuitization**
How much of personal retirement wealth should be allocated to life annuities and at what age should they be purchased?

- Yaari (RES, 1965),
- Brugiavini (JPubE, 1993)
- Gerrard, Haberman and Vigna (IME, 2004), Kingston & Thorpe (JPEF, 2005), Stabile (IJTAF, 2006)
- Milevsky, Moore and Young (MathFin, 2006), Milevsky and Young (JEDC, 2007)
- Horneff, Maurer, Stamos (JEDC, 2008), Horneff, Maurer, Mitchell and Stamos (JPEF, 2010)
- Koijen, Nijman and Werker (RF, 2010)
- other papers at this conference!

[2.] **Money's Worth Ratio (MWR) and Pricing Studies**

Do private-market life annuities provide "good value" around the world and how large is the adverse selection problem?

- Warshawsky (JRI, 1988)
- Finkelstein and Poterba (JPE, 2004)
- Mitchell, Poterba, Warshawsky and Brown (AER, 1999)
- Cannon and Tonks (FHR, 2004)
- Sheshinski (Princeton U. Press, 2008)
- Fong, Lemaire and Tse (WP, 2011)

[3.] **Annuity Puzzle and Solutions**

If annuities are so great, then why don't more people voluntarily purchase annuities, and what can we do about it?

- Davidoff, Brown and Diamond (AER, 2005)
- Dushi and Webb (JPubE, 2004)
- Lopez and Michaelidis (FRL, 2007)
- Butler and Teppa (JPubE, 2007)
- Pashchenko (WP, 2010)
- Inkman, Lopez and Michaelidis (RFS, 2011)
- Benartzi, Previtero and Thaler (JEP, 2011)
- Ameriks, et. al. (JF, 2011)

[4.] **Proper Valuation of Pension Liabilities**
A Defined Benefit (DB) pension plan creates deferred annuity-like liability for the sponsor and should be valued as such. Are sponsors doing this properly?

- Treynor (JF, 1976), Bulow (QJE, 1982)
- Bodie (FAJ, 1990), Ippolito (FAJ, 2002)
- Sundaresan and Zapatero (RFS, 1997)
- Brown and Wilcox (AER, 2009)
- Novy-Marx and Rauh (JF, 2011)

All of these research strands implicitly or explicitly assume that over short periods of time, the **only** change in prices should be due to interest (or valuation) rates.

The (*No Arbitrage*) Annuity Factor:

$$\bar{a}(x, g, r) = \int_0^\infty e^{-rt} p(x, g, t) dt.$$

The (pseudo) survival probability is:

$$p(x, g, t) = \begin{cases} 1 & t \leq g \\ e^{-\int_0^t \lambda(x+s)ds} & t > g. \end{cases}$$

Where g is the period certain (in years) and $\lambda(x + s)$ is any continuous mortality rate.

The *theoretical* duration of the life annuity factor is:

$$D(x,g,r) := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} \approx \frac{-\triangle \bar{a}(x,g,r)/\triangle r}{\bar{a}(x,g,r)}.$$

If (a_i, r_i) denotes the *observed* annuity factor and corresponding interest rate at time i, and (a_{i+1}, r_{i+1}) denotes the same pair in the next period, then:

$$\frac{-(a_{i+1} - a_i)}{a_i} = \frac{-\triangle a_i}{a_i} = D(r_{i+1} - r_i) = D\triangle r_i,$$

where D is now the *empirical* duration.

So, our plan is to run the regression:

$$\frac{-\triangle a_i}{a_i} = d_0 + d_1 \triangle r_i + e_i.$$

We would expect that $d_0 = 0$ and $d_1 = D(x, g, r)$, as long as we get r and $\triangle r$ calibrated correctly.

NUMERICAL EXAMPLE: Assume that $r = 4.35\%$ and the mortality rate $\lambda(t)$ obeys a Gompertz law of mortality with $m = 92.63$ and $b = 8.78$. This implies that $p(65, 35) = 10.3\%$, which is the survival probability for a healthy U.S. female from the Individual Annuity Mortality table. (Source: Milevsky and Young, 2007).

And, here are annuity duration values:

$$\begin{bmatrix} g = 10 \text{ (years)} & a(x,g,r) & \frac{\partial \bar{a}(x,g,r)}{\partial r} & D \text{ (Duration)} \\ \text{Age } x = 55 & \$17.02 & -220 & 12.9 \\ \text{Age } x = 65 & \$14.44 & -151 & 10.4 \\ \text{Age } x = 75 & \$11.51 & -90 & 7.8 \end{bmatrix}$$

The Data

- The database was created via a collaboration between CANNEX Financial Exchanges and QWeMA Group.
- It contains over three million individual quotes spanning seven years and twenty five life insurers.
- Quotes are classified by age; gender, guarantee period; mortality dependence, such as single life, joint life, and term certain; and qualified vs. non-qualified status.
- The database is updated weekly, and is validated and scrubbed for irregularities.

- We focused on qualified quotes and averaged the payout rates (minus outliers) across all U.S. insurance companies quoting on a given date.
- We then focused on the following six age groups 55, 60, 65, 70, 75, 80 and five guarantee periods 0, 5, 10, 15, 20.
- In total, each observation date consisted of a matrix of 30 numbers for males and 30 numbers for females.
- The monthly annuity payouts (MAP) were converted into annuity factors (AF) by annualizing the monthly payout and then dividing into the $100,000 premium.

Male Annuity Payout Rates at age 60, 65 and 70 with 0 PC:

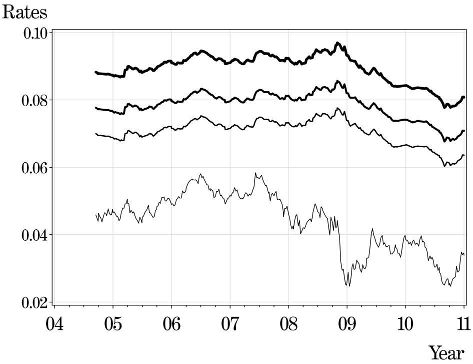

Figure: The lowest line is the **10 year USD swap rate** and the rates are plotted to increase in thickness as age increases

Female Annuity Payout Rates at age 60, 65 and 70 with 0 PC:

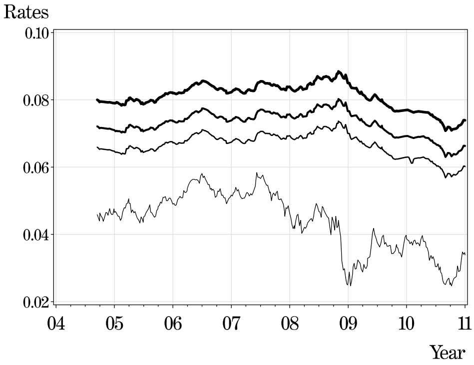

Figure: The lowest line is the **10 year USD swap rate** and the rates are plotted to increase in thickness as age increases

Table 1
Theoretical Annuity Factors ($)
Male Annuitant

	Guarantee Period				
Age	20	15	10	5	0
55	16.438	16.048	15.779	15.623	15.572
60	15.539	14.946	14.526	14.278	14.196
65	14.722	13.845	13.200	12.809	12.679
70	14.069	12.831	11.867	11.262	11.053
75	13.640	12.002	10.620	9.698	9.370
80	13.432	11.435	9.565	8.207	7.697

We used the earlier-mentioned Gompertz parameter values.

Table 2
Average of Observed Annuity Factors
Male Annuitant

	Guarantee Period				
Age	20	15	10	5	0
55	16.156	15.830	15.594	15.430	15.375
60	15.371	14.853	14.484	14.249	14.168
65	14.648	13.846	13.248	12.871	12.748
70	13.963	12.925	12.006	11.387	11.190
75	N/A	12.089	10.847	9.884	9.553
80	N/A	N/A	9.769	8.432	7.899

These are consistent with theoretical annuity factors.

Table 3
Summary Statistics for Interest Rates: Jan2004 to Dec2010

Variable (%)	Mean	Std	Min	Max	Skew	Kurt
10 Year Swap	4.368	.876	2.47	5.84	-0.45	-0.82
$\triangle$ *10 Yr*	-.0033	.139	-0.71	0.52	-0.16	2.49
30 Year Mort.	5.720	.683	4.17	6.80	-0.47	-0.93
$\triangle$ *30 Yr Mort.*	-.0025	.104	-0.44	0.52	0.43	5.47

(I'll discuss mortgage rates, later.)

Table 4
Summary Statistics of Weekly Percentage Changes in Annuity Factors, Male Annuitant

	Guarantee Period (Years)				
Age	20	15	10	5	0
55	-.0180	-.0241	-.0253	-.0215	-.0189
60	-.0202	-.0202	-.0193	-.0227	-.0149
65	-.0145	-.0156	-.0201	-.0140	-.0111
70	-.0388	-.0134	-.0112	-.0107	-.0099
75	N/A	-.0359	-.0124	-.0137	-.0123
80	N/A	N/A	-.0201	-.0158	-.0127

Notes: Standard errors close to 0.7%.

Regression Results Using 10 Year Swap Rates

- Recall that our primary regression concerns the empirical duration of the annuity contracts:

$$\frac{-\triangle a_i}{a_i} = d_0 + d_1 \triangle r_i + e_i.$$

  We expect that $d_0 = 0$. Expected values for d_1, based on the calibration, are presented in the next table.
- Note that theoretically $\triangle r$ is over an infinitely small period. In practice, we use one week changes, to start. Then we expand to wider windows.

Table 5
Theoretical Duration (Years) - Male

	Guarantee Period				
Age	20	15	10	5	0
55	12.0	11.9	11.9	11.9	12.0
60	11.0	10.7	10.7	10.7	10.8
65	10.0	9.6	9.4	9.5	9.6
70	9.3	8.5	8.2	8.2	8.3
75	8.9	7.7	7.0	7.0	7.1
80	8.6	7.1	6.0	5.8	5.9

Over one week the fit of the model is quite poor...

Table 4
Estimated Durations for Male Annuitant

	Guarantee Period				
Age	**20**	**15**	**10**	**5**	**0**
55	**1.21**	**1.21**	**1.25**	**1.31**	**1.24**
60	**1.14**	**1.10**	**1.01**	**1.12**	**1.10**
65	**1.29**	**1.01**	**0.92**	**0.99**	**0.96**
70	**0.32**	**0.85**	**0.80**	**0.83**	**0.78**
75	N/A	**0.16**	**0.68**	**0.68**	**0.61**
80	N/A	N/A	**-0.10**	**0.52**	**0.48**

Bolded coefficients are significant at the 5% level - all these coefficients are strongly significant so we reject the model.

...and the explanatory power is very low.

R^2s

	Guarantee Period				
Age	**20**	**15**	**10**	**5**	**0**
55	0.069	0.064	0.075	0.078	0.078
60	0.076	0.072	0.058	0.070	0.074
65	0.080	0.074	0.060	0.069	0.071
70	0.005	0.067	0.060	0.063	0.059
75	N/A	0.002	0.050	0.049	0.045
80	N/A	N/A	0.001	0.037	0.030

Note: We increased our window beyond one week, to a period of months and the R^2 never exceed 25%. The estimated duration values were < 3 years.

Are We Using the Correct Interest Rate?

- Insurance companies base pricing on the yield from their (highly regulated) investment portfolios.
- According to the NAIC, assets held by life insurance companies as of July, 2011 are:
  - corporate bonds (43.6%),
  - US government bonds (18.7%),
  - structured securities such as mortgage-backed and asset-backed bonds (18.5%) and
  - commercial mortgage loans (8.5%).
- The use of an interest proxy that better reflects the riskiness of portfolios held by insurance companies may be more appropriate.

30 Year Mortgage Rates

- Obtained from the weekly Federal Home Loan Mortgage Corporation's Primary Mortgage Market Survey.
- The rates are those that borrowers can expect for a 30-year fixed-rate mortgage loans on the survey day.
- The 30-year mortgage rates are less volatile than the 10-year swap rates.

R^2 as a Function of the Observation Lag, Males:

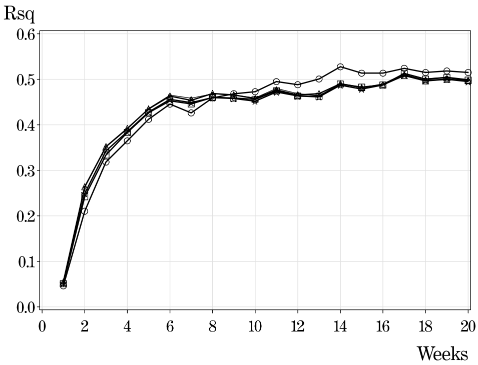

Figure: Age 65, across guarantee periods: 0 Years: $\diamond$; 5 Years: $\star$; 10 Years: $\square$; 15 Years: $\triangle$; 20 Years: $\circ$

R^2 as a Function of the Observation Lag, Females:

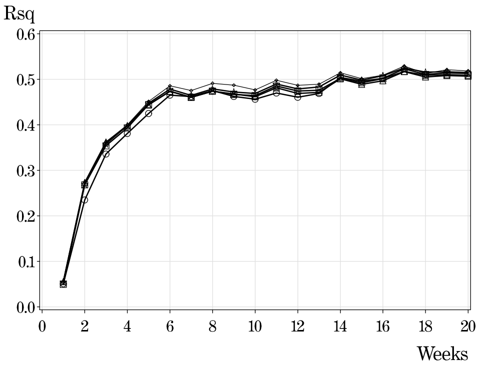

Figure: Age 65, across guarantee periods: 0 Years: ◇; 5 Years: ⋆; 10 Years: □; 15 Years: △; 20 Years: ○

Duration as a Function of the Obs. Lag, Males:

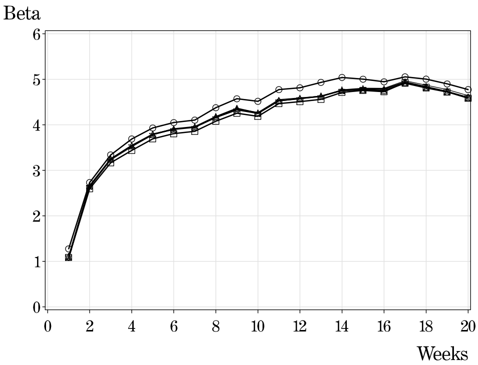

Figure: Duration coefficients, age 65, across guarantee periods: 0 Years: ⋄; 5 Years: ⋆; 10 Years: □; 15 Years: △; 20 Years: ○

Duration as a Function of the Obs. Lag, Females:

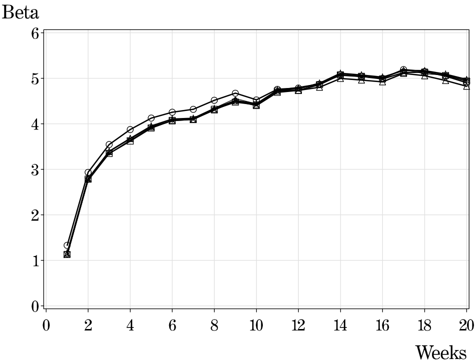

Figure: Duration coefficients, age 65, across guarantee periods: 0 Years: $\diamond$; 5 Years: $\star$; 10 Years: $\square$; 15 Years: $\triangle$; 20 Years: $\circ$

We do better with 30-year mortgage rates, but the duration values are still lower than expected...

- Prices may react to rate increases and decreases differently.
- We specify the regression equation as follows:

$$-\frac{\triangle a_i(x,g|k)}{a_{i-k}(x,g)} = \beta_k + \beta_{\triangle r_{t,Neg.Sign}(30|k)} \cdot \triangle r_i(30|k) \cdot I_{\triangle r,neg}$$

$$+\beta_{\triangle r_t(30|k)} \cdot \triangle r_i(30|k) + \varepsilon_i,$$

MALES: 30-Year Mortgage Rates, Asymmetry

Table 8, Measurement Period $k = 16$ weeks

Panel A
Estimated Durations
When Interest Rates Increase

	Guarantee Period (Years)				
Age	20	15	10	5	0
55	**7.19**	**7.08**	**7.22**	**7.32**	**7.24**
60	**6.87**	**6.77**	**6.83**	**6.99**	**6.89**
65	**6.35**	**6.21**	**6.16**	**6.22**	**6.31**
70	**5.44**	**5.67**	**5.56**	**5.58**	**5.56**
75	N/A	**4.56**	**5.07**	**5.03**	**4.94**
80	N/A	N/A	4.42	4.22	4.21

MALES: 30-Year Mortgage Rates, Asymmetry

Table 8, Measurement Period $k = 16$ weeks

Panel B
Estimated Durations
When Interest Rates Decline

	Guarantee Period (Years)				
Age	20	15	10	5	0
55	**4.51**	**4.50**	**4.59**	**4.73**	**4.62**
60	**4.25**	**4.13**	**4.31**	**4.33**	**4.14**
65	**4.06**	**3.78**	**3.78**	**3.83**	**3.81**
70	**3.81**	**3.47**	**3.43**	**3.51**	**3.68**
75	N/A	**3.23**	**3.04**	**3.08**	**3.35**
80	N/A	N/A	2.20	**2.79**	**3.08**

FEMALES: 30-Year Mortgage Rates, Asymmetry

Table 8, Measurement Period $k = 16$ weeks

Panel D
Estimated Durations
When Interest Rates Increase

	Guarantee Period (Years)				
Age	20	15	10	5	0
55	**7.49**	**7.45**	**7.49**	**7.62**	**7.47**
60	**7.31**	**7.19**	**7.18**	**7.26**	**7.12**
65	**6.58**	**6.46**	**6.49**	**6.60**	**6.56**
70	**5.65**	**5.82**	**5.88**	**5.73**	**5.88**
75	N/A	**5.22**	**4.63**	**5.17**	**5.23**
80	N/A	N/A	**4.05**	**4.05**	**4.16**

FEMALES: 30-Year Mortgage Rates, Asymmetry

Table 8, Measurement Period $k = 16$ weeks

Panel E
Estimated Durations
When Interest Rates Decline

	Guarantee Period (Years)				
Age	20	15	10	5	0
55	**4.50**	**4.48**	**4.48**	**4.52**	**4.69**
60	**3.96**	**4.07**	**4.18**	**4.30**	**4.26**
65	**3.95**	**3.90**	**3.99**	**3.99**	**3.97**
70	**3.87**	**3.49**	**3.58**	**3.62**	**3.67**
75	N/A	**3.47**	**3.22**	**3.20**	**3.25**
80	N/A	N/A	**3.22**	**2.19**	**3.14**

Conclusions

- Annuity prices (quotes, rates) do not respond to changes in interest rates in a way one might expect:
  - Empirical duration values are much smaller than predicted.
  - Prices take months to respond to interest rate changes.
  - Only a small portion of variability in prices is explained.
  - Price response is non-symetric.
- Overall, our results suggest some market timing benefits and suggest that further research is needed on developing a proper model for the dynamics of annuity prices.
  - **Caveats:** We use a point from the yield curve. Perhaps a multivariate regression on various yields from the curve would do a better job.