Pricing pension guarantees

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The guaranteed pension product

A guaranteed pension product is defined as a life annuity product that guarantees

- a yearly minimum return throughout the contract.
  → bounded downside financial risk with upside potential for the individual.
- fixed mortality rates agreed upon at contract initiation, usually according to a mortality table.
  → no longevity risk for the individual.

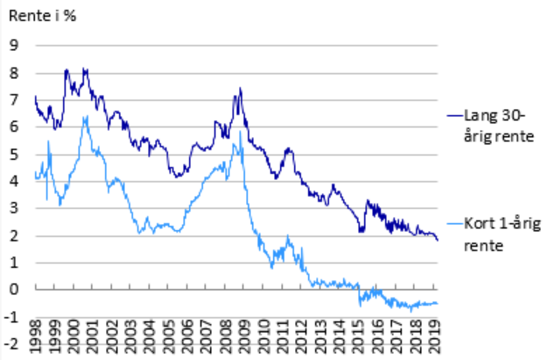
So there are two particular sources of risk transferred to the pension fund via the guaranteed pension contracts:

- Financial risk.
- Longevity risk.

Managing these risks is no problem if the risk-free interest rate does not decrease too much and life expectancy on average follows the implemented table, but...

Financial risk

Following the financial crisis, this happens...



Financial risk

Consequence: Because of the financial guarantee embedded in the guaranteed pension product, the pension fund risks being liable for covering an financial arbitrage, due to the risk-free return falling below the guaranteed.

Longevity risk

For longevity risk, we distinguish between

- Micro-longevity risk: That an individual possibly lives longer/shorter than the average life expectancy. Idiosyncratic risk that can be shared among individuals.
- Macro-longevity risk: That the average life expectancy increases/decreases more than expected. A systematic risk that cannot be shared among individuals.

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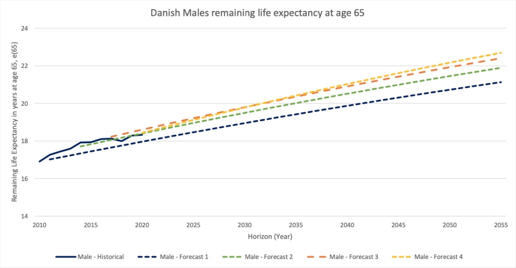
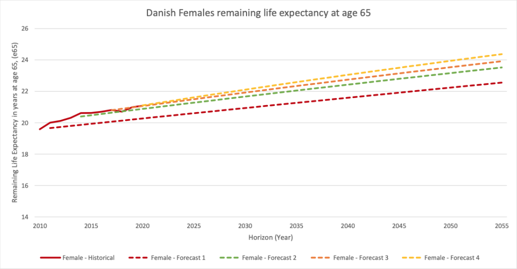
Longevity risk

The Danish population is living longer on average than expected





Longevity risk

Consequence: If the population lives longer on average than anticipated by the tables, pension funds that offer guaranteed products base their annuity calculations on a life expectancy that is too low, leading to annuity payments that are too high, which they are liable to pay for the remaining lifetime of each individual.

Solvency II

- Solvency II was initiated in 2000 and fully implemented in 2016.
- An improved regulatory solvency system with the aim of assessing the true risk profile of an insurance company/pension fund with more specific requirements to the capital buffer, including a requirement to include longevity risk.

Solvency II

Consequence: The capital requirements for pension funds supplying guaranteed products increases significantly due to the hard promises built into the guaranteed products and the associated risks.

Options to reduce risk

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- **Buy the guarantees back.**

Data

Level of Guarantee	Total	2%	3%	3.7%	4.25%
2011 Transition					
Individuals at risk	4,770	1,312	516	1,298	1,644
Individuals who transition	3,077	682	283	850	1,262
Transition pct.	64.51%	51.98%	54.84%	65.49%	76.76%
Compensation pct.	(22.8%+17.3%) = 40.1%	(22.2% + 17.3%) = 39.5%	(22.5% + 17.4%) = 39.9%	(22.7% + 17.5%) = 40.2%	(23.1% + 17.3%) = 40.4%
2012 Transition					
Individuals at risk	1,656	630	230	446	350
Missing individuals	37	0	3	2	32
Individuals who transition	494	182	62	129	121
Transition pct.	29.83%	28.89%	26.96%	28.92%	34.57%
Compensation pct.	(18.5% + 19.6%) = 38.1%	(21.4% + 17.6%) = 39.0%	(23.9% + 18.6%) = 42.5%	(23.1% + 19.2%) = 42.3%	(23.3% + 20.8%) = 44.1%
2018 Transition					
Individuals at risk	1,040	448	134	301	152
Missing individuals	122	0	34	16	77
Individuals who transition	198	96	21	61	20
Transition pct.	19.04%	21.19%	15.67%	20.27%	13.16%
Compensation pct.	27.4%	11.6%	18.8%	38.6%	76.5%

Missing individuals may occur due to death or from changing job. The unbalanced panel data set is more or less reflected in our Y variable as we loose individuals over time due to transitions. Compensation pct. reflects the empirical observed percentage given split into an individual bonus and collective bonus. For 2018 we find a total pct. which reflects the difference between pension wealth before and after transition as individuals were compensated by law, whereas for 2011 and 2012, it is the total of individual and a collective bonus.

Modelling framework: The individual

Consider an individual that is offered a transition at time ζ.

- The individual is of age z at time ζ and has retirement savings W_ζ.
- T denotes the retirement age and τ denotes the (random) age of death.

Modelling framework: The financial market

The financial market is assumed to consist of two assets (risk-free bond and risky stock) with dynamics

$$
\begin{aligned}
\mathrm{d}B_t &= rB_t\,\mathrm{d}t \\
\mathrm{d}S_t &= \mu S_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t
\end{aligned}
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and assume that same investment strategy is employed every period

→ $(I_t)_{t\in\mathbb{N}}$ is then an i.i.d. collection of random variables.

Modelling framework: The financial market

A simple portfolio of the two assets with weight θ on the risk-free bond and $1-\theta$ on the risky stock, we can write the i.i.d. return factors explicitly as

$$I_t = \mathrm{e}^{\theta r^f + (1-\theta)\mu - \sigma^2/2 + \sigma(W_t - W_{t-1})} \sim \text{lognormal}\big(\theta r^f + (1-\theta)/\mu - \sigma^2/2, \sigma^2\big).$$

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1. *Accumulation phase*: begins at contract initiation and lasts until $\min\{T, \tau\}$. During this phase the individual pays a yearly premium y^a to the pension product. The individuals wealth develops according to

$$W_t = I_t W_{t-1} + y^a \qquad \text{for } t \in \{\zeta, \ldots, \min\{\tau, T-1\}\}.$$

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2. *Decumulation phase*: Begins at retirement time T and lasts until τ. During this phase the individual receives pension payment y_t^d in year t. The individuals wealth develops according to

$$W_t = I_t W_{t-1} - y_{t-1}^d \qquad \text{for } t \in \{T+1, \ldots, \tau\}.$$

Modelling framework: Annuity calculation

We calculate the pension annuity payment as the time t constant value that matches the actuarial present value with the current wealth, i.e.

$$y_t^d = \frac{W_t}{\sum_{i\geq 1} {}_i p_{z+t-\zeta} \mathbb{E}[I_1]^i}$$

where

- $\mathbb{E}[I_1] = \mathbb{E}[I_t]$ for all $t \in \mathbb{N}$ are the expected return factors.
- ${}_i p_{z+t-\zeta}$ is the probability that the considered individual of age $z+t-\zeta$ at time t survives further i years.

Both are pension product dependent (guarantee or not).

Guaranteed pension product

— *Financial*

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Two cases can occur

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1. **Sustainable guarantee**: $r^g \leq r^f$.

Guaranteed pension product

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1. **Sustainable guarantee**: $r^g \leq r^f$. Split investment into two accounts managed separately; a part α of all wealth is invested in the risk-free asset with return factor I^f (deterministic), the remaining $1-\alpha$ is invested in a simple portfolio with return factor I_t^s (random log-normal). This strategy has return factor $I_t = \alpha I^f + (1-\alpha) I_t^s$ (random shifted log-normal) $\geq I^g$ if $\alpha = I^g / I^f$.

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1. **Sustainable guarantee**: $r^g \leq r^f$.
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2. **Unsustainable guarantee**: $r^g > r^f$. As this guarantee offers a certain return higher than the risk-free asset, we assume that there is no upside potential. The return factors are therefore equal to the guaranteed I^g.

Guaranteed pension product

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1. **Sustainable guarantee**: $r^g \leq r^f$.
   → Financial guarantee is fully hedged
2. **Unsustainable guarantee**: $r^g > r^f$.
   → Financial guarantee delivers arbitrage and is non-hedgeable.

Guaranteed pension product

Longevity

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$$\mu^{w}(x) = 0.0005 + 10^{0.038x-4.272} \qquad \text{for women}$$

$$\mu^{m}(x) = 0.0005 + 10^{0.038x-4.120} \qquad \text{for men}$$

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For numerical computation, we can use these directly to compute the survival probabilities by ${}_i p_t = \mathrm{e}^{-\int_t^{t+i} \mu(x)\,\mathrm{d}x}$ needed for the present actuarial values. For theoretical analysis, we are looking into approximations of the survival probabilities (Weibull distribution).

Unguaranteed pension product

Financial

If we consider a pension product with no financial guarantee, return factors $(I_t)_{t\geq 0}$ are simply those of a simple portfolio of the two market assets with weights $(\theta, 1-\theta)$ on the risk-free asset and risky stock, respectively. Hence, they are i.i.d. lognormals.

Unguaranteed pension product

Longevity

- If there is no longevity guarantee, the pension fund is allowed to deviate from the tables and use an approved mortality model.
- The pension fund can thus update its survival probability forecasts if they observe deviations, causing an update in the annuity pension payments to the individuals.

Unguaranteed pension product

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Let ${}_ip_t$ be the probability forecast that an individual of age t survives further i years according to some mortality model (e.g. standard Lee-Carter). Let ${}_ip_t^{(k)}$ be the same but based on the sample k years later.

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Define the deviation

$$ {}_{h-t}d_{x,t} = \log\left(\frac{{}_{h-t}p_{x,t}^{(k)}}{{}_{h-t}p_{x,t}}\right) $$

This deviation in survival probability forecasts causes a change in pension annuity payments to

$$ y_t^{d,(k)} = \frac{W_t}{\sum_{i\geq 1} {}_ip_{z+t-\zeta}^{(k)}\mathbb{E}[I_1]^i} = \frac{W_t}{\sum_{i\geq 1} e^{{}_{h-t}d_{x,t}}\, {}_ip_{z+t-\zeta}\mathbb{E}[I_1]^i} $$

Pension payment profiles

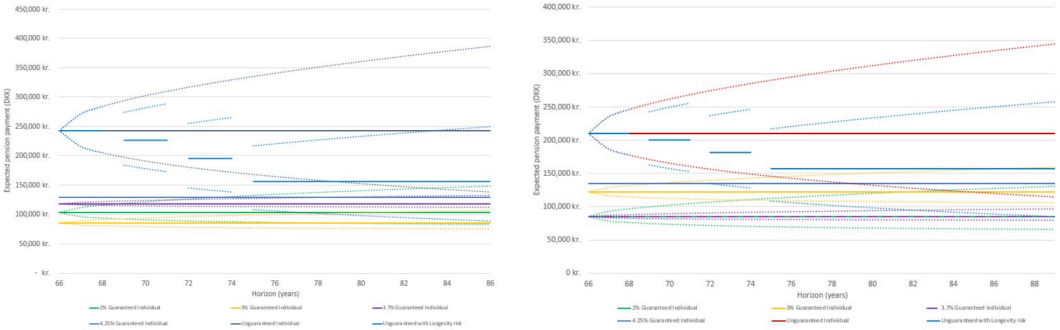

Figure: Pension payment profiles for males (left) and females (right).

Aim

- Propose a decent mortality model/survival probability model that captures longevity and has some degree of theoretical applicability. For example by studying the random deviations ${}_{h-t}d_{x,t}$ for a chosen mortality model.
- Compute the compensation of the embedded guarantees (financial and longevity) by considering a certainty equivalent framework, i.e. what compensation makes individuals indifferent between keeping their guaranteed pension product and transferring to an unguaranteed product.
- Consider an intermediate options (financial guarantee, no longevity guarantee) or (longevity guarantee, no financial guarantee) in order to see how the compensation distributes between the two built-in guarantees. This enables a discussion of add-ons.

Thank your for the attention.