

# **Life-time financial plan optimization with separate aversion against risk of short-term and long-term goals**

Radosław Pietrzyk

Paweł Rokita

Wrocław University of Economics

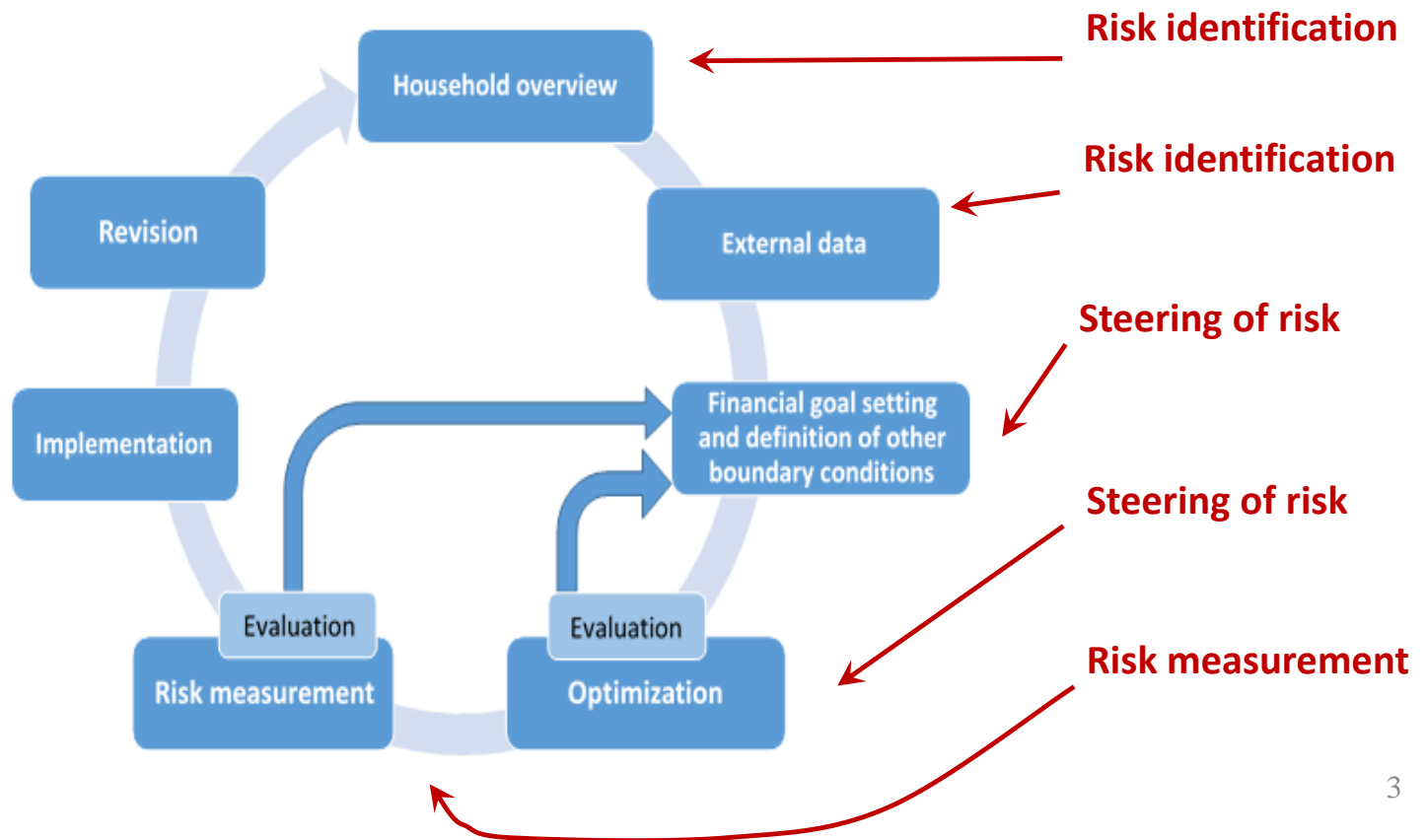
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# Risk management process

1. Definition of risk management objectives
2. Identification of risk types and risk factors
3. Risk measurement
4. Risk steering
5. Control of risk and risk management process

# Elements of RM in our financial plan optimization process (so far)

- A cyclical process
- Some elements of risk management are present on some stages of it



# Financial planning process with a risk model incorporated into the financial plan optimization procedure



# Integrated risk measure

- We considered so far several approaches to integrated risk measurement
  - Integration pertains to:
    - all types of risk
    - all financial goals of the household
    - all sources of financing
    - all periods through the whole life cycle
- This worked good when used to evaluate and compare risk of whole financial plans

# Integrated risk measures

- Long-time (whole life) measures based on final wealth
  - **Residual Wealth at Risk**
  - Residual Wealth Volatility
  - Residual Wealth Aspiration Level
- Long-time (whole life) measures based on threats along the line
  - Measures that identify threats to realization of the whole financial plan
    - **Lifetime Cumulated Net Cash Flow at Risk**
    - Incremental Shortfall
    - Shortfall Scenario Probability
  - Measures of bankruptcy risk
    - **Household Default Probability**

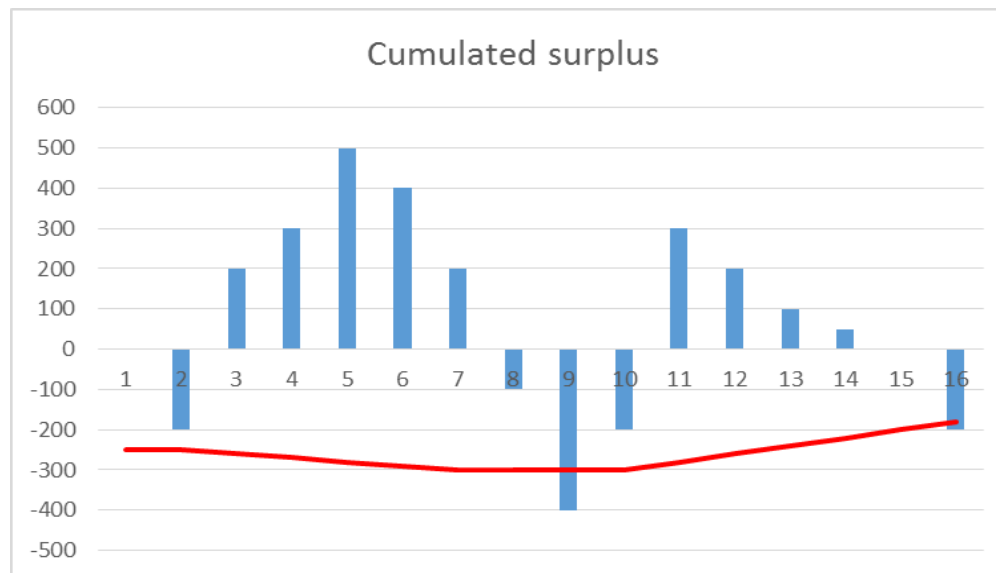
# Household Default Probability

- Firstly, determine a **bankruptcy threshold**
- Here, the term „bankruptcy” does not mean a bankruptcy of the household itself (its insolvency) but rather a **bankruptcy of the financial plan** (its failure)
  - the household may be still solvent, but may find itself in a need of putting some assets up for sale

unplanned, unwanted  
(and maybe also hasty) ⇒ failure of the plan  
emergency selling off

# Household Default Probability

- Bankruptcy threshold
  - Minimum cumulated net cash flow (**maximum cumulated financial shortfall**), beyond which the household is **not able to find financing for covering of the shortfall**





# Household Default Probability

- **The bankruptcy threshold is defined for the starting moment  $t_0$**  (e.g., depends on creditworthiness of the household at this moment); then, the threshold **changes in time as situation of the household changes**
- Scenarios in which cumulated shortfall exceeds the bankruptcy threshold are called **bankruptcy scenarios**; and they are defined as:

$$T^* = Z^* : \exists_{t=1, \dots, T_B^*} CS p_t^{(z^*)} < DTh_t^{(z^*)}$$

# Household Default Probability

- **Probabilities of bankruptcy scenarios**

These are probabilities of such scenarios in which cumulated shortfall (negative cumulated net cash flow) exceeded the defined bankruptcy threshold

$$p_{T_i^*} = \begin{cases} p_{z_i^*} & \text{if } \exists_{t=1, \dots, T_B^*} CS p_t^{(z_i^*)} < DTh_t^{(z_i^*)} \\ 0 & \text{if } \forall_{t=1, \dots, T_B^*} CS p_t^{(z_i^*)} \geq DTh_t^{(z_i^*)} \end{cases}$$

# Household Default Probability

- Probabilities of bankruptcy scenarios are summed
- **Probability that any of the bankruptcy scenario will be realized is treated as the measure of risk:**

$$HDP = \sum_{i=1}^n p_{T_i^*}$$

# Short-term and long-term measures

- There is no indication to assume that preferences of a household in respect of the short-term and long-term risk are identical
- Constructing short-term and long-term risk measures will allow to augment the optimization procedure by new constraints
- Let us distinguish two groups of financial goals:
  - Short-term (up to 1 year)
  - Long-term (more than 1 year)

# Household Default Probability

- **Short term.** Scenarios in which the bankruptcy threshold will be exceeded during the first year are defined as:

$$\mathbf{T}^1 = \mathbf{Z}^* : CS p_1^{(\mathbf{z}^*)} < DTh_1^{(\mathbf{z}^*)}$$

- Probability of a short-term bankruptcy scenario is:

$$p_{\mathbf{T}_i^1} = \begin{cases} p_{\mathbf{z}_i^*} & \text{if } CS p_1^{(\mathbf{z}_i^*)} < DTh_1^{(\mathbf{z}_i^*)} \\ 0 & \text{if } CS p_1^{(\mathbf{z}_i^*)} \geq DTh_1^{(\mathbf{z}_i^*)} \end{cases}$$

# Household Default Probability

- Probabilities of bankruptcy scenarios are summed
- **Probability that a bankruptcy will occur in a short term:**

$$HDP_1 = \sum_{i=1}^n p_{\mathbf{T}_i^1}$$

# Household Default Probability

- **Long term.** Scenarios in which the bankruptcy threshold **will not** be exceeded during the first year **but it will be exceeded during any of the next years** in the future:

$$\mathbf{T}^2 = \mathbf{Z}^* : CS p_1^{(\mathbf{z}^*)} \geq DTh_1^{(\mathbf{z}^*)} \wedge \exists_{t=2, \dots, T_B^*} CS p_t^{(\mathbf{z}_i^*)} < DTh_t^{(\mathbf{z}_i^*)}$$

- Probability of a long-term bankruptcy scenario:

$$p_{\mathbf{T}_i^2} = \begin{cases} p_{\mathbf{z}_i^*} & \text{if } CS p_1^{(\mathbf{z}_i^*)} \geq DTh_1^{(\mathbf{z}_i^*)} \wedge \exists_{t=2, \dots, T_B^*} CS p_t^{(\mathbf{z}_i^*)} < DTh_t^{(\mathbf{z}_i^*)} \\ 0 & \text{otherwise} \end{cases}$$

## How to use the short-term and long-term risk measures in a financial plan optimization procedure?

- It is a natural human tendency to perceive short term risk and to be concerned about it in the first instance, and then, not earlier than that, to consider long-term risks
- We assume that a household will see the risk that threatens accomplishment of its short term goals in the first place, then it will take long-term risk into account, and then – (maybe, if advised so) it will start to analyse integrated risk of the whole-lifecycle financial plan
- Thus, the households should have instruments to (separately) express their preferences in this respects



## Short-term and long-term risk measures in a financial plan optimization procedure

### Approach 1

- (1) Optimization without constraints on risk
- (2) Calculation of  $HDP$ ,  $HDP_1$  and  $HDP_2$
- (3) Decision whether to accept the plan or to start a new iteration of financial goals revision, modification and optimization of the plan
  - The decision (3) is not automated (an intervention of the household and financial advisor is needed),
  - The values of risk measures are just one of the outcomes from the optimization procedure (1) and the risk was not a criterion of this optimization; it may be, in turn, a criterion of the decision (3)

## Short-term and long-term risk measures in a financial plan optimization procedure

### Approach 2 (variant A)

- Weighted probability of bankruptcy is calculated
- $\xi$  - preference of short-term risk protection (over long-term protection)
- The household declares some maximum accepted level of this weighted probability ( $p^*$ ) and a maximum accepted level of bankruptcy probability for the whole plan ( $p^{**}$ ) – one can not be inferred from the other

$$\xi HDP_1 + (1 - \xi) HDP_2 \leq p^*$$

$$HDP \leq p^{**}$$

- These declared levels become boundary conditions of the optimization procedure

## Short-term and long-term risk measures in a financial plan optimization procedure

### Approach 2 (variant B)

- The household separately declares upper limits of the short-term and long-term bankruptcy probabilities

$$\left( HDP_1 \leq p^{***} \right) \wedge \left( HDP_2 \leq p^{****} \right)$$

- The plan is optimized under these two constraints at the same time

## Short-term and long-term risk measures in a financial plan optimization procedure

- Approaches 1 and 2 give different results
- In the **approach 1**:
  - Risk is **not measured** during automated optimization
  - Risk is measured only for the plan that is obtained as an outcome from the optimization
  - The resulting financial plan is then analysed in a manned revision step (mainly in respect of risk), and it is accepted or rejected
  - if the optimization result is rejected (due to high risk of the plan), the bunch of **goals** of the household must be **modified**
- In the **approach 2**:
  - Risk is **measured** during the automated optimization
  - The limits imposed on the risk measures are treated as optimization constraints
  - An optimal solution (if exists) is generated for a **fixed bunch of financial goals**
  - The bunch of financial **goals** is **revised and modified** by the household only in the case **when no optimal solution is feasible**

# Summary

- All measures used here are integrated measures of risk (integrating all types of risk that have been incorporated in the underlying household net cash flow model)
- Short-term and long-term measures allow to treat short-term and long-term risk separately, this, in turn, allows for the situation when short-term risk aversion differs from long-term one
- The measures are based on bankruptcy probability concept, which should be understandable for households; and it is also easy to set constraints on it
- Setting constraints on short-term and long-term risk measures by the household members allows to express their risk aversion towards these two risks
- The constraints on the values of the risk measures are also the way in which the risk is taken into account in the optimization procedure

Thank You