Evaluating Hedge Effectiveness for Longevity Annuities

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Outline

Longevity annuity

- deferred annuity where regular payments do not start until the annuitant attains certain high age, say 80;
- pure survivor benefit, i.e. the annuitant will receive nothing on death;
- cost-efficient protection against the risk of outliving savings;
- also known as longevity insurance, advanced-age deferred annuity, or deep-deferred annuity.

Research Motivations

- Low supply of longevity annuities in the private annuity market
- Intensive longevity risk
  - Cash flows are highly related to high age mortalities
  - High age mortality data are sparse and not reliable
- There is concern with hedging the longevity risk of this annuity

Research objectives

Compare hedge effectiveness of different hedging strategies for longevity annuities.

- Different hedging tools
  - deferred longevity bond
  - key q-forwards
  - s-forwards
- Impact of various risk factors on hedge effectiveness .
  - parameter risk
  - model risk
  - sample risk

Mortality models

- M1: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)} k_t$
- M2: $\ln m_{x,t} = \beta_x^{(1)} + \beta_x^{(2)} k_t + \beta_x^{(3)} \gamma_{t-x}$
- M3: $\ln m_{x,t} = \beta_x^{(1)} + n_a^{-1} k_t + n_a^{-1} \gamma_{t-x}$
- M5: $\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)} (x - \bar{x})$
- M6: $\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + \gamma_{t-x}$
- M7: $\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + k_t^{(3)} ((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}$
- M8: $\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + \gamma_{t-x} (x_c - x)$
- Plat model by Plat (2009): $\ln m_{x,t} = \beta_x^{(1)} + k_t^{(1)} + (x - \bar{x}) k_t^{(2)} + \gamma_{t-x}$

Data

- US males, 1963-2013, age 60-94, from Human mortality database
- BIC of these models

Model	$BIC * (\times 10^4)$
M1	2.9788
M2	1.6037
M3	2.2333
M5	4.7268
M6	1.7848
M7	1.7312
M8	1.7920
Plat	1.6001

Models selected

- Without model and parameter uncertainty
  - Based on BIC values, we select the Plat model and use it to simulate future mortality rates.
  - Extrapolate α_x in Plat model to high ages using polynomial function
- With model and parameter uncertainty
  - M2 model, an alternative to the Plat model, will be selected in some generated scenarios.
  - For M2 model, extrapolate $\beta_x^{(2)}$ using Guassian function, and $\beta_x^{(1)}$ and $\beta_x^{(3)}$ by polynomials.

Hedging objectives

- Risk to be hedged
  longevity risk in longevity annuity: US males, 65-year old at the beginning of 2013, payment starts at 85
- Hedging objective: minimize variance of annuity liability
- Partial risk reduction
  To mitigate longevity risk partially, by reducing the uncertainty in annuity liability.

Hedging instruments

- Annuity population: US males, 65-year old at the beginning of 2013, payment starts at 85
- Strategy 1: a deferred longevity bond
  - A type of bond whose future coupons are based on the percentage of the stated population group who are alive on the coupon payment dates.
  - A 0-year deferred bond has its first payment at the end of the first year.
  - A 20-year deferred bond has the first payment at the end of the 21st yrs (age 86). If its maturity is 50 years, then the payments are at time 21, 22, ... 50, respectively.

Hedging instruments

- Strategy 2: Key q-forwards
  A bucket of q-forwards with the maturity matched to key q-durations
  - q-forward: a zero-coupon fixed-for-floating mortality swap
  - key q-duration: A portfolio's price sensitivity to a shift in a key mortality rate.
- Strategy 3: s-forward(s)
  A single or bucket of s-forwards
  - s-forward: a zero-coupon fixed-for-floating survival swap

Hedge effectiveness assessment

- Static hedge
- Prospective hedge effectiveness assessment
- Risk metric
  variability in the present value of all unexpected cash flows with and without hedging strategy
- Basis for comparison: reduction in variation

$$R = 1 - \frac{\text{Var}(L(n) + h^* H(n))}{\text{Var}(L(n))},$$

where h^* is the optimal hedge ratio, $L(n)$ and $H(n)$ are the annuity liability and hedge value at the end of deferred period.

Hedge effectiveness assessment

- Hedge ratio calibration: h^*
  - $h^* = -\dfrac{\mathrm{Cov}(L(n), H(n)))}{\mathrm{Var}(H(n))}$, for deferred longevity bond hedge and s-forward hedge
  - key-q duration method for q-forward hedge.
- Simulation method:
  - assume that death count follows Poisson distribution
  - a portfolio of 1000 annuities sold
  - use the model with lowest BIC when incorporating model risk
- Valuation model: same as the simulation model
- Other assumption: interest rate 2%

A deferred longevity bond: maturity

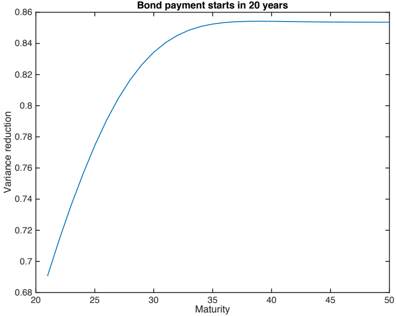


Figure: 20-year deferred bond with various maturities

- The greatest reduce is 0.8463, achieved by a 20-year deferred 20-payments longevity bond;
- The line is quite flat when maturity is higher than 35 years.

A deferred longevity bond: deferral period

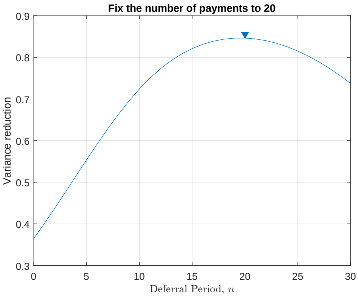
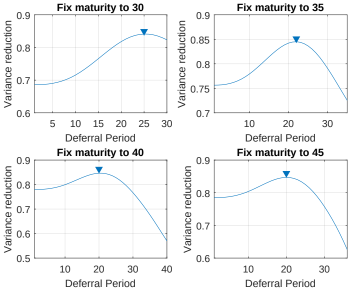


Figure: *n*-year deferred bond with 20 payments

- Variance reduction is highest when the bond has similar length of deferral period with annuity.

Deferral period v.s. Maturity

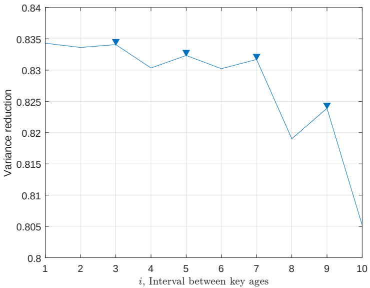


- With a fixed maturity, a longer deferral period for the bond means less number of payments.
- When we increase maturity, the optimal deferral period gradually moves from 24 towards 19.
- Highest variance reduction in the four cases are very close

q-forwards

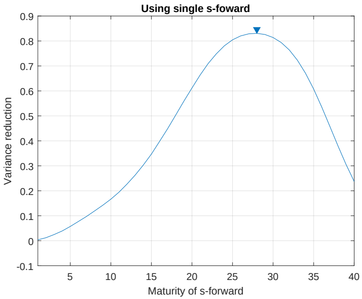
- Underlying cohort is 65 yr old male
- Use Key-q duration method to compute the amount of q-forward required.
- The maximum key age used here is 95.
  Denote the interval between key ages by i. The key ages used are $65, 65+i, 65+2i, \ldots, 65+ni$, where n is maximum integer such that $65+ni \leq 95$.
- In practice, using $i=5$ or $i=10$ is fairly reasonable.

q-forwards



- Hedging effectiveness using $i = 5$(having 7 q-forwards), or 6 (having 6 q-forwards) is very close to $i = 1$.
- Even using $i = 9$, where q-forward reference ages are 65, 74, 83, and 92, hedging effectiveness is only reduced by 0.015.
- When allowing higher maximum key ages, higher variance reduction can be achieved.

s-forward: only one



- Maturity 28 gives the highest variance reduction, 0.8390.
- When using two s-forwards, the highest variance reduction is 0.8485. Therefore, single s-forward is sufficient.

Comparison

- Deferred longevity bond, key q-forwards and s-forward can achieve similar hedging effectiveness.
- Variance reductions using the three different hedging strategies are very close.
- Effective hedge can be achieved by simple hedging structures, for example, a 24-year deferred 6-payment longevity bond, a single s-forward, a bucket of 6 q-forwards in our case study
- It appears that s-forward is more favorable, because s-forward has simpler structure.

Incorporating model and parameter uncertainty

Following Brouhns et al. (2005):

1. Bootstrap n samples, $d_{x,t}^{(1)}, d_{x,t}^{(2)}, \ldots, d_{x,t}^{(j)}, \ldots, d_{x,t}^{(n)}$, from Poisson distribution with mean equal to $d_{x,t}$, the observed number of deaths for age x in year t.
2. For the jth sample, we fit each mortality model to the pseudo data , select the best fitted mortality model by the BIC, and estimate the parameters.

Mortality forecasts with and without uncertainty

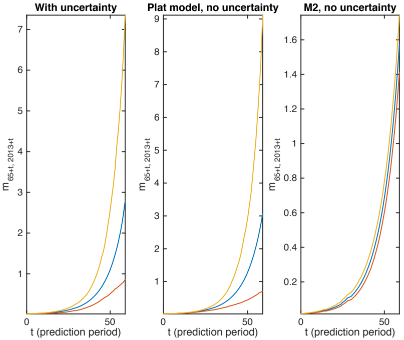
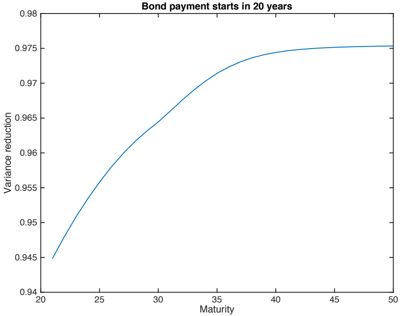


Figure: Mortality forecasts, mean, and 95% prediction interval

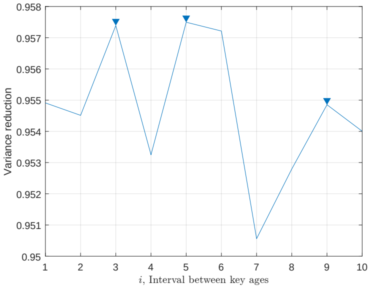
- 403 out of the total 1000 simulated pseudo data select the M2 model, and others choose the Plat model.

A deferred longevity bond



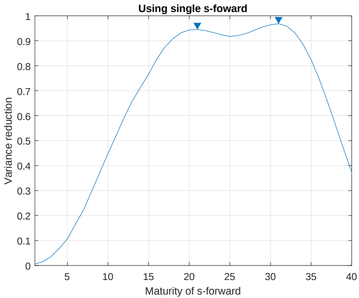
- Variance reduction converges to 0.976, higher than the case without model and parameter uncertainty.
- Variance increased in annuity liability after incorporating model and parameter risk, while a deferred longevity bond can achieve desirable variance reduction.

The q-forward



- Variance reduction is also higher than the case without model and parameter uncertainty.
- A bucket of q-forwards with a simple structure can achieve significant variance reduction.

The s-forward



- Higher variance reduction than the case without model and parameter uncertainty.
- A s-forwards with 21-year maturity can desirably reduce variance in longevity annuity liability

Small sample/Poisson risk

Portfolio size	1000	2000	3000	4000	5000
Variance reduction	0.8487	0.9268	0.9447	0.9598	0.9689

- The larger the sample size, the higher the hedge effectiveness.

Conclusions

- Deferred longevity bond, q-forward and s-forward all provide effective hedge for longevity annuities, when the objective is to minimize the variance of present value of future liabilities
- The market might over react to the risk in longevity annuities.
- More to consider: basis risk, recalibration risk, etc.

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