Pricing Pension Buy-ins and Buy-outs[1]

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Pension De-risking

- Around 40% of senior financial executives in mid-sized and large companies with at least $250 million defined benefit (DB) plan assets indicated that they will give "serious consideration" to pension risk transfer in 2014 and 2015 (Walts 2013)
- Driven by growing pension deficits
  - latest market downturn and low interest rate environment: plans of FTSE 100 companies were only 91% funded in 2013
  - new pension accounting standards: IAS19 revision leads to £2 billion lower in the total 2012 profits of FTSE 100 companies
  - prolonged life expectancy of retirees

Key Pension De-risking Tools

- Pensioner **buy-in**: purchase of a bulk annuity policy with an insurance company as an investment to match part (or full) of a pension plan's liabilities, typically pensions in payment.
- Full **buy-out**: each individual pensioner is issued a policy so that their pension is provided directly by an insurance company; the obligation for the pension plan ceases.
- **Longevity swap**: allows a pension plan to transfer the risk of members living longer than expected to a third party (the counterparty), whilst retaining direct control of the assets.

Buy-in v.s. Buy-out

- Both strategies remove the following Risks:
  - interest rate risk
  - inflation risk
  - asset risk
  - longevity risk
- Buy-out: pension liabilities are **completely removed** from the pension firm's balance sheet.
- **Credit risk** of Buy-in: pension liabilities are still on the pension firm's balance sheet; the pension firm is subject to the default risk of the insurer.

Literature Review

- Pension and life insurance embedded options: Blake (1998), Grosen and Jogensen (2000, 2002), Marshall et al. (2010), Gerber et al. (2013)
- Longevity and mortality risk securitization: Lin and Cox (2005), Cox and Lin (2007), Milidonis et al. (2011), Cox et al. (2010, 2013), Lin et al. (2014)
- Insurance insolvency risk: Cummins (1988), Phillips et al. (1998), Gerber and Shiu (1998), Grosen and Jogensen (2002)

Objective and Methodology

- **Objective**:
  - to provide a feasible model to price pension buy-ins and buy-outs.
  - to explain the price differences between buy-ins and buy-outs.

- **Methodology**: analyze separately the investment risk premium, the longevity risk premium and the credit risk premium.
  - Investment risk: analogous to put options sold by the insurer to the pension firm
  - Longevity risk: calibrate the market price of longevity risk by Wang transform
  - Credit risk: can be viewed as a series of one-year insolvency put options.

DB Pension Liabilities

- Present value of a firm's future benefit obligations

$$PL_t = N(t) \cdot Pa_{x_0+t}, \quad t = 1, 2, \cdots$$

  - $N(t)$: members of a retired cohort survived at time t (with age x_0 at time 0)
  - P: promised annual payment to each pensioner who survives at the end of each year
  - a_{x_0+t}: immediate life annuity factor for age $x = x_0 + t$
- Immediate life annuity factor a_x

$$a_x = a_{x_0+t} = \sum_{s=1}^{\infty} v^s \, {}_s\hat{p}_{x,t}$$

  - $v = 1/(1 + r_p)$: discount factor using the pension valuation rate r_p.
  - ${}_s\hat{p}_{x,t}$: conditional expected s-year survival rate for age x at time t given the past mortality tables.

Lee and Carter (1992)'s Mortality Model

- One-year death rate $q_{x,t}$ for age x $(x = 0, 1, 2, \cdots)$ in year t $(t = 1, 2, \cdots, K)$

$$\ln q_{x,t} = c_x + b_x \gamma_t + \epsilon_{x,t},$$

$$\gamma_t = \gamma_{t-1} + g + e_t, \qquad e_t \sim N(0, \sigma_\gamma)$$

  - c_x and b_x: age-specific parameters
  - g: drift rate
  - $\epsilon_{x,t}$ and e_t: normal errors with mean zero
- U.K. male population mortality tables from 1950 to 2003 are used
- Year 2003 is our base year $t = 0$

Financial Market Model

- Pension fund assets: S&P 500 index $A_{1,t}$, Merrill Lynch corporate bond index $A_{2,t}$ and 3-month T-bill $A_{3,t}$.
- Processes of $A_{i,t}, i = 1, 2, 3$, as a geometric Brownian motion:

$$A_{i,t+\Delta}|\mathcal{F}_t = A_{i,t} \exp\left[(\alpha_i - \frac{1}{2}\sigma_i^2)\Delta + \sigma_i \Delta W_{it}\right]$$

  - $\mathcal{F}_t$: the information set up to time t
  - α_i and σ_i: drift and instantaneous volatility of asset i
  - W_{it}: standard Brownian motion with mean 0 and variance t
- Assets i and j are correlated with

$$\mathrm{Cov}(W_{it}, W_{jt}) = \rho_{ij}\sigma_i\sigma_j\ t, \qquad i = 1, 2, 3;\ j = 1, 2, 3;\ i \neq j$$

- Annual data from 1975 to 2003 are used to estimate parameters

Valuation of Investment Risk

- Pension liabilities are evaluated annually as long as there are survivors in the retired cohort
- Dynamics of pension assets at $t = 1, 2, \cdots$

$$PA_{t^+} = \max\{PA_t - N(t) \cdot P, PL_t\}$$

  - PA_t: value of pension assets at time t
  - PA_{t^+}: value of the pension assets after annuity payments and supplementary contributions (if there are any)
  - $PA_0 = PL_0$: initial pension asset value
- Equivalent to a series of one-year put options on the pension plan

Pension Assets between Valuation Dates

- Process of pension assets between annuity payment dates

$$d\log PA_t = \left(\sum_{i=1}^{3}\pi_i(t)\left(\alpha_i - \frac{1}{2}\sigma_i^2\right) + \gamma_\pi^*(t)\right)dt + \sum_{i=1}^{3}\pi_i(t)\sigma_i dW_{it}$$

  - $\pi(t) = (\pi_1(t), \pi_2(t), \pi_3(t))'$: weights of the pension portfolio at time t
  - $\gamma_\pi^*(t) = \frac{1}{2}\left[\sum_{i=1}^{3}\pi_i(t)\sigma_i^2 - \sum_{i,j=1}^{3}\pi_i(t)\pi_j(t)\rho_{ij}\sigma_i\sigma_j\right]$: instantaneous excess growth rate of the pension assets at time t (e.g. Fernholz 2002)
- $\pi(t)$ are further assumed to be constant throughout the buy-out contract

Investment Risk Premium

- Risk-neutral price of the funding guarantee option of the buy-outs, given $N(t)$, is

$$PV_{invest}(N(\cdot)) = \sum_{t=1}^{\tau_N} v_t \cdot E^{\mathbb{Q}}\left[(PL_t + N(t) \cdot P - PA_t, 0)^+\right] - v_{\tau_N+1} \cdot E^{\mathbb{Q}}\left[PA_{\tau_N+1}\right]$$

  - $\tau_N = \min\{\lfloor t \rfloor : N(t) = 0\}$: number of integer years that the last pensioner of the retired cohort can survive
  - v_t: discount factor based on the risk-free rate r_t for each year

- Ultimate investment risk premium of buy-outs

$$P_{invest} = E\left[PV_{invest}(N(\cdot))\right] / PL_0$$

Longevity Risk Premium

- Two-factor Wang transform (Wang, 2002)

$$F^*({}_tq_x) = {}_tq_x^* = Q[\Phi^{-1}({}_tq_x) - \lambda]$$

  - Φ and Q: standard normal and t-distribution
  - $\lambda > 0$: market price of longevity risk; calibrated from observed prices of pure longevity securities

- Bulk annuity price based on the transformed survival probabilities

$$a_x^* = \sum_{t=1}^{T} v_t \cdot {}_tp_x^* = \sum_{t=1}^{T} v_t \cdot \left(1 - Q[\Phi^{-1}({}_tq_x) - \lambda]\right)$$

- Longevity risk premium of buy-outs

$$P_{longevity} = \frac{a_x^*}{a_x} - 1$$

Total Risk Premium of Buy-out

- Assume the independence of investment risk and longevity risk
- Total risk premium of buy-out

$$P_{total,buyout} = P_{invest} + P_{longevity}$$

- Other costs and fees can be added accordingly

Insolvency Risk

- Credit Risk of buy-in: default of buy-in insurer
- Total asset and liability process (e.g., Cummins (1988))

$$dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t}, \quad dL_t = \mu_L L_t dt + \sigma_L L_t dW_{L,t}$$

  - μ_A (μ_L): instantaneous growth rate of total assets (liabilities)
  - σ_A (σ_L): instantaneous total asset (total liability) volatility
  - $W_{A,t}$ ($W_{L,t}$): standard BM with $dW_{A,t} \cdot dW_{L,t} = \rho_{AL} dt$
- CAPM model to price total asset and liability accounts

$$\mu_A = r + \theta_A, \qquad \mu_L = r_L + \theta_L$$

  - r_L: inflation rate of total liabilities
  - σ_A (σ_L): instantaneous total asset (total liability) volatility
  - θ_A and θ_L: market risk premia for holding insurance assets and liabilities

Asset-Liability Ratio Process

- Asset-liability ratio ξ_t under risk-neutral measure $\mathbb{Q}$

$$\xi_t \equiv \frac{A_t}{L_t} = \xi_0 \exp\left\{\left[\left(r - \frac{\sigma_A^2}{2}\right) - \left(r_L - \frac{\sigma_L^2}{2}\right)\right] t + (\sigma_A W_{A,t} - \sigma_L W_{L,t})\right\}$$

where $\xi_0 = 1/\alpha$ is the initial asset-liability ratio.

- Observed default time τ: the first valuation date that ξ_t less than 100% is observed.
- Default value at τ
  - no authority benefit protections (some states in U.S., e.g. New Jersey)

$$e^{-r\tau} PL_\tau (1 - \xi_\tau, 0)^+$$

  - with authority benefit protections (e.g., recovery rate $\varphi = 0.9$ in U.K.)

$$e^{-r\tau} PL_\tau \left[(1 - \xi_\tau, 0)^+ - (\varphi - \xi_\tau, 0)^+\right]$$

Decomposition of the Insolvency Put

- Decomposition of the buy-in insolvency put
  - viewed as a series of one-year put options
  - Each one-year option only covers the default event observed in that year's audit
  - Put options in later years will only be triggered when no default event occurs in prior years
- Easily adapted to other periodic audit cases (e.g., quarterly)

Pricing Formulas

Proposition: Prices of one-year insolvency put options without authority benefit protection

(i) Buy-in insolvency put of the first year, $Put_{credit,1}$,

$$E^{\mathbb{Q}}\left[e^{-r} PL_1 \left(1-\xi_1, 0\right)^+\right] = n(0)\, \hat{p}_{x_0,0} \cdot Pa_{x_0+1} \cdot e^{-r+\left(\mu+\frac{\sigma^2}{2}\right)} \cdot \text{Put}\left(\xi_0, 1, 1, \mu+\frac{\sigma^2}{2}, \sigma\right),$$

Put (S_0, K, T, r, σ): B-S price of a put option

(ii) Buy-in insolvency put of year t $(t = 2, 3, \cdots)$

$$\begin{aligned} Put_{credit,t} &\equiv E^{\mathbb{Q}}\left[e^{-rt} PL_t \left(1-\xi_t, 0\right)^+ \cdot 1(m^Y_{t-1} \geq \log\alpha)\right] \\ &= n(0) \cdot {}_t\hat{p}_{x_0,0} \cdot Pa_{x_0+t} \cdot e^{-rt+\left(\mu+\frac{\sigma^2}{2}\right)} \cdot \text{Put}^*, \end{aligned}$$

$$\text{Put}^* = \int_0^\infty \text{Put}\left(\xi_0 e^y, 1, 1, \mu+\frac{\sigma^2}{2}, \sigma\right) \Pr\left(m^Y_{t-1} \geq \log\alpha,\ Y_{t-1} \in dy\right).$$

Credit Risk Premium of Buy-in

- Buy-in credit risk premium without authority benefit protection

$$P_{credit} = Put_{credit}/PL_0 = \sum_{t=1}^{\infty} Put_{credit,t}/PL_0$$

- Buy-in credit risk premium with authority benefit protection

$$P_{credit}(\varphi) = \left(Put_{credit} - Put_{credit}\big|_{K=\varphi}\right)/PL_0$$

- Total risk premium of buy-in

$$P_{total,buyin} = P_{total,buyout} - P^*_{credit} = P_{invest} + P_{longevity} - P^*_{credit}$$

Risk Parameters and Assumptions

- MetLife Assurance Limited (MAL)
  - U.K. based, a subsidiary of MetLife Inc., established in 2007
  - Total assets/liabilites a/o 2012: $3.31 /$2.93 billion
  - Pension Assets: 4.22% stocks, 92.94% bonds, and 2.84% cash or its equivalence
  - perform the analysis as if it were in operation in 2004 with $\xi_0 = 1.10$
- Pension valuation rate: $r_p = 5.12\%$
- Risk-free interest rates: term structure of U.K. gilt curve in November 18, 2004
- Liability inflation rate: $r_L = 1.3\%$
- Market price of longevity risk: $\lambda_{EIB} = 0.1140$, based on the European Investment Bank (EIB) bond issued in November 2004

Prices for Hypothetical Buy-out and Buy-in Contracts

- Hypothetical Buy-out and Buy-in Contracts
  - At time 0, all plan participants reach the retirement age $x_0 = 65$
  - The pension cohort has the same mortality experience as the U.K. male population
  - At time 0, 10,000 pensioners with annual survival benefit £60,000 per pensioner
- Simulations
  - 5,000 scenarios of the mortality rates
  - For each mortality scenario, 1,000 scenarios were generated to simulate the value of pension assets
- Numerical Outcome
  - Investment risk premium: 4.11%
  - Longevity risk premium: 3.32%
  - Credit risk premium: 0.17%

Credit Risk Premiums by PIC, MAL and PICA

Table 1: Credit Risk Premiums $P_{credit}(\varphi)$ of Buy-in Bulk Annuities Issued by PIC, MAL and PICA

A/L Ratio	PIC ($\sigma = 0.0962$)		MAL ($\sigma = 0.0668$)		PICA ($\sigma = 0.0091$)	
(ξ_0)	$r_L = 0.013$	$r_L = 0.030$	$r_L = 0.013$	$r_L = 0.030$	$r_L = 0.013$	$r_L = 0.030$
1.05	1.21%	1.64%	0.41%	0.74%	6.84×10^{-21}	2.45×10^{-11}
1.10	0.77%	1.16%	0.17%	0.40%	4.24×10^{-36}	2.87×10^{-18}
1.20	0.36%	0.67%	0.03%	0.14%	–	–
1.30	0.18%	0.41%	0.01%	0.05%	–	–
1.40	0.09%	0.26%	1.9×10^{-5}	0.02%	–	–

- PIC: Pension Insurance Corporation, U.K. based
- PICA: Prudential Insurance Company of America, New Jersey based

Credit Risk Premiums by Changing φ

Table 2: Credit Risk Premiums $P_{credit}(\varphi)$ of Buy-in Bulk Annuities at Different Recovery Rates φ

	MAL ($\sigma = 0.0668$)		PIC ($\sigma = 0.0962$)	
Recovery Rate (φ)	$\xi_0 = 1.05$	$\xi_0 = 1.10$	$\xi_0 = 1.05$	$\xi_0 = 1.10$
0.95	0.36%	0.15%	0.89%	0.59%
0.90	0.41%	0.17%	1.21%	0.77%
0.85	0.42%	0.17%	1.30%	0.82%
0.80	0.42%	0.17%	1.32%	0.83%
0.75	0.42%	0.17%	1.32%	0.83%

Risk Premiums by Changing r_p

Table 3: Risk Premiums of MAL's Buy-out/Buy-in Bulk Annuities at Different Pension Valuation Rates r_p

Valuation Rate (r_p)	P_{invest}	$P_{longevity}$	$P_{credit}(\varphi)$	$P_{total,buyout}$	$P_{total,buyin}$
5.00%	3.09%	4.44%	0.17%	7.53%	7.36%
5.12%	4.11%	3.32%	0.17%	7.43%	7.26%
5.20%	4.78%	2.34%	0.17%	7.12%	6.95%
5.30%	5.64%	1.37%	0.17%	7.01%	6.84%

Conclusion

- We provided a pricing framework to quantify the risks embedded in the pension buy-in and buy-out transactions.
- The key price difference of buy-in and buy-out may be explained by the involved credit risk of buy-in insurer
- Risk management implications for buy-in insurers:
  - Collection of risk capital
  - Importance of asset liability management

Thank You!