# Mortality Table Graduation

# and Unisex Mortality Tables

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# Background

Continuous Mortality Investigation (CMI) in the UK:

- Life Office Mortality Committee (LOMC)
- Self-Employed Pension Schemes Committee (SAPS)
- Critical Illness Committee (CI)
- Income Protection Committee (IP)

Chairman of LOMC 2002 – 2011, produced “00” Series of graduations:

- Data from 1999 – 2002
- Twelve tables for assurance contracts
- Twenty-eight tables for pension/annuity contracts

# Overview

- Data and data quality
- Methods available
- Consistency and ‘reasonableness’
- Modelling and co-graduation
- Unisex mortality tables

# Occurrence/Exposure (O/R) Rates

All CMI graduations have estimated mortality rates at individual ages $x$ using some form of occurrence/exposure (O/R) rate:

$$\frac{\text{No. of deaths at age } x}{\text{Person-years exposed to risk at age } x}.$$

For many years the quantity estimated has been the force of mortality denoted $\mu_x$, not the annual rate of mortality denoted $q_x$.

Mortality tables of $q_x$ are obtained numerically as a last step:

$$q_x = 1 - \exp\left( - \int_0^1 \mu_{x+t} \, dt \right).$$

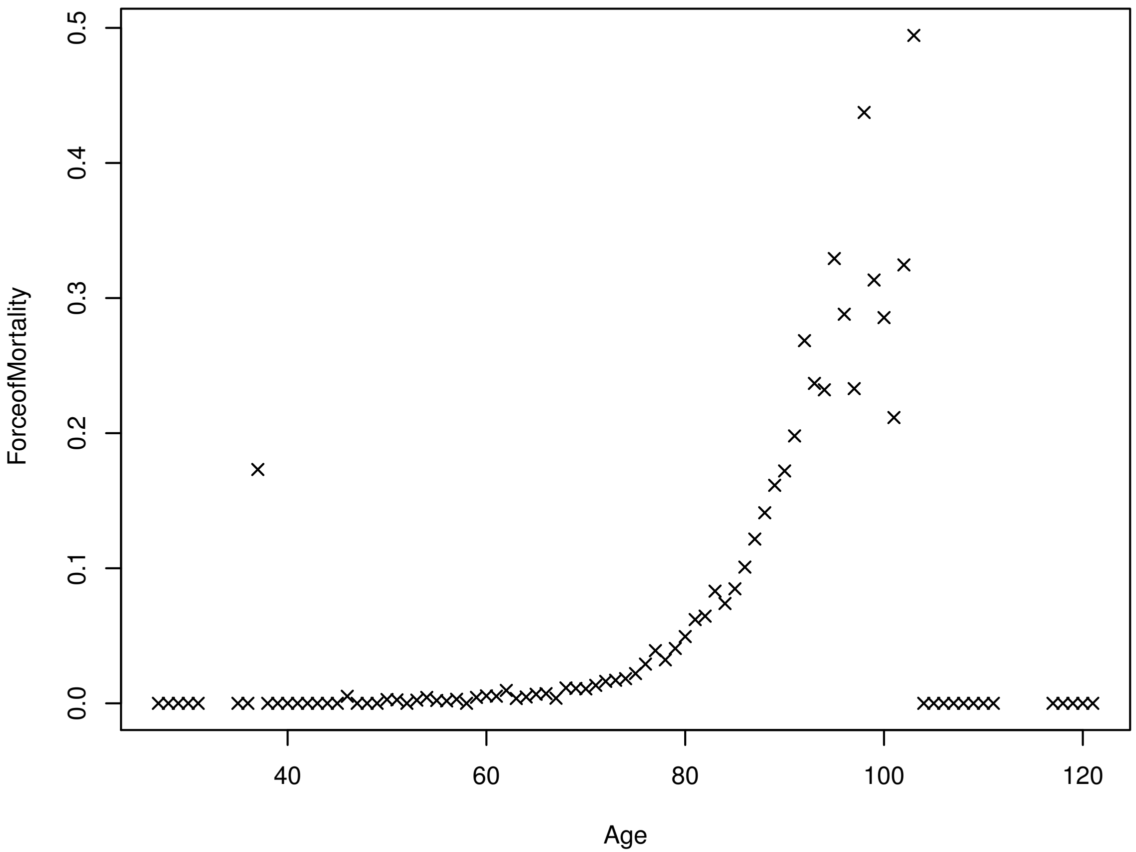
# Data

Basic data requirement are the two figures in the O/R rates.

These are aggregate quantities summed over all persons age $x$ in the experience.

Traditionally, life offices supplied the CMI with these totals, easily extracted from ordinary business systems, called scheduled data.

Recently, the CMI has requested individual policy data, meaning a separate record for each policy (proxy for person).

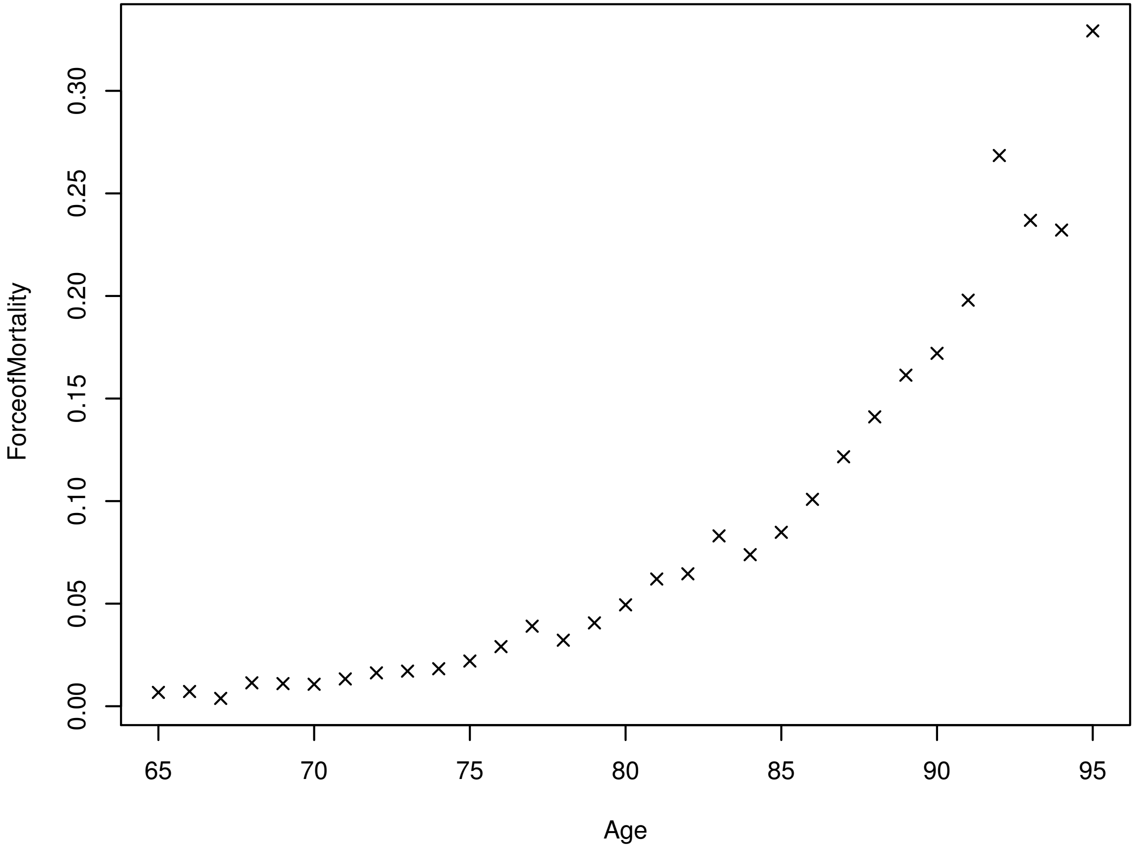


0.5
0.4
0.3
0.2
0.1
0.0
ForceofMortality
40
60
80
100
120
Age

# Selection of Age Range

- Data below age 65 are unreliable, zero deaths at many ages.
- This is pensioner data, so younger ages are unimportant and should not influence the graduation unduly.
- Data above age 95 are unreliable — small exposures and reporting of deaths is questionable.
- High ages are relevant for annuities but not influential.

Result: Limit the age range graduated, in this case 65–95.



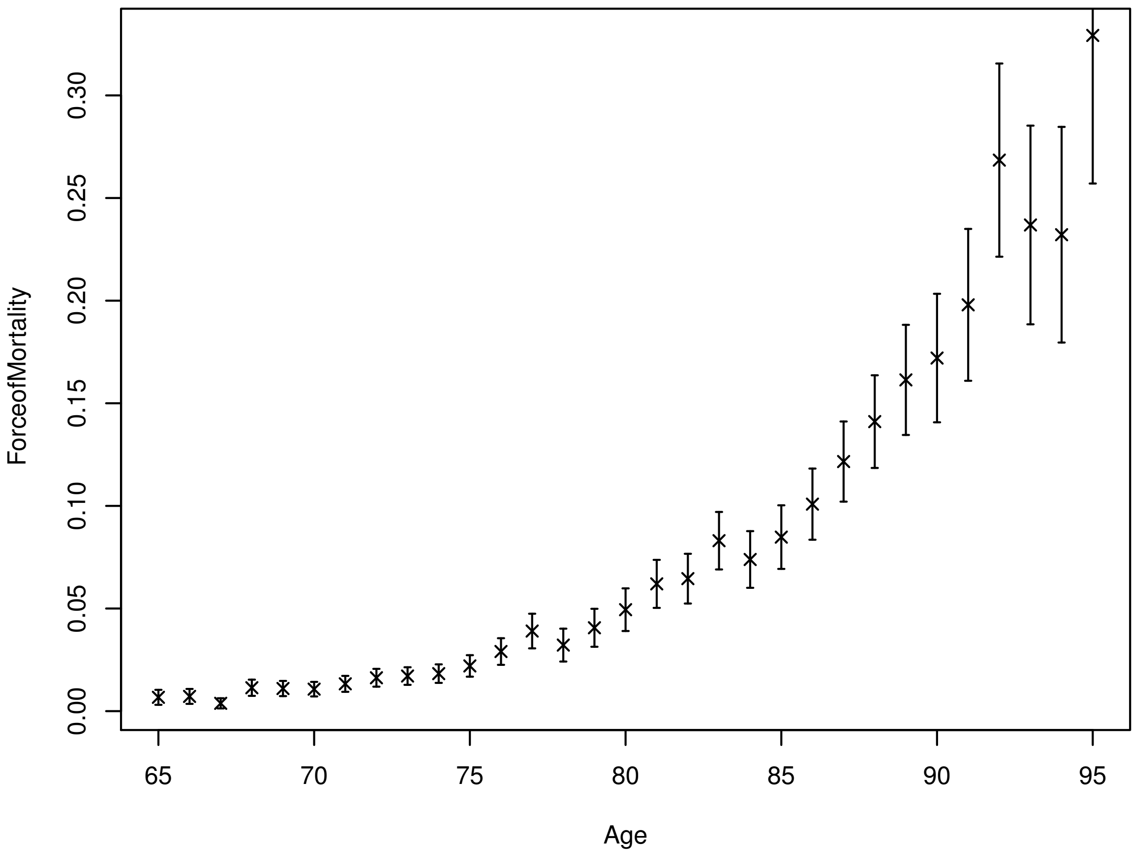
ForceofMortality
0.00
0.05
0.10
0.15
0.20
0.25
0.30
65
70
75
80
85
90
95
Age

# The Poisson Model

- Poisson distribution often used to represent the number of events occurring at rate $\mu$ over a given time $E$

$$\text{No. events} \sim \text{Poisson}(\mu E)$$

- Time $E$ can be person-years exposed if we assume homogeneity
- Poisson model provides:
  - the continuous-time limiting case of the Binomial model
  - a good approximation to the theoretical likelihood
- MLE is $\hat{\mu}_{x+1/2}$ = Deaths/Exposure, the O/E rate, under homogeneity


ForceofMortality
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65
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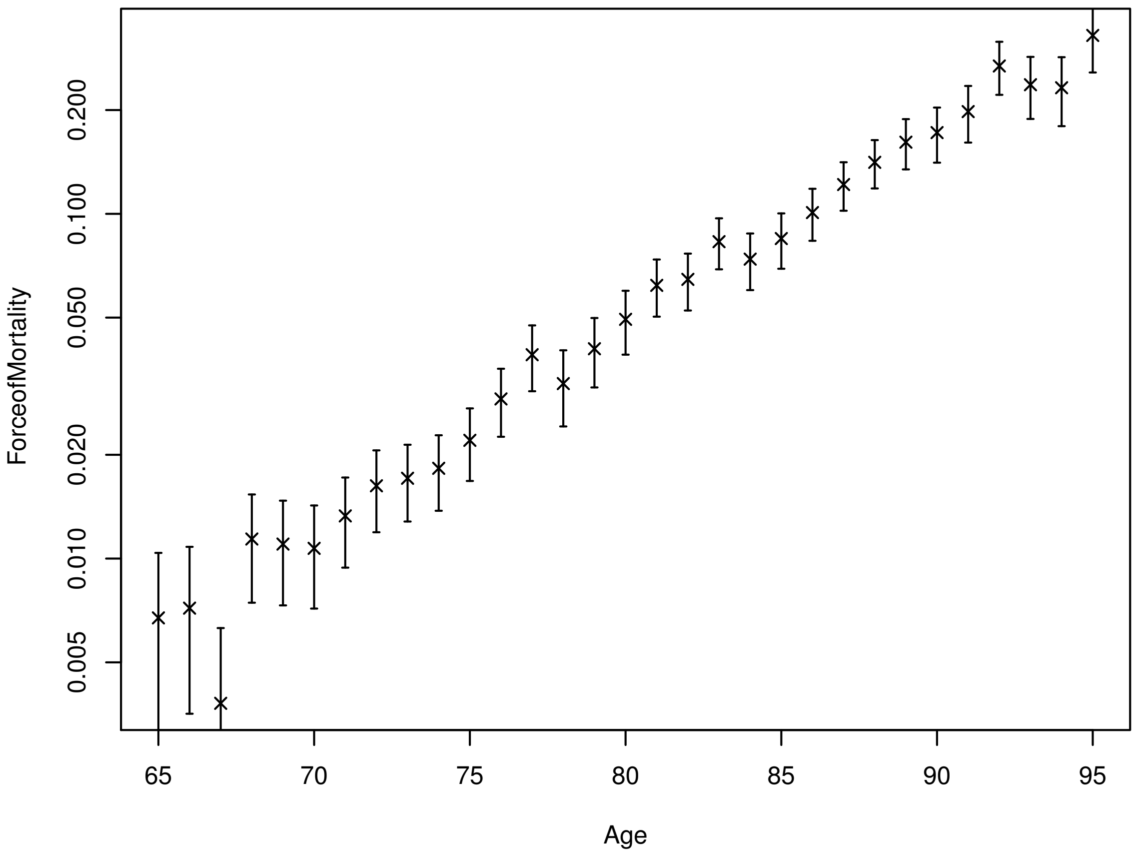
95
Age


ForceofMortality
0.005
0.010
0.020
0.050
0.100
0.200
65
70
75
80
85
90
95
Age

# Methods

## Parametric methods

Fit formulae using standard statistical procedures.

The Poisson model allows us to write down the likelihood of all the observations so we can use the maximum likelihood method of fitting.

- Available in all statistics packages (R, SAS, Stata).
- Optimal among all methods of fitting, in a theoretical sense.
- Sampling distribution (approximately Normal) easily obtained as well as parameter estimates.
- Criteria of good fit *versus* smoothness (i.e. number of parameters) available to aid model selection.

# Typical Parametric Functions

Gompertz (1825) noticed that mortality rates increase exponentially at adult ages:

$$\mu_x = \beta\, e^{\gamma x}$$

Makeham (1859) added a constant to represent accidents as well as ageing:

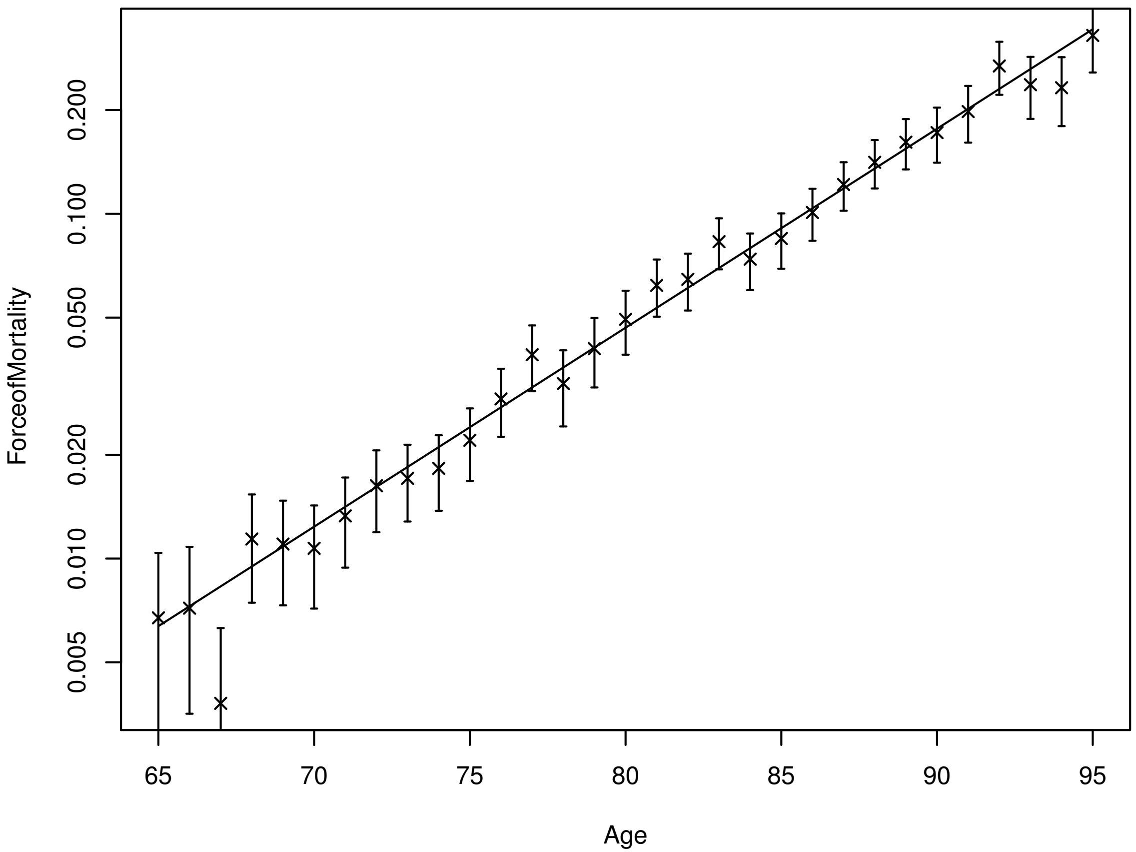
$$\mu_x = \alpha + \beta\, e^{\gamma x}$$

Perks (1932) noted that mortality decelerates at the highest ages:

$$\mu_x = \frac{\beta\, e^{\gamma x}}{1 + \beta\, e^{\gamma x}}$$

Forfar, McCutcheon & Wilkie (1988) generalised to the Gompertz-Makeham family:

$$\mu_x = \text{polynomial}_1 + \exp\big(\text{polynomial}_2\big)$$


ForceofMortality
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0.010
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Age

# Choice of Function

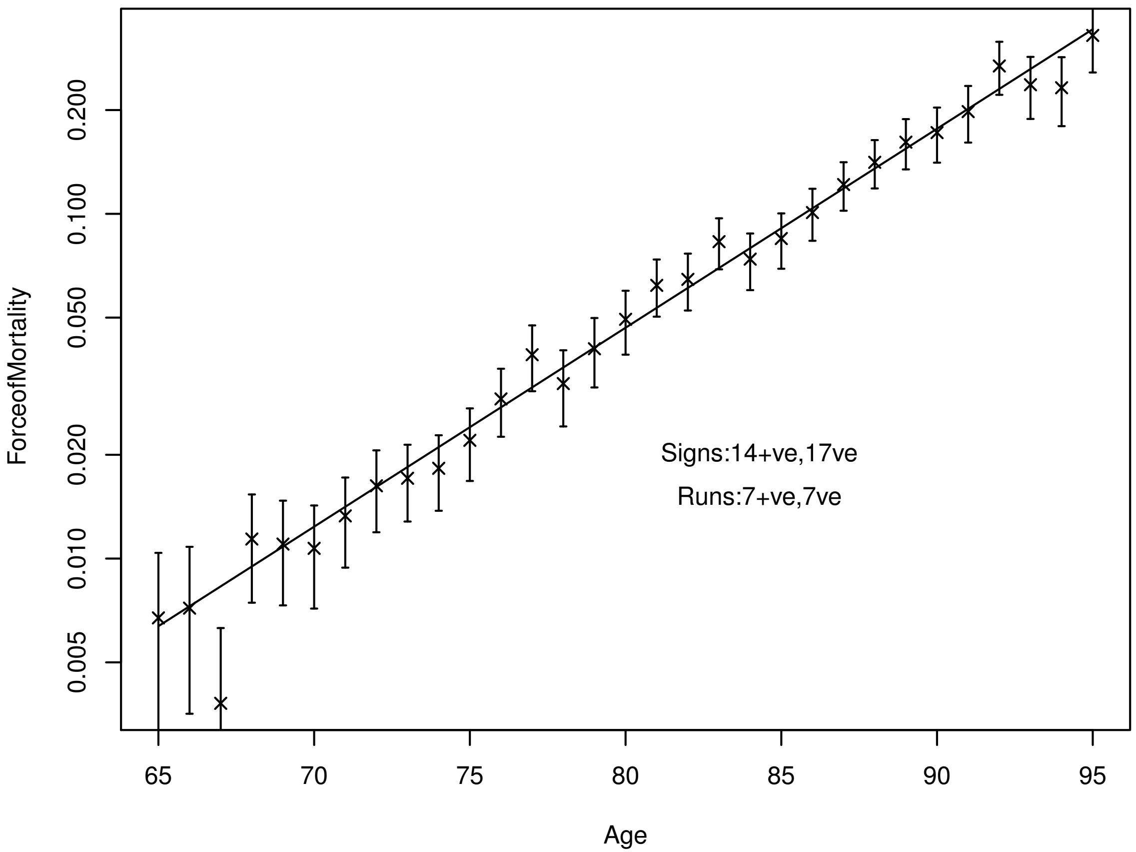
Graduation is a trade-off between smoothness and goodness of fit.

Smoothness is usually guaranteed given a Gompertz-Makeham function with a small number of parameters.

Goodness of fit is conventionally measured by statistical tests applied to the deviations of the crude rates from the graduated rates:

- Signs test: roughly equal numbers of positive and negative deviations.
- Runs test: roughly equal numbers of positive and negativeve groups.

Overall fit measured by chi-squared test but this tends to show poor fits for large data sets.



ForceofMortality
0.200
0.100
0.050
0.020
0.010
0.005
Signs:14+ve,17ve
Runs:7+ve,7ve
65
70
75
80
85
90
95
Age

# Choice of Function

Information criteria may help in selecting a model.

- Akaike Information Criterion (AIC)
- Bayesian Information Criterion (BIC)

All have the general form:

$$\text{log-likelihood} - \text{penalty function.}$$

But for large data sets any information criterion may suggest adding more and more parameters and judgement is needed to decide when to stop.

Alternatively, choose a non-parametric method which smooths the data without fitting a function:

- Kernel smoothing
- Spline smoothing (used for English Life Tables).

# Low and High Ages

Having fitted at ages where data are reliable, how to extend the graduation to lower or higher ages?

High ages: The function chosen may behave reasonably up to ages 110–120. Often the case with a small number of parameters.

Or, the choice of function can combine smoothness, goodness-of-fit and behaviour beyond the fitted ages (used by CMI for the “92” Series tables).

Or an assumption may be based on research into centenarians, such as $\mu_{120} = 1$, and the rates converged smoothly on that limit (used by CMI for the “00” Series tables).

Low ages: Comparisons with other experiences may be used, e.g. National or Assured Lives, possibly with some assumption about ordering.

# Consistency and Reasonableness

We may have to graduate several experiences, e.g. male and female, smoker and non-smoker, temporary and permanent assurances.

Experience/intuition/practicality often suggest that we import/impose two kinds of structure on multiple graduations.

- Smoothness for example with respect to age or duration. This leads to graduation in one or more dimensions.
- Ordering for example $\mu_x^{male} > \mu_x^{female}$ or $\mu_x^{smoker} > \mu_x^{non-smoker}$ or $q_{[x]} < q_{[x-1]+1}$.

Ordering of different tables may influence methods of extending tables to higher and lower ages.

We have two approaches:

- Stratification
- Model-building

# Stratification

Stratification means fitting entirely separate models to different parts of the experience, maintaining homogeneity with respect to some feature. Examples from CMI practice are:

- Males and females
- Smokers and non-smokers
- Each life office investigation
- SAPS experience by pension amount (four bands)

Advantages include goodness-of-fit if separate experiences are large enough.

Disadvantages include experiences crossing over so that expected orderings are reversed, and sub-optimal use of the data. Too many variables fragments the data into small samples.

# Model-Building

Model-building means fitting a single model jointly instead of stratifying. It is not necessary to pursue homogeneity with respect to key features.

- Instead of fitting $\mu_x^{male}$ and $\mu_x^{female}$ separately, fit $\mu_{x,s}$ where $s$ is sex. Sex appears as a discrete covariate.
- Instead of stratifying SAPS pension size, fit $\mu_{x,s}$ where $s$ is pension size. Pension size appears as a continuous covariate.

Advantages include making optimal use of all the data, preserving expected orderings, handling small data sets and using likelihood tools to assess the importance of covariates.

Disadvantages include poor fits with very large data sets and assumption or choice of model structure.

# Cox-Type Models

Cox-type models split the hazard rate into the product:

$$\mu(y, z) = \mu_0(y)\, e^{\beta_1 z_1 + \beta_2 z_2 + \cdots + \beta_n z_n}$$

where $\mu_0(y)$ is the baseline hazard, a function of age $y$, and the exponent of the second term is an ordinary linear regression model.

In survival analysis, the regression parameters ($\beta$s) can be estimated by partial likelihood without fitting the baseline hazard $\mu_0(y)$ at all. This is very useful in testing for differences between (e.g.) treatments. Actuaries usually will want to fit the baseline hazard too.

The baseline hazard can be anything, e.g. Gompertz-Makeham.

# Cox-Type Models

Stratification: fit $\mu_y^{male}$ and $\mu_y^{female}$ separately.

Cox-type model: fit $\mu_{y,s} = \mu_0(y)e^{\beta_1 s}$ where $s$ is 0 (male) or 1 (female).

Advantages: Allows basic shape of $\mu_0(y)$ to be very flexible, allows importance of covariates to be assessed/ranked, all required statistical properties of the fit are available.

Disadvantages: Hazards are proportional:

$$\frac{\mu_y^{male}}{\mu_y^{female}} = \text{constant}$$

so model only suitable if experiences have similar basic shapes.

# Current Issues

- Generality: models specified in terms of hazards (transition intensities) generalize to many problems of interest to the CMI, especially CI and IP.
- Choice between stratification and model-building is influenced by many practical considerations.
  - The form of data (per-policy data will allow more choice).
  - The amount of data: very large experiences will usually not fit a pre-specified model well.
- Stratification is limited to a small number of variables which may conflict with homogeneity.
- Model-building removes need for homogeneity at the expense of imposed/assumed structure.

# Issues

- Standard statistical software is usually well-adapted to mainstream use of models in medical statistics.
- Significant computing capability (R, C++) may be needed to apply ‘standard’ models to actuarial data and problems.
- No single approach is correct, different approaches are available for use when appropriate.
  - Sometimes stratification will work best, e.g. with extremely large data sets.
  - Sometimes model-building will work best, e.g. smaller or heterogeneous data sets with more than a very few covariates.

# Unisex Life Tables

Test-Echats case: Since 2012 all insurance prices in the EU must be unisex.

Insurers may still know a policyholder's gender and reserving need not use unisex mortality tables.

The CMI continues to collect data separately for men and women and will continue to graduate separate mortality tables. It is anticipated that offices will use these as a basis for reserving.

The CMI has not so far considered producing any unisex mortality tables.

The only question then for the life insurance company is how to combine male and female tables for pricing?

# Research

Swiss Re (2012) using German assurance data found that the proportion of males in portfolios varied by every risk factor considered:

Company, age at entry, age attained, sum insured, distribution channel, duration, postcode.

Unisex premium rates had to be tailored to every company and line of business.

# Gender Among Other Risk factors

Underwriting annuities for factors other than gender is becoming more common. Some are of equal significance.

| Factor | Change | Annuity Rate | Change (%) |
|---|---|---|---|
| Base Case | | 13.39 | |
| Gender | Female $\rightarrow$ Male | 12.14 | $-9.3$% |
| Lifestyle | Top $\rightarrow$ Bottom | 10.94 | $-9.9$% |
| Duration | Short $\rightarrow$ Long | 9.88 | $-9.7$% |
| Pension Size | High $\rightarrow$ Low | 9.36 | $-5.2$% |
| Region | South $\rightarrow$ North | 8.90 | $-4.9$% |
| | | | $-33.6$% |

Source: Richards & Jones (2004) based on 2002 UK data.

# What Actually Happened?

November 2012: Prudential UK launched unisex annuities.

- Female rates improved by 4.5% (age 6) to 6.5% (age 65).
- Male rates worsened by 2.5% (age 60) to 1.5% (age 65)

Meanwhile, annuity rates were falling for a fifth consecutive year because of falling interest rates and improving longevity generally; they fell by about 8% in 2012.