

# **Annuity Choices and Longevity Bonds: A Developing Country Application**

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## **Abstract**

We use a variable that comes from a dynamic programming model developed by Mitchell et al. (1999) and Brown (2001), called Annuity Equivalent Wealth, in order to capture the benefits of having access to the annuity market. We introduce longevity bonds as assets for insurance companies, and we expect an increase in the annuitization probability.

**Keywords:** Annuities, Pensions, Welfare.

**JEL:** G22, G23, I38, H55, J32

## **1. Introduction**

In 1981, Chile converted its old Pay-as-You-Go system into a new system based on individual accounts in which private companies, called Pension Fund Administrators (AFPs), are in charge of collecting and investing the mandatory contributions from affiliates during their working lives. At the time of retirement, a person can choose either a pension on a phased withdrawal in which the retiree not only keeps the control over the funds but also retains the longevity risk and bears the capital market risk associated with the rate of return of the investments or as an annuity that will give a stream of income for the length of purchaser's life but transfers the property of the funds to an insurance company. The annuity protects the retiree from longevity risk, but the decision to annuitize is irreversible. The normal retirement age is 60 for females and 65 for males. Thus, the experience of more than three decades of the Chilean private pension system provides a unique experience to study the determinants of annuitization. Whereas

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annuities are not frequently used as pension payment in the world, Chile has one of the highest rates of annuitization (Rocha and Thorburn, 2006).

The longevity risk of individuals has been underestimated, as survival probabilities have improved across the developed world (Ngai and Sherris, 2011).

This study provides evidence from Chile, an emerging economy that has developed successful public policies in the last decades (Claessens et al., 2010). We use dynamic approach to capture the benefits of having access to the annuity market. The way to compute these benefits are summarized by the Annuity Equivalent Wealth (AEW) that a person requires to be compensated in the case that the annuity markets were absent.

This paper is organized in six sections, including this introduction. Section 2 discusses the literature. Section 3 describes the dataset. Section 4 describes the methodology to be employed. Section 5 reports the empirical results and section 6 concludes.

## **2. Literature Review**

Many studies have emphasized the effect of adverse selection on annuities (Abel (1986), Finkelstein and Poterba (2004), McCarthy and Mitchell (2010)). Abel (1986) examines the implications for individual consumption and bequest motives. Using an overlapping generation model in which individuals are utility maximizers but are uncertain regarding the date of death, he shows that an actuarially fair social security will increase the rate of return on purchased annuities reducing the effects of adverse selection. Finkelstein and Poterba (2004) examine annuity policies in the U.K. and show that, in both the compulsory and the voluntary markets, there is strong evidence that longer-lived individuals buy more back-loaded annuities. McCarthy and Mitchell (2010) provide international evidence of adverse selection in annuities.<sup>2</sup> Other studies have emphasized the Money's Worth Ratios (MWR), calculation of the expected present discounted value of payouts for annuities in relation to the premium cost of the annuity as a way to understand the value of purchasing an annuity (Mitchell et al 1999, Mitchell and Ruiz, 2011). Brown (2003) acknowledges that annuities are valuable longevity insurance for individuals with uncertain lifetimes. In his model, however, mandating annuities for

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<sup>2</sup> The countries studied are the United States, the United Kingdom, and Japan.

all individuals would not be optimal because of the existence of different mortality distributions across groups and purchase costs.

Brown and Poterba (2000) find that the utility gain from annuitization is bigger for single individuals than for couples. As more potential buyers are couples, this fact could help to explain the low level of annuitization in the country given the effects of adverse selection in the case they decide to annuitize. They also find no evidence of bequest motives as an important variable.

Using dynamic programming techniques, Brown (2001) use a measure called the Annuity Equivalent Wealth first used by Mitchell et al. (1999) to report the utility value of gaining access to an actuarially fair annuity market. This variation in utility arises from differences in marital status, risk aversion, mortality risk, planning horizon and health states. He finds no empirical support of bequest motives. Buttler and Teppa (2007) study the choice between an annuity and a lump sum using the AEW measure for Swiss pension funds. They use administrative records and find that the sponsor influences the annuity decision. They also find that low accumulation of retirement assets is strongly associated with the choice of a lump sum. This paper is close related with the idea of our study. However, their sample lack of non-pension wealth and has limited individual information.

Mackenzie (2006) recognizes that the optimal portfolio should include some part in annuities and some precautionary savings. Dus, Maurer and Mitchell (2005) examine different alternatives for the retirement asset decumulation process using a risk-value approach. Here, return is the expected level of benefits and risk pertains to uncertainty of reaching the desired level of consumption. They mention that phased withdrawal plans have the advantages of flexibility and bequest motives, but they also require retirees to formulate asset allocation and withdrawal rules which can be complex and expose the retiree to longevity and capital market risk. Horneff et al. (2010) point out that utility maximizers will gradually annuitize the portfolio, even if retirees do not have a strong bequest motives. Then, there is an annuity puzzle given by the low rate of annuitization in some countries and the economic advantage of taking this decision.

A pioneer proposal for Mortality-linked securities was introduced by Blake and Burrows (2001). They suggest a survivor bond for hedging the longevity risk, and this

should be issued by the government based on population mortality. The later was also reported by Brown and Orszag (2006). Longevity bonds are paid according to the survival experience of a cohort (Dowd et al., 2006; Blake et al., 2006).

### **3. Data**

We examine the empirical evidence in Chile by using the Encuesta de Protección Social (Social Protection Survey; EPS; Department of Economics at the University of Chile and the University of Pennsylvania, 2006), which has 16,443 respondents. This survey is unique in the Latin American economy because it includes socioeconomic, demographic, and other information provided by the interviewees. A panel of international experts participated in the design of the survey to ensure representative information at a national level. This survey consists of respondents who are 18 years and older, representing a total population of about 12.5 million people, of whom 50.9% are women and 49.1% men. Explanatory variables for pension benefits are related with retiree age, gender, Marital status, Pension Balance. Non Retirement Wealth as higher asset people have outside the pension system, lower probability to annuitize we can expect. The reason is they are able to diversify better the assets and then try to keep the capital market risk and leave a bequest in case of die sooner.

Another important variable is risk aversion. People who are risk-averse prefer to smooth consumption streams, so we expect they will annuitize their balances. Also, as the longevity risk is not covered by PW, the possibility of running out funds will increase the value for them of being an annuitant. This variable was also included in the Brown (2001) study. Using a question related to preferences for safety in the job, we create a dummy called risk averse, with value 2 if people choose two conservative options, 1 if choose one conservative option, and 0 otherwise. Our hypothesis is risk-averse people prefer to smooth consumption, so they will value higher an annuity. Thus, the expected sign for changes in probability of annuitizing with respect to risk aversion is positive.

### **4. Methodology**

The methodology to be employed models choice behavior as influenced by the difference in expected utility between purchasing an annuity and taking PW. Our objective is to derive a measure of the annuity Equivalent Wealth (AEW) to incorporate in an empirical model of the annuity choice.

#### 4.1 The Model

Assuming that individuals are expected utility maximizers, it is optimal for an individual to choose an annuity instead of a PW payment at retirement if  $E(U(\text{Annuity})) \geq E(U(\text{PW}))$ .

**Expected Utility under Annuity.** If the individual chooses to buy an annuity, he signs a contract in which he pays an initial up-front sum to the insurance company, in exchange for a lifelong income annuity stream. Then, the expected discount utility is:

$$EU(\text{Annuity}) = \sum_{t=1}^{T-\text{age}+1} \frac{p_t U(c_t)}{(1+\rho)^t} + \frac{(1-p_t)p_t^w \alpha d U(0.6C_t)}{(1+\rho)^t} \quad (2)$$

where  $T$  is the maximum possible life-span of an individual,  $\text{age}$  is the retiree's age at time  $t$ ,  $U(C_t)$  represents the utility level defined over the consumption in period  $t$ ,  $p_t$  is the probability of remaining alive at period  $t$ ,  $\rho$  the utility discount rate,  $d$  is a dummy that takes the value of one if the person is male, and  $\alpha$  represents the importance the individual places on leaving a pension for his widow (Annuities and PW give them a benefit equivalent to 60% of their husbands' pension). We are assuming that there are no children who can receive the survivorship benefit. We can see directly from the specification that a female does not leave a survivorship pension for her husband ( $d = 0$ ).

Consumption streams are determined when the individual decides to purchase an annuity. We will assume that the only annuity available is a real annuity (the pension is adjusted to the consumer's price index). Then the income stream the person will receive by purchasing the annuity is:

$$C_t = \begin{cases} C_t & \text{if } C_t \geq \text{MPG}_t \\ \text{MPG}_t & \text{if } C_t < \text{MPG}_t \end{cases} \quad (3)$$

**Expected Utility under PW.** Under this payout option, individuals can decide how much of the pension accrual they will consume each period, and the amount that they will save in order to increase future consumption. We assume that individuals reinvest in the same AFP portfolio in which they have their balances as of the retirement date. We will solve by using dynamic programming techniques for the optimal path of consumption. Let  $U(C_t)$  represents the utility level defined over consumption in period  $t$ ,  $\rho$  is the utility discount rate, and  $d$  is a dummy that takes the value of one when the individual is male. Then, the problem is to maximize the discounted utility until the age of death considering survivorship benefits. The maximization is the following:

$$\max_{C_t} \left[ \sum_{t=1}^{T-age+1} \frac{p_t U(c_t)}{(1+\rho)^t} + \frac{(1-p_t)p_t^w \alpha [dU(0.6C_t) + (1-d)U(W_t)]}{(1+\rho)^t} \right] \quad (4)$$

Subject to the constraints:

$$W_t \text{ is given at } t = 0 \quad (5)$$

$$W_t \geq 0 \text{ for all } t \quad (6)$$

$$W_{t+1} = (1+r)(W_t + P_t - C_t) \quad (7)$$

$$P_t = \frac{B_t}{12 \cdot CNU_{i,t,r_p}} \quad (8)$$

In these constraints,  $W_t$  is the AFPs balance the individual has available to draw down in period  $t$ ,  $C_t$  is the consumption level at  $t$ ,  $S_t$  is the amount of the PW benefit saved in order to increase future consumption,  $P_t$  is the benefit level at  $t$  and  $CNU_{i,t}$  is the capital necessary unitary to finance a unit of pension for the individual  $i$  at time  $t$ . We can define the equation (4) as a value function. Then, this value function will satisfy the following recursive Bellman equation:

$$\max_{C_t} V_t(W_t) = \max_{C_t} U(C_t) + \frac{P_{t+1}}{1+\rho} V_{t+1}(W_{t+1}) \quad (9)$$

The advantage of the Bellman equation compared to solve the full maximization presented in equation (4) is the fact that the full maximization problem is reduced to a series of two-period problems. The latter can be solved numerically by solving backwards from the last period. The maximization is subject to the constraints in equations (5) to (8).

The strategy followed by Mitchell et al (1999) and Brown (2001) was to resolve the problem in the case in which annuities are not available and there is no bequest motive. Then,  $\alpha$  is constrained to be zero. We can find the amount in wealth that an individual lacking access to the annuity market would pay for having that access. Thus, we can find  $\Delta W$  as:

$$V(W^* + \Delta W / \alpha = 0, \text{ for all } t) = V^* \quad (10)$$

The specific mechanism for solving these two dynamic programming is presented in appendix.

Following Mitchell et al (1999) and Brown (2001) we can define the “Annuity Equivalent Wealth” (AEW) as:

$$AEW = \frac{W^* + \Delta W}{W^*} \quad (11)$$

$V^*$  is the utility level that the individual reaches when he is constrained to take the PW. Thus, AEW represents the maximum mark-up over the actuarially fair premium of an annuity that the individual would be willing to pay, giving the utility level that could be attained without access to an annuity market.

#### 4.2 Model Calibration (Pending)

## 5. Results (Pending)

## 6. Conclusions (Pending)

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## Appendix

Solving the dynamic problem

In this section we add the utility maximization process we follow in order to find the annuity equivalent wealth (AEW) for each individual. The process shows the procedure for both pension payouts.

For the Annuity:

$$V_A(W_t, B, p_t) = \underset{c_t}{Max} \left\{ U(C_t) + \frac{1}{1 + \rho} \left\{ p_{t+1} V_A(W_{t+1}, B, p_{t+1}) + (1 - p_{t+1}) p_{t+1}^w d_w \tilde{V}_A(\tilde{W}_{t+1}) + \right. \right. \\ \left. \left. + (1 - p_{t+1})(1 - d)b \tilde{V}_A(W_{t+1}) \right\} \right\}$$

Subject to the constraints:

$$W_{t+1} = (1 + r)(W_t + A_t - C_t)$$

$$W_{t+1} \geq 0$$

$$\tilde{W}_t = W_t + PV(0.6A_t)$$

$$A_t = \frac{B_{t0}(1 - x)}{12 \cdot CNU_{i,t,r_A}} + xB_{t0}L_{i,t}$$

Where W is the cash in the “bank”, A is the annuity payment, C is the consumption, PV is the present value and L is the longevity bond payment.

For the Phased withdrawal:

$$V_P(W_t, B, p_t) = \underset{c_t}{Max} \left\{ U(C_t) + \frac{1}{1 + \rho} \left\{ p_{t+1} V_P(W_{t+1}, B, p_{t+1}) + (1 - p_{t+1}) p_{t+1}^w d_w \tilde{V}_P(\tilde{W}_{t+1}) + \right. \right\} \\ \left. \left. + (1 - p_{t+1})(1 - d)b \tilde{V}_P(H_{t+1}) \right\} \right\}$$

Subject to the constraints:

$$W_{t+1} = (1 + r)(W_t + P_t - C_t)$$

$$W_{t+1} \geq 0$$

$$\tilde{W}_t = W_t + PV(0.6P_t)$$

$$P_t = \frac{B_t}{12 \cdot CNU_{i,t,r_p}}$$

$$H_t = W_t + B_t$$

Where W is the cash in the “bank”, A is the annuity payment, C is the consumption, PV is the present value and P is the pension amount under phased withdrawal rule.