Market-Based Solutions for the Management of Longevity Risk

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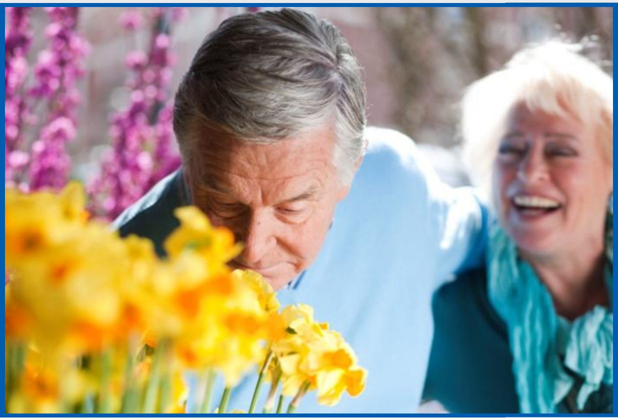

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LIFE INSURANCE | PENSIONS | ASSET MANAGEMENT

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- Pricing of Longevity Risk
- Lee Carter Model
- Synthetic Portfolio Hedge

Pricing of Longevity Risk

- The fundamental principle of insurance pricing for diversified and independent insurance risks is based on the idea that as the number of insurance contracts that are written becomes large then the premium loading that must be applied to all contracts becomes small.

- What is true in this classical case is manifestly false in many important insurance applications. For example, the very highly correlated risks inherent in variable annuity product guarantees have been the undoing of many well-established life insurers.

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Pricing of Longevity Risk

- In the case of longevity, the correlated nature of these risks is again a cause for concern.
- For an insurer engaged in accepting pension plan risk, a central issue is whether the risks for a book of these transactions has a manageable risk profile.
- When the insurer accepts the pension plan risk there is a quantifiable amount of economic capital that is needed to cover the possible contingencies embedded in the plan.

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Pricing of Longevity Risk

- By this we mean, that depending on realised longevity trends and the random fluctuations about these trends for the lives in the pension plan; that the insurer may wind up paying close to the expected cash flows or may pay a large and extreme amount close to the 99th percentile of the cash flow distribution.

- The insurer will hold sufficient capital to meet such an unfavourable outcome. Hence the term “economic capital”.

- This capital must remain in place so that the transaction if fully collateralised.

Pricing of Longevity Risk

- Of course, the insurer cannot charge anywhere near such an extreme amount to accept the pension plan risk. Indeed, why would the party that was previously liable for the pension plan payments agree to pay close to the worst possible outcome to get rid of this uncertain liability?

- Consequently, the insurer will have to charge an amount for accepting the pension plan risk that exposes the insurer to a significant probability of incurring a loss on the individual transaction.

Pricing of Longevity Risk

- This is not a problem for the insurer providing that the insurer has sufficient capital to replicate a large number of these blocks and providing that there is material diversification of risk that is achieved by replicating a large number of these blocks.

- In the classical case of independent risks, the insurer need only charge a small amount above the expected loss on a large book of these individual transactions in order to enjoy a very high probability of making a profit. Furthermore, in this classical case, when a loss is made it comes from a very light tail.

Pricing of Longevity Risk

- In the longevity market, there is little or no diversification in writing more business.
- Consequently, the insurer needs to be able to raise large amounts of contingent capital in order to service the market for accepting pension plan risk.
- Such an insurer will take prudent measures to balance their global book and to take advantage of natural hedging.
- The development of a viable capital markets solution is a major step forward.
- This is why the AEGON – Deutsche Bank deal is so important.

Pricing of Longevity Risk

- Let us illustrate our discussion with a simple example.

- We assume that there is a closed pension plan consisting of 1000 newly retired US males each at age 65.

- The pension plan pays until the pensioner reaches age 85, at which time all payments cease. [20-yr annuity immediate.]

- We base our pricing example on the one-factor Lee-Carter model and on a modified version of that model.

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Lee-Carter Model

- The Lee-Carter model remains the benchmark model for mortality modelling.

$$\mu(x,t) = \exp\left(a_x + b_x k_t\right)$$

- The parameters a and b are age dependent parameters and k is a time dependent process the reflects the temporal variation in mortality.

- Estimated over an age range and a time range [t_0,t_N].

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Lee-Carter Model

- In order to use the model for the risk-management simulation of mortality, one must adopt a stochastic process for k that permits simulation going forward.

- The determination of such a stochastic process is an involved issue and one that is not fully settled in the literature, particularly in the case of the multi-factor version of the model.

- However, for the moment, let us suppose that such a stochastic process is adopted.

- We then simulate mortality as:

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Lee-Carter Model

$$\mu(x, t_N + s) = \exp\left(a_x + b_x k_{t_N+s}\right) \equiv \mu(x, t_N) \exp\left(b_x \tilde{k}_s\right)$$

$$\tilde{k}_s \stackrel{\text{def}}{=} k_{t_N+s} - k_{t_N}$$

- Traditionally, the series k is treated as a random walk beyond the last data point.

$$\tilde{k}_s = \mu s + \sigma W_s$$

$$\mu(x, t_N + s) = \mu(x, t_N) \exp\left(b_x \mu s + b_x \sigma W_s\right)$$

Lee-Carter Model

- The model used by AEGON to price longevity risk is of a multi-factor log-linear structure and employs several careful adjustments to volatility and base curves.
- A simple idea that produces useful results is to alter a factor model by introducing a simple but flexible drift dynamic.
- The drift of the random walk process is allowed to change value.
- The drift is permitted to take on values from 0% to over 100% of its historical values, representing periods of no mortality improvement or accelerated mortality improvement.

Lee-Carter Model

- Each permissible value of mortality improvement is tagged with a duration range.
- Migration dynamics must be specified to direct where mortality improvements evolve to when they leave a given state.
- A Markov chain could be used.
- For our pricing, we used durations corresponding to draws from a simple collection of uniform random variables.

$$\mu(x, t_N + s) = \mu(x, t_N) \exp\left(b_x \int_0^s \mu_u \, du + b_x \sigma W_s \right)$$

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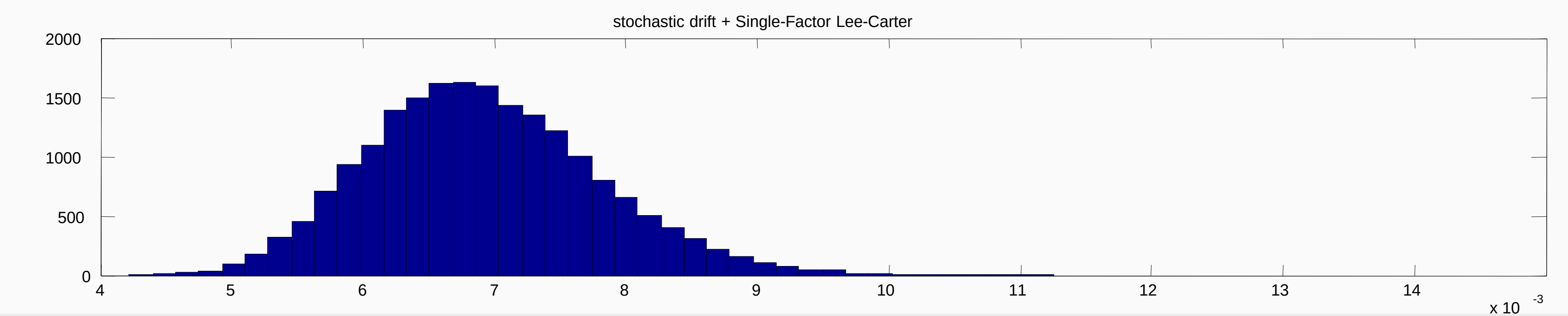
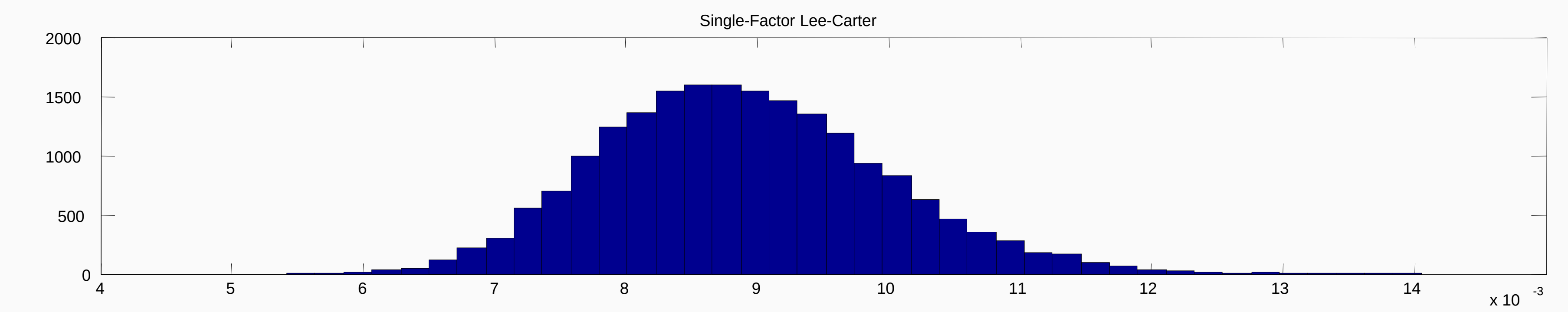
Lee-Carter Model

- The effect of the stochastic drift is shown in the following. Improvements are 100%, 130% and 160% of historical.





Pricing of Longevity Risk in a Lee-Carter framework

- This has the desirable feature of generating systemic pathwise changes in mortality dynamics that have a trend different from that embedded in the base Lee-Carter model.

- We priced the pension plan risk assuming a cost of capital of 5% and a risk-free interest rate of 3%.

- The statistics for the pricing exercise are shown in the following table. The table focuses on the relationship between the present value of benefit payments and the premium for the block.

Pricing of Longevity Risk in a Lee-Carter framework

	Stochastic Drift Model	Lee-Carter	Lee-Carter (trend only)
Prob Net CF (CoC Premium) < 0	16.24%	15.87%	16.03%
Economic Capital	7,787,624	7,872,423	6,647,807
Expected Payment per Policy	235,351	234,374	234,384
CoC Premium per Policy	238,466	237,523	237,043
Indifference Premium for Block	237,662,219	236,653,433	236,053,000
Indifference Premium per Policy	237,662	236,653	236,053
CoC Block Premium	238,465,748	237,523,360	237,043,208
Block zeta_0.995	243,138,323	242,246,814	241,031,892
Min	220,551,357	219,849,566	223,540,628
Max	248,889,941	246,830,241	245,681,935
Average	235,350,699	234,374,390	234,384,085
StDev	3,146,445	3,132,375	2,674,556

- The CDF's for the PV of Benefit Payments for the first and second columns are shown in the following chart.

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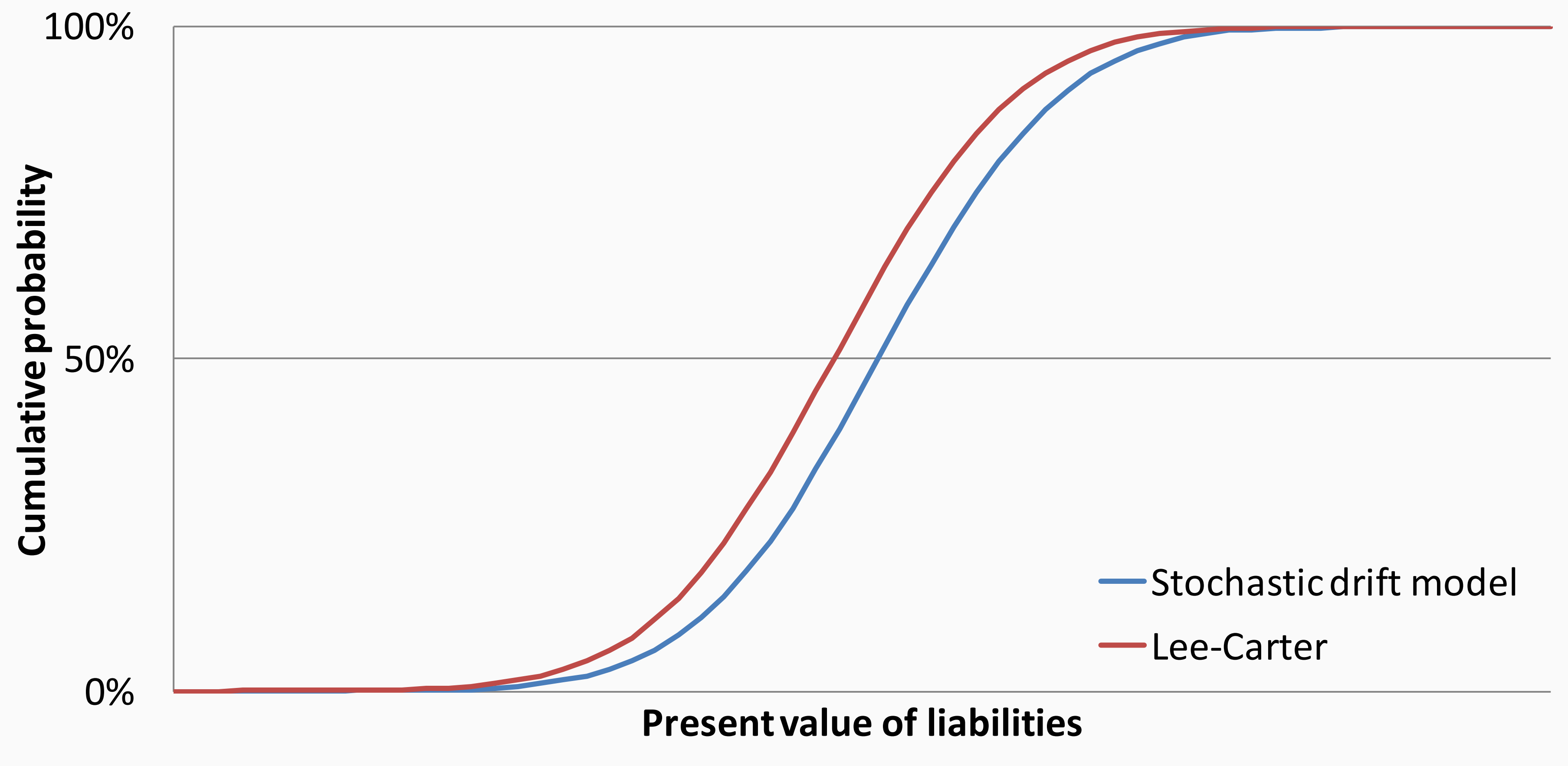
Pricing of Longevity Risk in a Lee-Carter framework



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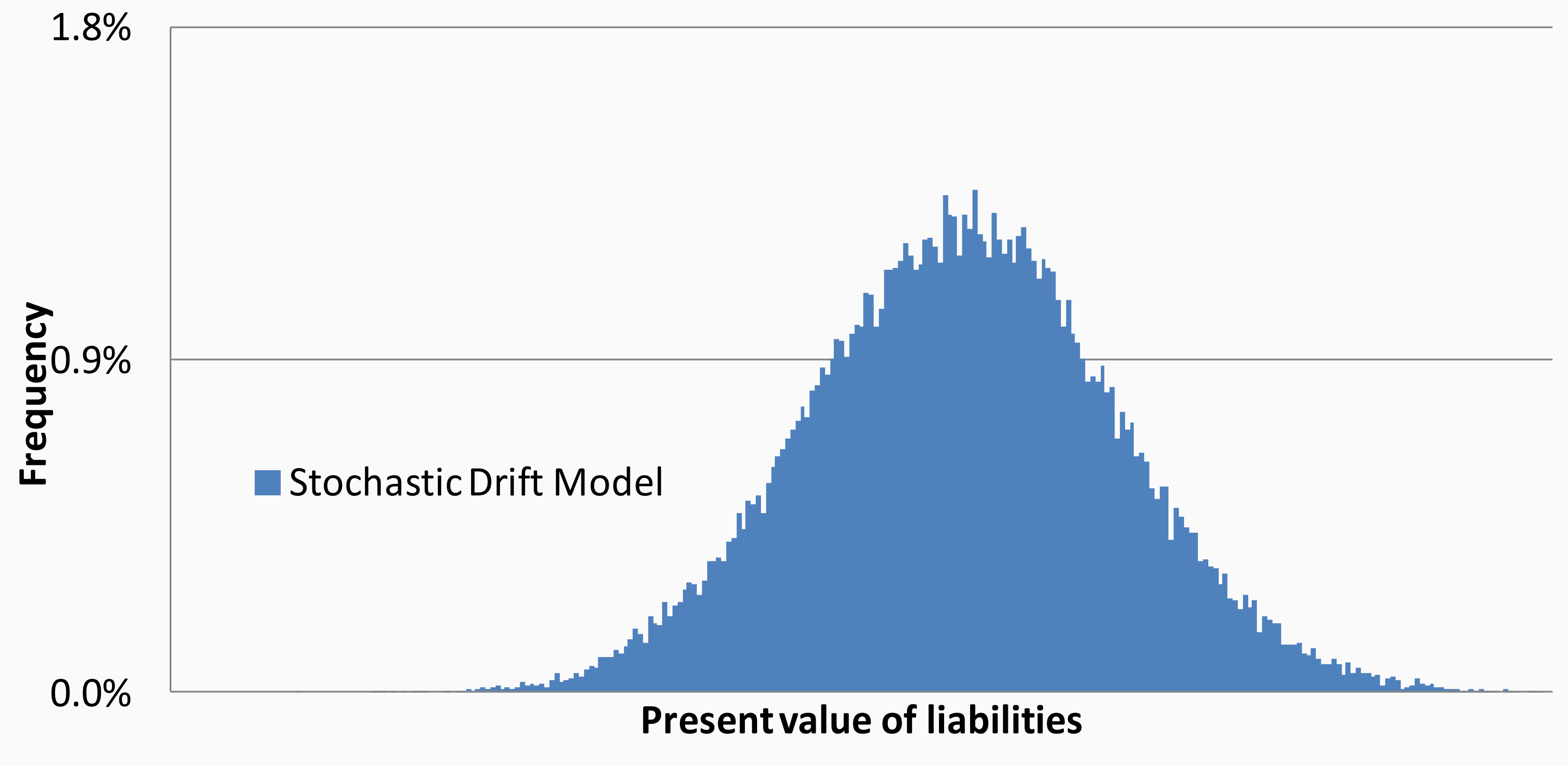
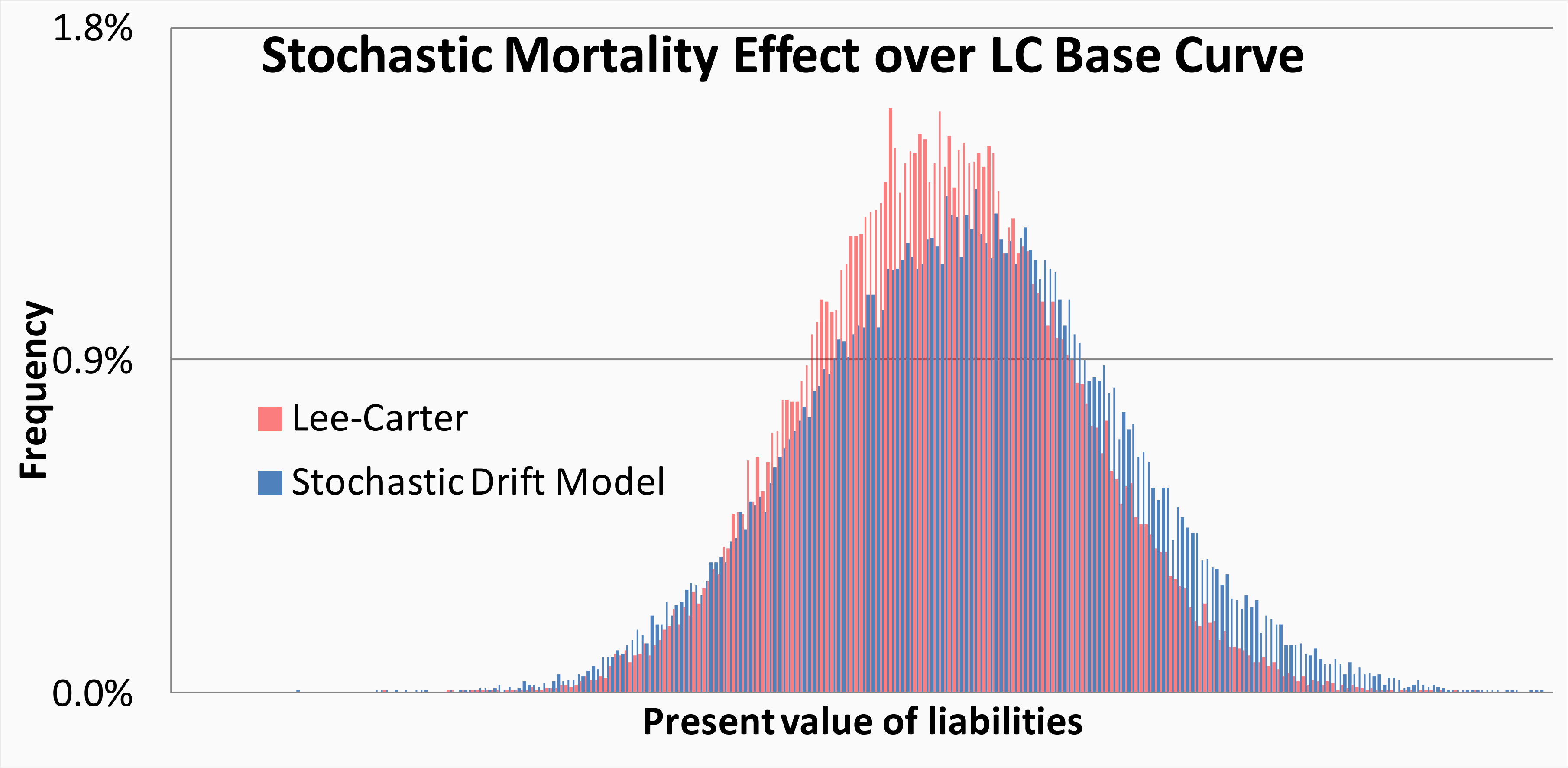
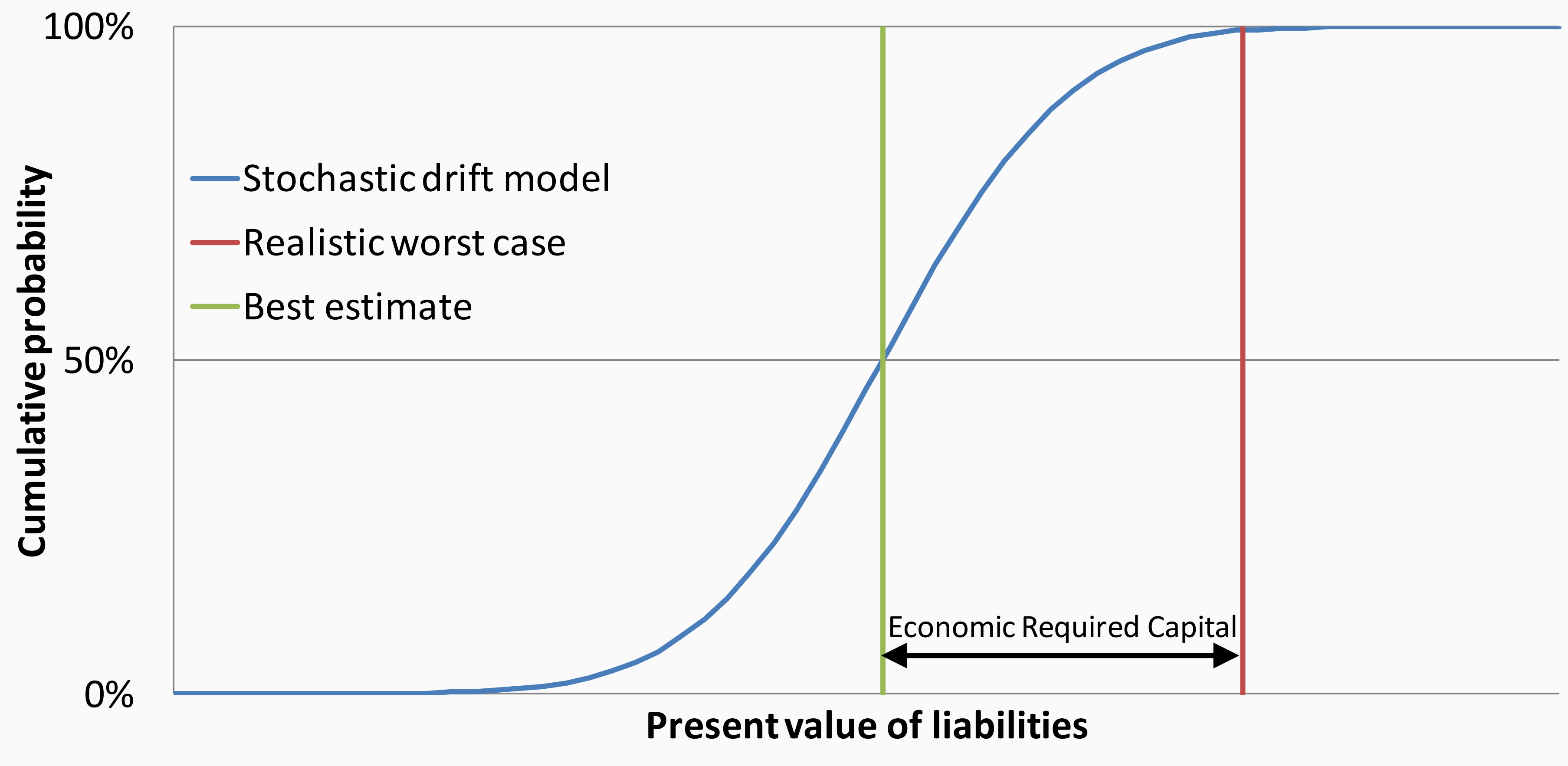
Pricing of Longevity Risk in a Lee-Carter framework



Pricing of Longevity Risk in a Lee-Carter framework



Pricing of Longevity Risk in a Lee-Carter framework



Economic capital may be viewed as the longevity risk in the pension plan.

Pricing of Longevity Risk in a Lee-Carter framework

- Evidently, the stochastic mortality model that is used to analyse the block is going to have a major impact on the economic capital required to fully collateralise the deal.

- If we assume that a reasonable estimate has been made of the economic capital then the opportunity for a capital market solution exists if the capital markets are able to provide capital relief at a workable price.

- Rather than directly tailored solutions, one might attempt a synthetic portfolio hedge.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A different solution: Synthetic Portfolio Hedge

- Start with underlying portfolio
- Apply experience adjusted population mortality to the synthetics portfolio going forward
- Works exactly as an indemnity hedge except that public statistics are used going forward except for actual plan experience

Advantages

- Avoids data issues between the cedent and risk takers
- Mirrors structures attractive to capital markets investors
- As a result, the solution is fairly cost effective

Disadvantages

- Some mismatch between future actual plan experience and future experience rated population experience will inevitably exist

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A different solution: Out-of-the-Money Synthetic Portfolio Hedge

- Similar to synthetic portfolio hedge but only pays if future experience rated population experience exceeds a certain threshold

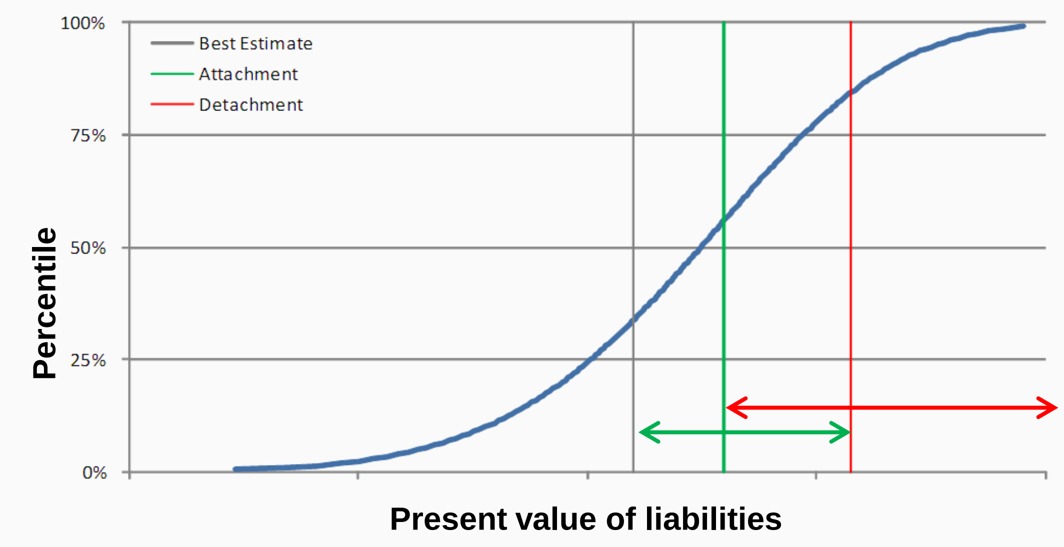
- Essentially a call option on plan cash flows
  - Plan begins to receive option payoff if cash flows exceed a certain threshold

Advantages

- Retains upside, but protects more

Disadvantages

- Plan also retains some downside



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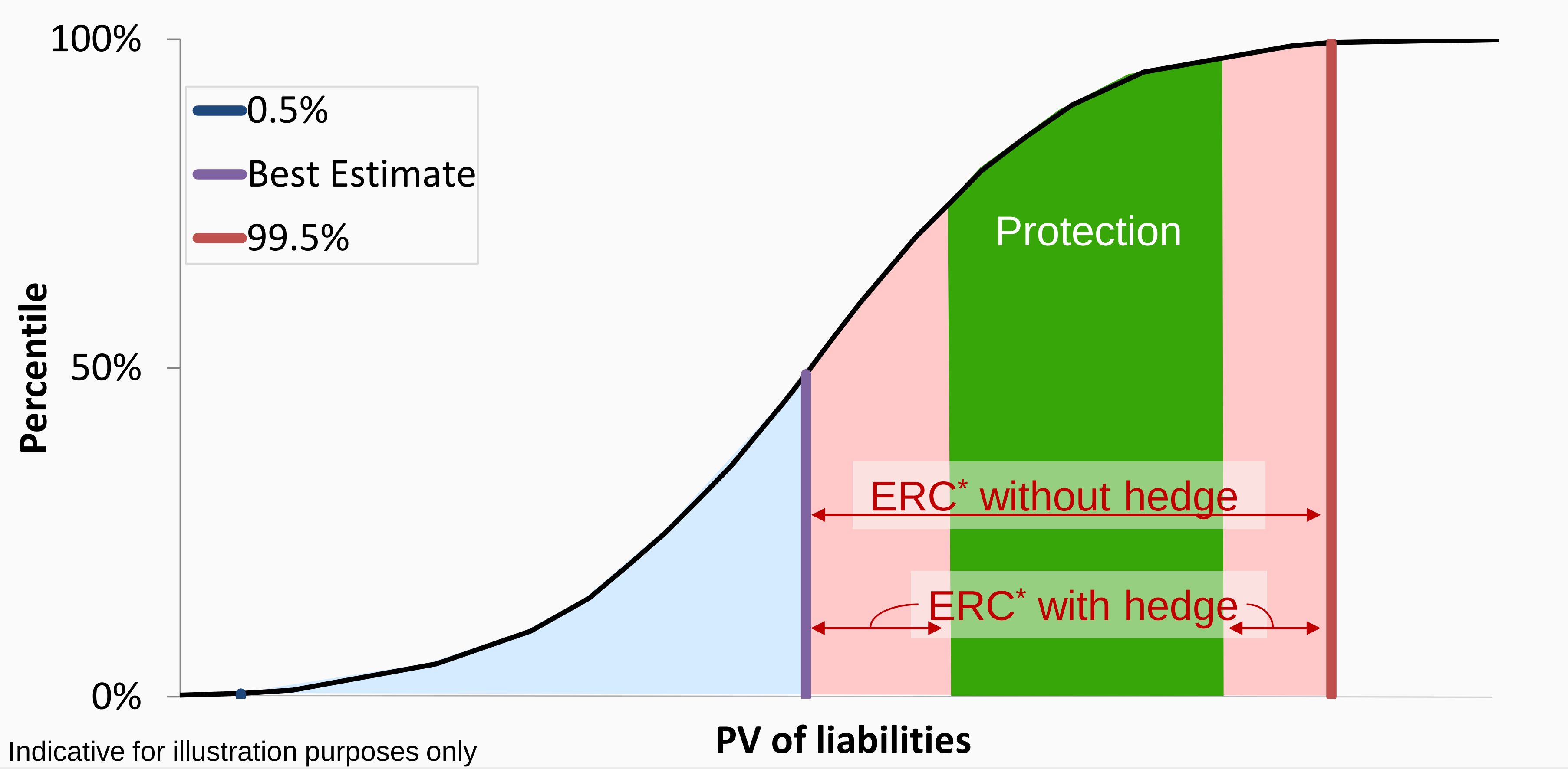
Hedge target concept



* Economic Required Capital

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Thank you

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