Single and cross-generation natural hedging of longevity and financial risk

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Contents

Outline

- we choose an affine continuous-time **cohort-based** model for mortality intensity
- we couple it with a **standard model** for stochastic interest rates
- we compute the sensitivities of annuities and death assurances with respect to **longevity and financial** risk factors
- we consider hedging **within the same cohort and across cohorts**
- we show how to compute Delta-Gamma hedged portfolios (solving a linear system of equations), **exploiting natural hedging**

Natural hedging

- Unexpected changes in mortality rates affect both the portfolios of annuity providers and life insurers, but have **opposite effects** on them.
- Natural hedging can exploit the **natural offsetting between contracts** written on the death and on the life of insureds to reduce exposure to longevity/mortality risk.
- We explore **in which cases and how** naturally hedged portfolios can be set up, when not only longevity risk but also interest rate risk affects insurance liabilities.

Related Literature

- In a previous paper (Luciano et al.,2012) we presented a Delta-Gamma hedging technique focusing on the use of longevity bonds as hedging instruments.
- Previous literature on natural hedging:
  1. highlighted the importance natural hedging can have: Cox and Lin (2007) provide evidence that annuity providers with naturally hedged portfolio mixes have a competitive advantage
  2. explored an immunization approach, which is closely related to our approach: Wang et al. (2010) develop an immunization approach, longevity only
  3. highlighted the importance of considering longevity as well as financial risk hedging: Hari et al. (2011) find substantial importance of longevity risk in annuity providers portfolios; Stevens et al. (2011) stress the importance of financial risk on the riskiness of the annuity-death benefit mix

Longevity risk

- Death arrival is modelled as the first jump time of a doubly stochastic process.
- Let $\lambda_x(t)$ be the mortality intensity of generation x at time t. We assume that, under a risk neutral measure (under the assumption of a constant risk premium) its dynamic is:

$$d\lambda_x(t) = a\lambda_x(t)dt + \sigma_x dW_x(t) \tag{1}$$

- This cohort-based model has been shown to be appropriate to model human mortality (Luciano and Vigna, 2008) and has nice tractability properties.

Financial risk modelling

- As customary, we assume that no arbitrage holds in the financial market. For simplicity, we let the market be complete.
- We model the forward interest-rate $F(t,T)$ as a (constant parameter) Hull-White one under $\mathbb{Q}$, namely

$$dF(t,T) = A'(t,T)dt + \Sigma(t,T)dW_F'(t)$$

with

$$\Sigma(t,T) = \Sigma \exp(-g(T-t)), \quad \Sigma > 0, g > 0$$

and $A'(t,T)$ which is tied to $\Sigma(t,T)$ by the usual HJM relationship.

- We assume independence between longevity and financial risk

Longevity Risk exposure of pure endowments

We compute the Greeks of pure endowment reserves – evaluated at fair-value – with respect to the longevity risk factor or "forecast error":

$$I(t) := \tilde{\lambda}(t) - f(0,t)$$

which is the difference between the actual and the 'forecasted' value of the intensity.
These Greeks are:

$$\Delta^M = \frac{\partial S}{\partial I} = -SX \leq 0$$

$$\Gamma^M = \frac{\partial^2 S}{\partial I^2} = SX^2 \geq 0$$

where

$$X(t,T) := \frac{\exp(a(T-t)) - 1}{a}$$

Financial Risk exposure of pure endowments

Analogously, we derive the exposures with respect to the financial risk factor

$$K(t) := \tilde{r}(t) - F(0,t)$$

which is the difference between the actual and the realized interest rate. The Greeks are

$$\Delta^F = \frac{\partial B}{\partial K} = -B\overline{X} \leq 0$$

$$\Gamma^F = \frac{\partial^2 B}{\partial K^2} = B\overline{X}^2 \geq 0$$

where $\overline{X}$ is defined similarly to X.

Greeks for annuities and death assurances

- Under the assumption of independence between longevity and financial risk, the first and second order sensitivities to the risk factor for the reserves of annuities and death assurances can be derived based on the ones of pure endowments. For annuities we have:

$$\Delta_L^M(t,T) = -\sum_{u=1}^{T-t} B_{t,u} S_{t,u} X_{t,u} = \sum_{u=1}^{T-t} B_{t,u} \Delta_{t,u}^M \leqslant 0$$

$$\Gamma_L^M(t,T) = \sum_{u=1}^{T-t} B_{t,u} S_{t,u} [X_{t,u}]^2 = \sum_{u=1}^{T-t} B_{t,u} \Gamma_{t,u}^M \geqslant 0$$

$$\Delta_L^F(t,T) = -\sum_{u=1}^{T-t} B_{t,u} S_{t,u} \bar{X}_{t,u} = \sum_{u=1}^{T-t} S_{t,u} \Delta_{t,u}^F \leqslant 0$$

$$\Gamma_L^F(t,T) = \sum_{u=1}^{T-t} B_{t,u} S_{t,u} [\bar{X}_{t,u}]^2 = \sum_{u=1}^{T-t} S_{t,u} \Gamma_{t,u}^F \geqslant 0$$

Greeks for death assurances

For death assurances we have:

$$\Delta_D^M = \sum_{u=1}^{T-t} B_{t,u}(\Delta_{t,u-1}^M - \Delta_{t,u}^M)$$

$$\Gamma_D^M = \sum_{u=1}^{T-t} B_{t,u}(\Gamma_{t,u-1}^M - \Gamma_{t,u}^M)$$

$$\Delta_D^F = \sum_{u=1}^{T-t} [S_{t,u-1} - S_{t,u}]\, \Delta_{t,u}^F \leqslant 0$$

$$\Gamma_D^F = \sum_{u=1}^{T-t} [S_{t,u-1} - S_{t,u}]\, \Gamma_{t,u}^F \geqslant 0$$

- When considering portfolio mixes of annuities and death assurances we highlight the presence of offsetting terms in portfolio exposures.

D-G Hedging: deriving the hedges/1

- Hedges can be found solving the system of equations

$$\begin{cases} -n_H \Delta_H^{M,j}(t, T_H) + \sum_{i=1}^{N} n_i \Delta_i^{M,j}(t, T_i) = 0, \; j = 1, ..., J \\ -n_H \Delta_H^{F}(t, T_H) + \sum_{i=1}^{N} n_i \Delta_i^{F}(t, T_i) = 0 \\ -n_H \Gamma_H^{M,j}(t, T_H) + \sum_{i=1}^{N} n_i \Gamma_i^{M,j}(t, T_i) = 0, \; j = 1, ..., J \\ -n_H \Gamma_H^{F}(t, T_H) + \sum_{i=1}^{N} n_i \Gamma_i^{F}(t, T_i) = 0 \end{cases}$$

  or part of it, depending on the hedger's objective (Delta or Delta-Gamma hedge of one or both risks).
- N is the number of hedging instruments, n_H is the position to hedge.

D-G Hedging deriving the hedges/2

- While the financial risk factor is unique, the longevity risk factors are J, one for each cohort.
- It is possible to add a self-financing condition, i.e. the premiums collected are used to set up the hedge:

$$\Pi = n_H Z_H - \sum_{i=1}^{N} n_i Z_i$$

- When a unique solution to the system exists, we interpret a negative coefficient n_i as a short position (the insurer issues the product), a positive coefficient as a long position (reinsurance/the insurer buys a mortality-linked derivative).

UK-Calibrated application

Table: Reserves and Greeks for annuities and death assurances

Maturity	Whole-life annuity	10-year DA	20-year DA
Generation 1935 (Gx)			
Price	12.66	15.53	33.59
Δ_i^M	-269.54	1240.69	2181.05
Γ_i^M	16164.35	-20053.31	-107139.46
Δ_i^F	-95.82	-83.22	-308.34
Γ_i^F	1007.17	537.53	3406.96
Generation 1945 (Gy)			
Price	13.09	12.94	30.05
Δ_i^M	-323.48	1355.29	2619.28
Γ_i^M	24847.66	-23225.97	-146827.81
Δ_i^F	-100.92	-70.48	-285.16
Γ_i^F	1075.37	459.63	3211.46

Intra-generational hedging

- As expected, first and second order hedges of DAs and annuities have opposite signs, BUT
- the products have exposure of the same sign with respect to financial risk.
- A first conclusion is that true natural hedging – achieved without resorting to reinsurance or derivatives – is in theory possible to achieve only when hedging longevity risk only.

Hedging across generations

- We consider the intensities of two cohorts driven by correlated brownian motions:

$$d\lambda_{Gx} = a_x \lambda_x dt + \sigma_x dW_x(t), \quad (2)$$

$$d\lambda_{Gy} = a_y \lambda_y dt + \sigma_y dW_y(t). \quad (3)$$

- We identify a common risk factor across generation and a residual one, idiosyncratic.
- We exploit the correlation to hedge against longevity risk, deriving sensitivities with respect to these risk factors;
- this allows us to extend the approach in Luciano et al. (2012) to the use of imperfectly correlated hedging instruments

UK-Calibrated application

Table: Hedging strategies for an annuity on generation Gx

Instrument	$10 - y_{Gx}$	$15 - y_{Gx}$	$5 - y_{Gy}$	$10 - y_{Gy}$	$15 - y_{Gy}$	$20 - y_{Gy}$	$30 - y_{Gy}$
Strategy							
D MF	-0.22			-3.15	2.03		
DG MF	0.45	-0.45	-22.93	19.22	-8.09	1.92	
D M SF	2.58			-3.83	1.04		
DG M SF	1.90	-1.45		121.43	-108.61	28.2	
D MF SF	-62.14	41.88		70.75	-45.55		
DG MF SF	0.45	-0.45	-0.42	2.38	-3.77	2.73	-0.49

The Table reports the hedging strategies when at least one product written on generation Gy is available on the market. Notice that C=100, R=1.

Cross-generation natural hedging

- The financial risk factor is the same across cohorts.
- The identification of the common and the idiosyncratic longevity risk factor is important because it allows us to use products written on other cohorts to "complete" the market and find a unique solution to the hedging problem.
- Even when it is not possible to completely hedge the overall risk exposure, it may be possible to hedge against the common risk factor and have an assessment of the unhedged exposure.
- The estimation of ρ is an important practical issue, in particular if the hedged portfolio has products written on more than one cohort.
- In the example above, natural hedging can be exploited only when Delta-hedging against longevity risk.

Conclusions

- We studied natural hedging of longevity and financial risk;
- we considered both cohort-homogenous and cohort-heterogeneous portfolios;
- we obtained closed-form sensitivities and hedges up to the second order, using a reliable continuous-time diffusive model for longevity and a standard model for financial risk;
- in our application, we showed that natural hedging (without reinsurance or use of mortality-linked derivatives) is possible only when hedging the portfolio against longevity risk.