JOURNAL OF APPLIED ECONOMETRICS
J. Appl. Econ. **14**: 309–318 (1999)

TESTING FOR A UNIT ROOT IN THE VOLATILITY OF ASSET RETURNS

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SUMMARY

It is now well established that the volatility of asset returns is time varying and highly persistent. One leading model that is used to represent these features of the data is the stochastic volatility model. The researcher may test for non-stationarity of the volatility process by testing for a unit root in the log-squared time series. This strategy for inference has many advantages, but is not followed in practice because these unit root tests are known to have very poor size properties. In this paper I show that new tests that are robust to negative MA roots allow a reliable test for a unit root in the volatility process to be conducted. In applying these tests to exchange rate and stock returns, strong rejections of non-stationarity in volatility are obtained. Copyright © 1999 John Wiley & Sons, Ltd.

1. INTRODUCTION

Ever since the seminal paper of Engle (1982), an enormous literature has developed on models with time-varying heteroscedasticity. A number of recent surveys of this literature are available, such as Bollerslev *et al.* (1992, 1994). Researchers have found these models very useful for characterizing the persistence in volatility and the fat tails of time series of asset returns. Much of the literature has considered ARCH/GARCH models in which the variance of the time series at date t is known, conditional on information dated $t-1$ and earlier. More recently, researchers have considered models in which the variance at date t is random, even after conditioning on information dated $t-1$ and earlier. These models of stochastic volatility are natural discrete time analogues of the continuous time models used in modern finance theory, and may fit the data better than ARCH/GARCH models. But they have the disadvantage of being hard to estimate precisely because the variance at date t is not a function of the parameters and observed data alone.

The persistence in the volatility of asset returns appears to be very high. This observation has motivated consideration of the IGARCH model by numerous authors, reviewed in Bollerslev *et al.* (1994). It has also led many researchers to consider a stochastic volatility model in which the volatility process is non-stationary (e.g. Hansen, 1995; Harvey *et al.*, 1994; Ruiz, 1994). The stochastic volatility model implies that the log of the squared time series is an ARMA process, the largest autoregressive root of which is the same as the largest autoregressive root of the volatility process. It is possible to test for a unit root in the unobserved volatility process by testing for a unit root in the log of the squared time series. This test is very easy to conduct and does not require distributions to be specified for the error terms (unlike for estimation of a stochastic volatility model). Unfortunately, as observed by Harvey *et al.* (1994), this process has a large

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CCC 0883-7252/99/030309–10$17·50

Received 5 January 1998
Revised 22 June 1998

negative moving average root and standard unit root tests are known to suffer from extreme size distortions in the presence of negative MA roots (Schwert, 1989; Pantula, 1991). But recently Perron and Ng (1996), building on work of Stock (1990, unpublished manuscript), have proposed modified unit root tests which are robust to large negative MA roots. I propose using these to test for a unit root in the log of the squared time series and hence to test for a unit root in the volatility process.

The plan of the remainder of this paper is as follows. In Section 2, I describe the model and the proposed method of testing for a unit root in a stochastic volatility process. Sections 3 and 4 contain Monte Carlo results. In Section 5, I report the results of an application of this method to exchange rate and stock return data. Strong rejections of the hypothesis of non-stationarity in the volatility process are obtained for all the time series considered. Section 6 presents conclusions.

2. THE PROPOSED METHOD FOR CONSTRUCTING TESTS

Consider the standard autoregressive stochastic volatility (ARSV) model which specifies that $\{y_t\}_{t=1}^T$ is a time series of returns such that

$$y_t = \sigma_t \varepsilon_t$$

The model further specifies that ε_t is i.i.d. with mean zero and variance 1, $\log(\sigma_t^2) = \mu + h_t$, $a(L)h_t = \eta_t$ and $a(L) = b(L)(1 - \alpha L)$ is a pth-order autoregressive lag polynomial such that $b(L)$ has all roots outside the unit circle. The parameter α is the largest autoregressive root of the volatility process. It is assumed that η_t is i.i.d. with mean zero and variance σ_η^2 and is distributed independently of ε_t. Clearly,

$$\begin{aligned} a(L)\log(y_t^2) &= a(1)\mu + \eta_t + a(L)\log(\varepsilon_t^2) \\ a(L)\log(y_t^2) &= \omega + \eta_t + a(L)\xi_t = \omega + x_t \end{aligned} \tag{1}$$

where $\xi_t = \log(\varepsilon_t^2) - E(\log(\varepsilon_t^2))$, $\omega = a(1)(\mu + E(\log(\varepsilon_t^2)))$ and $x_t = \eta_t + a(L)\xi_t$.

A number of approaches have been proposed to estimate the parameters of a stochastic volatility model. These methods can be computationally expensive and require distributional assumptions to be made concerning ε_t and η_t, usually specifying that both are normal. In practice, these estimators are applied imposing that $p = 1$, though this is principally a matter of computational convenience. These estimators include the quasi maximum-likelihood (QML) estimator which maximizes the likelihood function, calculated from the Kalman filter treating ξ_t as though it were Gaussian (Harvey *et al*., 1994; Ruiz, 1994). Ruiz considers both the model in which it is assumed that $\alpha < 1$ and the model in which it is imposed that $\alpha = 1$. Another approach is the GMM estimator which minimizes the distance between a vector of sample moments of the data and its population counterpart (Melino and Turnbull, 1990). More computationally intensive strategies have been proposed by Danielsson (1994), Jacquier *et al*., (1994) and Shephard (1993).

If the researcher is just interested in deciding whether $\alpha = 1$ or not, then a much simpler approach is available. This does not require distributional assumptions to be made concerning the error terms (unlike for estimation) and is simple to apply for any value of p. The time series $x_t = \eta_t + a(L)\xi_t$ has a Wold representation and, from inspection of its autocovariance function, this is an MA(p) reduced form. It follows from equation (1) that $\log(y_t^2)$ is a stationary

Table I. Values of the moving average root θ in the reduced form

	$\sigma_\eta^2 = 0{\cdot}01$	$\sigma_\eta^2 = 0{\cdot}05$	$\sigma_\eta^2 = 0{\cdot}1$	$\sigma_\eta^2 = 0{\cdot}15$	$\sigma_\eta^2 = 0{\cdot}2$
$\alpha = 0{\cdot}8$	− 0·7956	− 0·7790	− 0·7606	− 0·7441	− 0·7291
$\alpha = 0{\cdot}85$	− 0·8439	− 0·8223	− 0·7994	− 0·7797	− 0·7623
$\alpha = 0{\cdot}9$	− 0·8908	− 0·8612	− 0·8327	− 0·8094	− 0·7895
$\alpha = 0{\cdot}95$	− 0·9333	− 0·8911	− 0·8567	− 0·8304	− 0·8086
$\alpha = 1$	− 0·9560	− 0·9043	− 0·8674	− 0·8402	− 0·8179

ARMA(p, p) process if $|\alpha| < 1$ but is an ARIMA($p - 1$,1p) process if $\alpha = 1$ (α is the largest autoregressive root of $\log(y_t^2)$). So the researcher may test the hypothesis $\alpha = 1$ by testing for a unit root in $\log(y_t^2)$ using, in principle, any one of the unit root tests available in the econometric literature.

Unit root tests have been used in this way, for example by Harvey *et al*. (1994). However, they pointed out that these unit root tests should be expected to have very poor size properties and so they attached little significance to the rejection of the unit root null that they obtained using exchange rate data. The reason why these unit root tests should be expected to suffer from serious size distortions is because $\log(y_t^2)$ has an ARMA or ARIMA reduced form, but with a large negative moving average root. In the case $p = 1$ (so that $a(L) = 1 - \alpha L$) a simple calculation reveals that $x_t = \eta_t + \xi_t - \alpha\xi_{t-1}$ has an MA(1) reduced form with a moving average parameter

$$\theta = [-1 - \alpha^2 - q + \sqrt{(1 + \alpha^2 + q)^2 - 4\alpha^2}]/2\alpha$$

where $q = \sigma_\eta^2/\sigma_\xi^2$ and $\sigma_\xi^2 = \mathrm{Var}(\xi_t)$. Table I shows the value of the moving average parameter θ, for a number of values of α and σ_η^2, assuming that ε_t is normally distributed (so that $\sigma_\xi^2 = \pi^2/2$). The presence of a large negative moving average root is well known to cause serious size distortions in standard unit root tests (Schwert, 1989; Pantula, 1991) in finite samples. Accordingly, standard unit root tests applied to the log of squared time series on asset returns may in principle be interpreted as testing for a unit root in the volatility process, but should in practice suffer from serious finite sample size distortions even in the large sample sizes that are available for asset returns data.

But recently Perron and Ng (1996), building on the work of Stock (1990, unpublished manuscript), have proposed modified unit root tests which have much better finite sample properties in the presence of large negative MA roots. I accordingly test the hypothesis that $\alpha = 1$ against the alternative $|\alpha| < 1$ by applying these tests to $\log(y_t^2)$. The three test statistics are

$$MZ_\alpha = [T^{-1}(v_T - \bar{v})^2 - s^2]\left[2T^{-2}\sum_{t=1}^{T}(v_t - \bar{v})^2\right]^{-1}$$

$$MSB = \left[s^{-2}T^{-2}\sum_{t=1}^{T}(v_t - \bar{v})^2\right]^{1/2}$$

$$MZ_t = MZ_\alpha \cdot MSB$$

where $v_t = \log(y_t^2)$, $\bar{v} = T^{-1}\sum_{t=1}^{T} v_t$ and s^2 is the autoregressive spectral density estimate obtained from the autoregression

$$v_t = a_0 + a_1 v_{t-1} + \sum_{j=1}^{k} a_j \Delta v_{t-j} + e_t$$

where $k = o(T^{1/3})$. The ability of the tests to control the size well in the presence of large negative MA roots hinges critically on this choice of spectral density estimator. Under the null that $\alpha = 1$, as $T \to \infty$,

$$MZ_\alpha \Rightarrow [W^\mu(1)^2 - W^\mu(0)^2 - 1]\left[2\int_0^1 W^\mu(r)^2 dr\right]^{-1}$$

$$MSB \Rightarrow \left[\int_0^1 W^\mu(r)^2 dr\right]^{1/2}$$

$$MZ_t \Rightarrow [W^\mu(1)^2 - W^\mu(0)^2 - 1]\left[2\int_0^1 W^\mu(r)^2 dr\right]^{-1/2}$$

where $W^\mu(r)$ is a demeaned standard Brownian motion on the unit interval. The tests MZ_α, MSB and MZ_t are all one-sided, which reject if the test statistic is less than some critical value.

Under the sequence of local alternatives $\alpha = 1 + c/T$, as $T \to \infty$,

$$MZ_\alpha \Rightarrow [J_c^\mu(1)^2 - J_c^\mu(0)^2 - 1]\left[2\int_0^1 J_c^\mu(r)^2 dr\right]^{-1}$$

$$MSB \Rightarrow \left[\int_0^1 J_c^\mu(r)^2 dr\right]^{1/2}$$

$$MZ_t \Rightarrow [J_c^\mu(1)^2 - J_c^\mu(0)^2 - 1]\left[2\int_0^1 J_c^\mu(r)^2 dr\right]^{-1/2}$$

where $J_c^\mu(r)$ is a demeaned standard Ornstein–Uhlenbeck process (see Stock, 1994). So these unit root tests have power against local alternatives in a T^{-1} neighbourhood of unity. This contrasts with maximum-likelihood tests for a unit root in a GARCH/IGARCH model which use root-T asymptotics and correspondingly have power only in a $T^{-1/2}$ neighbourhood of unity (Lumsdaine, 1996).

3. MONTE CARLO RESULTS

In this section I report the results of Monte Carlo experiments simulating the size and power of the proposed tests that $\alpha = 1$. The design of the experiment consists of the ARSV model, described in the previous section, where $p = 1$, η_t is normally distributed, $\sigma_\eta^2 = 0{\cdot}01, 0{\cdot}04$, $\mu = 0$, $\alpha = 0{\cdot}98, 0{\cdot}985, 0{\cdot}99, 0{\cdot}995, 1$ and ε_t has a t-distribution on 6 degrees of freedom (standardized to have variance one). The parameter μ is the mean of $\log(\sigma_t^2)$ for $\alpha < 1$. Most Monte Carlo experiments using the ARSV model choose negative values of μ, but in the present context the choice of μ does not matter. This is because all the test statistics use demeaned log-squared returns, so the results are numerically invariant to adding any constant to $\log(y_t^2)$ and so are numerically invariant to μ. Only values of α that are very close to one are considered: otherwise all the tests have power that is virtually 100%. The values of σ_η^2 are consistent with those used in other Monte Carlo simulations when α is very close to 1. For example, for $\alpha = 0{\cdot}98$, Andersen and Sorensen (1996) use $\sigma_\eta = 0{\cdot}166$. The combination of a (near) unit root in the volatility with a much larger variance of η_t will generate an implausibly large amount of volatility clustering. If ε_t is normal, then although the unconditional distribution of returns is nonnormal, the returns will not have the very heavy tails that are typically observed in real data. This is why a non-normal specification is used for the distribution of ε_t. The results for Gaussian ε_t are, however, quite similar and are available from the author on request.

In obtaining s^2, the autoregressive spectral density estimator, the parameter k is chosen by the deterministic rule $k = 8(T/100)^{1/4}$, rounded up to the nearest integer. Two sample sizes are considered: 1000 and 3000. Each experiment involves 5000 replications and each test has a 5% nominal size.

The size and power of the tests are reported in Table II. In addition to the results using the tests MZ_α, MSB and MZ_t, I also simulate the size and power of some older unit root tests, using the familiar Z_α and Z_t statistics (Phillips and Perron, 1988) and the Augmented Dickey–Fuller test (ADF). For the Z_α and Z_t statistics, s^2 was used as the spectral density estimate. For the ADF test, k lags of the differenced data were added to the Dickey–Fuller regression.

Table II. Size and power of tests in ARSV model

	ADF	Z_α	Z_t	MZ_α	*MSB*	MZ_t
$T = 1000$						
$\alpha = 0{\cdot}98$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	99·20	99·46	97·50
$\alpha = 0{\cdot}98$, $\sigma_\eta^2 = 0{\cdot}04$	99·94	100·00	100·00	80·18	84·24	64·22
$\alpha = 0{\cdot}985$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	96·86	98·08	92·68
$\alpha = 0{\cdot}985$, $\sigma_\eta^2 = 0{\cdot}04$	99·12	100·00	100·00	62·54	67·62	44·50
$\alpha = 0{\cdot}99$, $\sigma_\eta^2 = 0{\cdot}01$	99·98	100·00	100·00	89·42	91·70	80·70
$\alpha = 0{\cdot}99$, $\sigma_\eta^2 = 0{\cdot}04$	93·06	100·00	100·00	37·74	44·02	23·40
$\alpha = 0{\cdot}995$, $\sigma_\eta^2 = 0{\cdot}01$	99·24	100·00	100·00	67·76	71·94	55·30
$\alpha = 0{\cdot}995$, $\sigma_\eta^2 = 0{\cdot}04$	68·06	100·00	100·00	16·82	20·86	9·66
$\alpha = 1$, $\sigma_\eta^2 = 0{\cdot}01$	78·16	100·00	100·00	28·72	32·14	20·28
$\alpha = 1$, $\sigma_\eta^2 = 0{\cdot}04$	30·28	98·46	100·00	4·60	5·84	2·20
$T = 3000$						
$\alpha = 0{\cdot}98$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	100·00	100·00	100·00
$\alpha = 0{\cdot}98$, $\sigma_\eta^2 = 0{\cdot}04$	100·00	100·00	100·00	99·98	100·00	99·76
$\alpha = 0{\cdot}985$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	100·00	100·00	99·98
$\alpha = 0{\cdot}985$, $\sigma_\eta^2 = 0{\cdot}04$	100·00	100·00	100·00	99·96	100·00	99·40
$\alpha = 0{\cdot}99$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	99·98	100·00	99·72
$\alpha = 0{\cdot}99$, $\sigma_\eta^2 = 0{\cdot}04$	100·00	100·00	100·00	99·34	99·94	94·86
$\alpha = 0{\cdot}995$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	98·78	99·78	93·74
$\alpha = 0{\cdot}995$, $\sigma_\eta^2 = 0{\cdot}04$	97·42	100·00	100·00	76·98	83·62	51·50
$\alpha = 1$, $\sigma_\eta^2 = 0{\cdot}01$	67·02	100·00	100·00	28·46	33·24	16·48
$\alpha = 1$, $\sigma_\eta^2 = 0{\cdot}04$	21·44	94·98	99·60	5·24	7·42	1·86

Note: The simulations when $\alpha = 1$ are size simulations, the remaining results refer to power simulations. The design of the experiment is as described in the text and ε_t is i.i.d. N(0, 1). The nominal size of all tests is 5%.

As was to be expected, Z_α and Z_t suffer from extreme size distortions, even when the sample size is 3000. Under the null, the unit root hypothesis is rejected far too often. The ADF test also rejects far too often under the null, though the size distortions are not quite as bad. The statistics MZ_α, MSB and MZ_t may also reject the null too often, but any size distortions are very much smaller than with the other tests. The test based on MZ_t appears to be most conservative and may have an empirical size below the nominal level. The smaller is σ_η^2, the closer to -1 is the MA root in the reduced form of $\log(y_t^2)$ and so the harder it is to control the size of the tests. For example, in the sample of size 3000, with $\sigma_\eta^2 = 0{\cdot}01$, MZ_t has an empirical size of 16·48% (compared with 67·02% and 100% for both Z_α and Z_t).

In the application to asset returns below, the hypothesis of a unit root in the volatility process is consistently rejected. In interpreting these results, concerns about controlling size are of primary importance. But the MZ_α, MSB and MZ_t tests have good power properties. In the sample of size 3000, if $\alpha = 0{\cdot}99$, the rejection probabilities are all three tests are always greater than 90%. Thus these tests are reliable tests of the hypothesis of non-stationarity in a stochastic volatility process. The size-unadjusted power of ADF, Z_a and Z_t is, of course, greater. Rejection rates of 100% were recorded in many cases. But this is of little consequence in view of the massive size distortions associated with these tests.

4. POWER AGAINST FRACTIONALLY INTEGRATED ALTERNATIVES

The unit root tests are designed to have power against the alternative that the volatility process is a stationary autoregression (the ARSV model with $\alpha < 1$). The squares, log-squares and absolute value of asset returns often have correlograms that decay very slowly, a fact noted in many recent papers including Dacorogna *et al.* (1993), Harvey (1998) and Breidt *et al.* (1998). This finding motivated Breidt *et al.* (1998) to propose a fractionally integrated stochastic volatility (FISV) model. A simple case of this model specifies that $\{y_t\}_{t=1}^T$ is a time series of returns such that

$$y_t = \sigma_t \varepsilon_t$$

where ε_t is i.i.d. with mean zero and variance 1, $\log(\sigma_t^2) = \mu + h_t$,

$$(1 - L)^d (1 - \alpha L) h_t = \eta_t$$

$(1 - L)^d$ denotes the fractional differencing operator and η_t is i.i.d. $N(0, \sigma_\eta^2)$ and is independent of ε_t. More generally h_t could be an arbitrary Gaussian fractional ARIMA process.

The tests for a unit root in the volatility process ought to have power against the FISV alternative provided that $\alpha \neq 1$ and $d \neq 1$. Accordingly, I also simulated the power of all the tests listed in the previous section against this alternative. The parameter values considered were $d = 0{\cdot}3, 0{\cdot}4$, $\alpha = 0{\cdot}96$ and $\sigma_\eta^2 = 0{\cdot}01, 0{\cdot}04$ while ε_t was specified to be t-distributed on 6 degrees of freedom, standardized to have variance 1 (the results for Gaussian ε_t are similar and available on request). As in the previous section, the sample size is 1000 or 3000, each experiment involves 5000 replications and each test has a 5% nominal size. The results are reported in Table III. Even though the modified unit root tests were designed in the context of the ARSV model, they can be seen to have good power against these FISV alternatives.

Table III. Power of tests in FISV model

	ADF	Z_α	Z_t	MZ_α	*MSB*	MZ_t
$T = 1000$						
$d = 0{\cdot}3$, $\sigma_\eta^2 = 0{\cdot}01$	99·52	100·00	100·00	63·48	69·12	45·94
$d = 0{\cdot}3$, $\sigma_\eta^2 = 0{\cdot}04$	76·04	100·00	100·00	37·36	44·22	19·82
$d = 0{\cdot}4$, $\sigma_\eta^2 = 0{\cdot}01$	81·94	100·00	100·00	23·14	28·22	11·76
$d = 0{\cdot}4$, $\sigma_\eta^2 = 0{\cdot}04$	36·74	100·00	100·00	18·36	23·54	8·76
$T = 3000$						
$d = 0{\cdot}3$, $\sigma_\eta^2 = 0{\cdot}01$	100·00	100·00	100·00	99·86	99·98	98·90
$d = 0{\cdot}3$, $\sigma_\eta^2 = 0{\cdot}04$	99·96	100·00	100·00	99·84	99·92	99·00
$d = 0{\cdot}4$, $\sigma_\eta^2 = 0{\cdot}01$	99·74	100·00	100·00	96·56	98·28	86·18
$d = 0{\cdot}4$, $\sigma_\eta^2 = 0{\cdot}04$	96·84	100·00	100·00	96·78	97·88	90·24

Note: The design of the experiment is as described in the text and ε_t is i.i.d. t distributed on 6 degrees of freedom, with variance standardized to 1. The autoregressive parameter α is equal to 0·96. The nominal size of all tests is 5%.

5. RESULTS ON EXCHANGE RATE AND STOCK RETURN DATA

I applied the tests for a unit root in volatility, discussed in this paper, to daily exchange rate and stock return data. The exchange rate series were obtained from Datastream and were the dollar/pound, dollar/mark and dollar/yen exchange rates (mnemonics BRITPUS, WGMRKUS and JAPYNUS, respectively). The data covered the entire years 1986–1996 inclusive. The exchange rate returns were then constructed as the first differences of the log exchange rates. These returns were demeaned. The stock return data consist of the daily changes of the log of the SP500 index (also demeaned), covering the period from 4 January 1982 to 23 September 1994. Figure 1 shows the sample autocorrelograms of the log-squares of each of these returns. The usual pattern of slowly decaying autocorrelations can be observed.

The procedures described in Section 2 were then applied to testing for a unit root in the volatility of each of these series. For comparison, the familiar Z_α, Z_t and ADF statistics were also used. The sample size was 2680 for the pound exchange rate, 2696 for the yen and mark exchange rates and 3209 for the stock return data. The results are reported in Table IV. In obtaining s^2, the autoregressive spectral density estimator, the results are reported for $k = 5$, 10, 15 and 20 (see Ng and Perron, unpublished manuscript 1997, for further discussion of the choice of k). For the Z_α and Z_t statistics, s^2 was used as the spectral density estimate. For the ADF test, k lags of the differenced data were added to the Dickey–Fuller regression.

The Z_α and Z_t statistics yield overwhelming rejections, but the enormous size distortions associated with these tests mean that this is of little consequence. The ADF statistics also yield rejections at all conventional significance levels, though are less extreme than the Z_α and Z_t statistics. But the ADF test also suffers from large size distortions. However, using the unit root tests that are robust to a large MA root, the hypothesis of a unit root in the volatility is clearly rejected at all conventional significance levels, regardless of the choice of k, for all four series. In the light of the fact that these tests control size reasonably well, this is strong evidence against the model of a unit root in the volatility process. The rejection of the null hypothesis is especially overwhelming in the case of the yen returns.

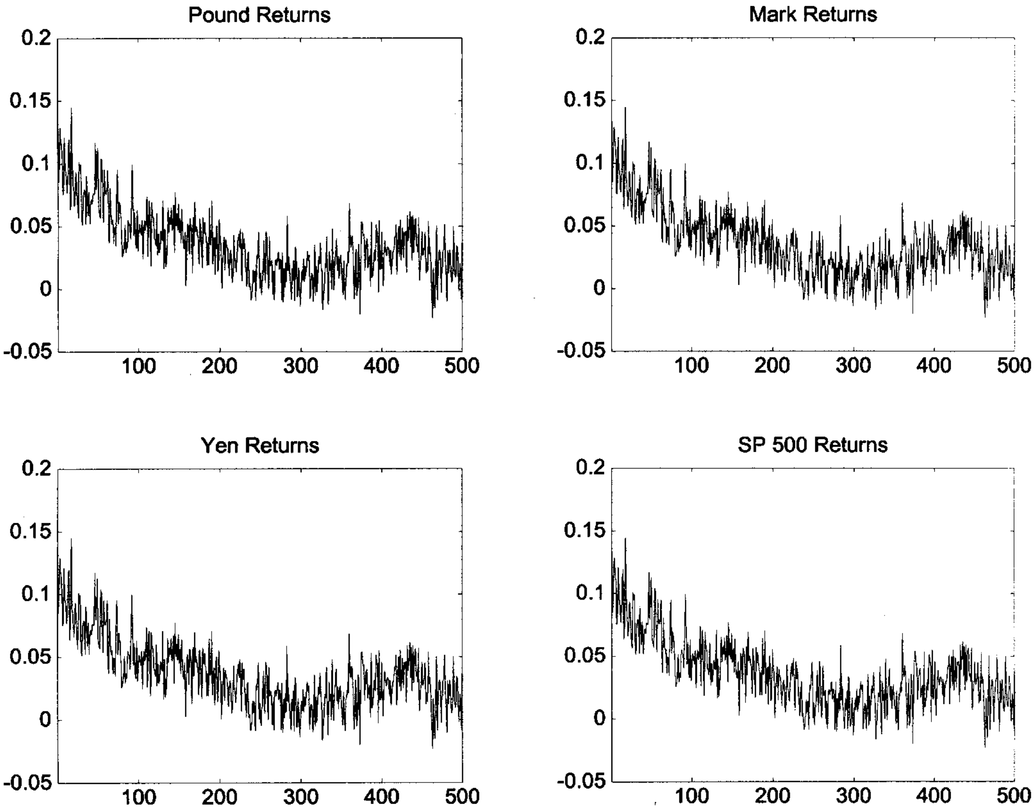

Figure 1. Sample autocorrelograms of log-squared returns

Harvey *et al.* (1994) propose a multivariate stochastic volatility model in which different time series have volatility processes that contain a common component that is a multivariate random walk. They apply this model to exchange rate data. The multivariate extension of the unit root tests proposed in this paper could be used to determine the number of common factors in such a model. However, the results obtained with the univariate data indicate that, at least for these time series, any common components in the volatility of different series are stationary and so that the factor model, as proposed by Harvey *et al.* does not provide a good representation of this data.

6. CONCLUSIONS

It is possible to test for a unit root in the volatility process of a stochastic volatility model by testing for a unit root in the log-squared time series. I have shown that standard unit root tests, applied to this problem, will have serious size distortions but that newly available unit root tests will not. These unit root tests do not require the researcher to make restrictive assumptions about the distribution of the error terms. Nevertheless, these new unit root tests still reject the hypothesis of nonstationary stochastic volatility in some exchange rate and stock return series. This indicates that while there is considerable persistence in the volatility of returns, a unit root in

Table IV. Unit root test statistics for log-squared asset returns

	ADF	Z_α	Z_t	MZ_α	*MSB*	MZ_t
Pound, $k = 5$	−15·42	−1354·5	−49·67	−370·90	0·037	−13·61
Pound, $k = 10$	−10·56	−1099·3	−71·99	−115·79	0·066	−7·59
Pound, $k = 15$	−8·17	−1034·6	−101·52	−51·17	0·098	−5·02
Pound, $k = 20$	−6·81	−1011·8	−132·45	−28·43	0·131	−3·72
Mark, $k = 5$	−18·68	−1866·7	−50·21	−690·43	0·027	−18·57
Mark, $k = 10$	−11·95	−1330·8	−75·51	−154·76	0·057	−8·78
Mark, $k = 15$	−9·51	−1247·7	−103·85	−71·66	0·083	−5·97
Mark, $k = 20$	−7·30	−1202·1	−164·91	−26·07	0·137	−3·58
Yen, $k = 5$	−18·59	−1868·0	−50·82	−675·59	0·027	−18·38
Yen, $k = 10$	−13·12	−1428·9	−65·70	−236·53	0·046	−10·88
Yen, $k = 15$	−10·56	−1321·9	−82·13	−129·53	0·062	−8·05
Yen, $k = 20$	−9·51	−1304·3	−87·18	−111·92	0·067	−7·48
SP 500, $k = 5$	−19·59	−2187·2	−60·58	−651·80	0·028	−18·05
SP 500, $k = 10$	−13·63	−1749·8	−84·50	−214·41	0·048	−10·35
SP 500, $k = 15$	−9·95	−1599·6	−141·21	−64·16	0·088	−5·66
SP 500, $k = 20$	−7·90	−1563·2	−209·61	−27·81	0·134	−3·73

Note: All test statistics are significant at the 1% level.

the stochastic volatility model is too extreme a specification. Models in which the volatility process is an AR(p) model with a very large root (but not a unit root) or in which the volatility process is fractionally integrated may provide a better representation of the data.

ACKNOWLEDGEMENTS

I am grateful to the coeditor and two anonymous referees for helpful comments on earlier versions of this manuscript. All errors are the sole responsibility of the author.

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