Streamlined mean-variance analysis for multiperiod models

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This document contains a brief rationale for the research and summary of the main results.

The methodology is based on maximization of mean-variance utility according to Markowitz [4], i.e., the investor wishes to maximize her mean return for a given level of portfolio variance. We work in a multiperiod setting with a fixed time horizon T and assume that the investor pre-commits to the optimal strategy as perceived at the start of the planning horizon at time 0.

The research identifies three quantities that conveniently encapsulate all information needed for the construction of the efficient investment frontier.

1) Opportunity process L measures the smallest second moment of a fully invested portfolio.
2) Tracking process $V(1)$ records intermediate wealth that minimizes the hedging distance to the constant payoff 1.
3) Squared hedging error $\varepsilon^2(1)$ gives the smallest hedging distance to the constant payoff 1.

With the three quantities in hand, the formula for the variance of a fully invested portfolio R on the efficient frontier is given by

$$\mathrm{Var}(R) = \frac{L_0\varepsilon_0^2(1)}{L_0V_0^2(1)+\varepsilon_0^2(1)} + \left(\frac{1}{1-L_0V_0^2(1)-\varepsilon_0^2(1)}-1\right)\left(\mathrm{E}[R]-\frac{L_0V_0(1)}{L_0V_0^2(1)+\varepsilon_0^2(1)}\right)^2. \qquad (\#\mathbf{1})$$

Formula **(#1)** has several advantages.

i. It is robust to specification of asset price processes.

ii. It does not rely on Markovian assumptions.

iii. It holds whether one works in discrete or continuous time.

iv. It applies equally in models with and without a risk-free asset.

Thanks to its versatility, formula **(#1)** brings together multiple disparate results such as [2–4, 6, 7].

In discrete time the suggested computational strategy has an additional advantage in that the three quantities $L, V(1)$, and $\varepsilon^2(1)$ can be obtained by a simple backward recursion. For practical illustrations along these lines, see [1, Section 6].

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