Interplay between longevity and mortality risks

Joint or stand-alone pricing

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Longevity vs. Mortality risks

- Longevity risk: policyholders live **longer** than expected.
  $\longrightarrow$ bad for term annuity providers.
  $\longrightarrow$ good for term insurance providers.

- Mortality risk: policyholders live **shorter** than expected.
  $\longrightarrow$ bad for term insurance providers.
  $\longrightarrow$ good for term annuity providers.

Longevity vs. Mortality risks

- Exploiting the offsetting relationship through:

  1. Natural hedging (*risk reduction*)
  2. Joint pricing (*competitiveness*)

Longevity vs. Mortality risks

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Longevity vs. Mortality risks

- **Natural hedging**: ex-post
- **Joint pricing**: ex-ante

Natural hedging

- **Optimal product mix** under different settings (premium loadings, interest rate risk, basis risk, multi-period framework, **...**), which **reduces the portfolio's risk exposure** according to some meaningful risk measure**.**

  - *Gründel et al. (2006), JRI.*
  - *Tsai et al. (2010), IME.*
  - *Wang et al. (2010), JRI.*
  - *Wang et al. (2013), IME.*
  - *Cox et al. (2013), JRI.*
  - *Gatzert and Wesker (2014), JRI.*
  - *Li and Haberman (2015), IME.*
  - *Luciano et al. (2017), JRI.*
  - *Wong et al. (2017), JRI.*
  - *...*

Competitiveness

- *Empirical* research: supports the intuition (Cox and Lin, 2007).

Competitiveness

Bayraktar and Young (2007), IME:

- Prices of **pure endowments** and **term insurances** are lower when they are priced jointly.

  $\longrightarrow$ insight on each business line?
  $\longrightarrow$ challenges for the insurer?

- Premium loading is exogenous, and independent of portfolio composition.

  $\longrightarrow$ Insurer's exposure depends on the **relative contribution** of each business line in the overall risk.

Under what conditions joint pricing can be recommended?

Pricing models

- Business line A, with per-policy value V_A.
- Business line I, with per-policy value V_I.

Pricing models

- Business line A, with per-policy value V_A.
  - Term annuity contract.
- Business line I, with per-policy value V_I.
  - Term insurance contract.

Pricing models

General comments on the setting

1. V_A and V_I are negatively dependent.
2. The insurer charges a risk premium determined from the <u>standard deviation</u> risk measure:

$$\varphi[X] = \pi_X + \gamma\sigma_X.$$

3. $\sigma_I > \sigma_A$.
4. We stand at the time the products are launched.

Pricing models

Some simplifying assumptions

1. One age group in each business line, and same contract (benefit, maturity) for each group.
2. No market risk, i.e. $v(0, k) = v^k$.

Pricing models

Loaded premiums

- Business line A:

$$P_A = \pi_A + \Psi_A.$$

- Business line I:

$$P_I = \pi_I + \Psi_I.$$

Pricing models

Stand-alone pricing

- Stand-alone loaded premium P_A^{sa} for business line A:

Pricing models

Stand-alone pricing

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Pricing models

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  - Risk reduction: $\zeta \in [0, 1]$.

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  - $\varphi\left[V_A - P_A^{sa}\right] = (1 - \zeta)\,\varphi\left[V_A - \pi_A\right].$

Pricing models

Stand-alone pricing

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  - Insurer's risk **with** risk premium: $\varphi\left[V_A - P_A^{sa}\right]$.
  - Risk reduction: $\zeta \in [0, 1]$.
  - $\varphi\left[V_A - P_A^{sa}\right] = (1 - \zeta)\,\varphi\left[V_A - \pi_A\right].$
  - $P_A^{sa} = \pi_A + \zeta\gamma\sigma_A.$

Pricing models

Stand-alone pricing

- Stand-alone loaded premium P_I^{sa} for business line I:

$$\varphi\left[V_I - P_I^{sa}\right] = (1-\zeta)\,\varphi\left[V_I - \pi_I\right].$$

$$\longrightarrow P_I^{sa} = \pi_I + \zeta\gamma\sigma_I.$$

Pricing models

Joint pricing

- Number of policies in business line A is N_A.
- Number of policies in business line I is N_I.
- **Remark**: N_A and N_I are unknown when the loaded premium is set.

Pricing models
Joint pricing

- **Conditional** analysis of the required risk premium.
- Notation : $n = \frac{N_I}{N_A+N_I}$.

Pricing models
Joint pricing

- Loaded premium P_A^{nh} for business line A with joint pricing:

$$P_A^{nh} = \pi_A + \Psi_{ptf}(n).$$

- Loaded premium P_I^{nh} for business line I with joint pricing:

$$P_I^{nh} = \pi_I + \Psi_{ptf}(n).$$

Pricing models

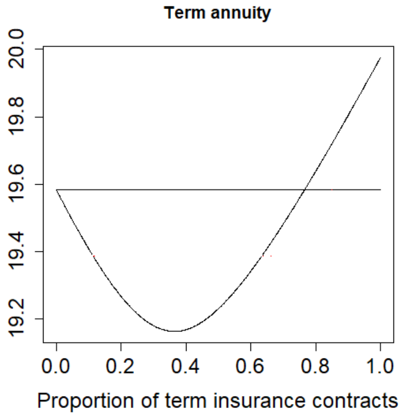
Joint pricing

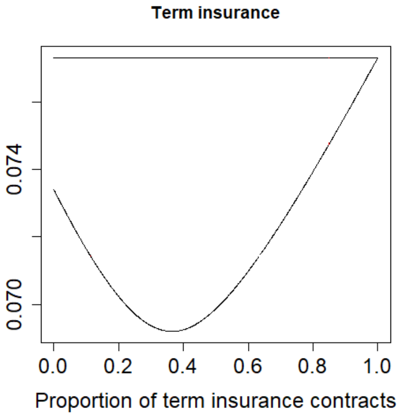
- Premium loading at portfolio level:

$$\Psi_{ptf}(n) = \zeta\gamma\sigma_{ptf}.$$

Conditional joint pricing

Required loaded premium per unit of benefit





Conditional joint pricing

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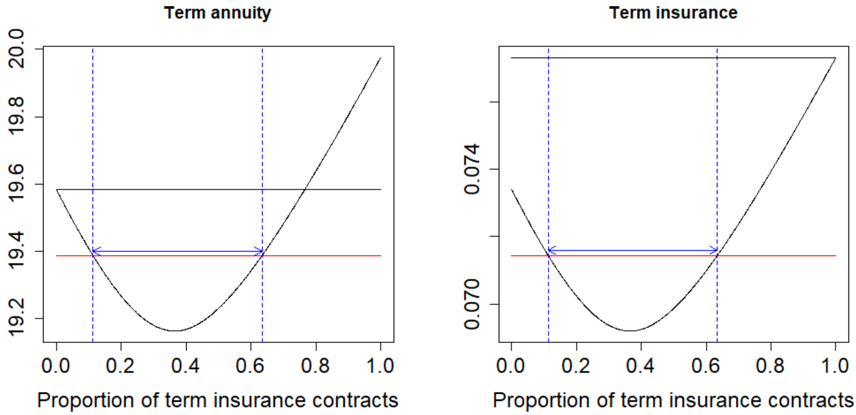


Conditional joint pricing

Summary

1. Competitiveness region.

2. Critical threshold.

3. Burden of portfolio monitoring.

Modeling the demand

- The competitive advantage depends on **portfolio composition**.
- Portfolio composition depends on the **competitive advantage**.

Modeling the demand

- Total number of policyholders: N_A^T and N_I^T.

- Total number of insurance companies: k_A and k_I.
  - One *joint pricer*.
  - $k_A - 1$ stand-alone pricers active on A.
  - $k_I - 1$ stand-alone pricers active on I.

Modeling the demand

- Number of policies sold by the joint pricer:

$$\begin{cases} N_A &= \frac{N_A^T}{k_A}\left(1 - \frac{k_A - 1}{k_A} q_A c_A^\star\right), \\ N_I &= \frac{N_I^T}{k_I}\left(1 - \frac{k_I - 1}{k_I} q_I c_I^\star\right), \end{cases}$$

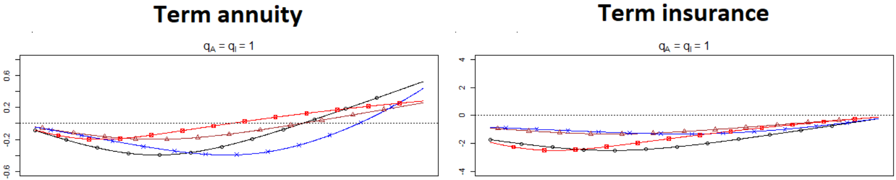
- $c_A^\star$ and $c_I^\star$ are given by:

$$\begin{cases} c_A^\star &= \frac{\pi_A + \Psi^\star}{P_A^{sa}} - 1, \\ c_I^\star &= \frac{\pi_I + \Psi^\star}{P_I^{sa}} - 1. \end{cases}$$

- q_A and q_I are the reaction factors.

Total collected premiums

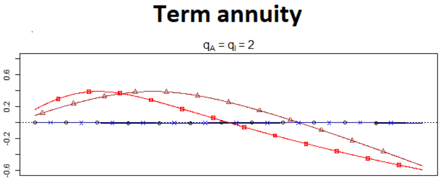
Low reaction

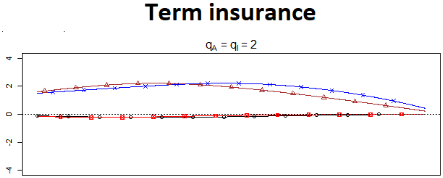


- $k_A = k_I = 2$
- $k_A = k_I = 4$
- $k_A = 2$ and $k_I = 4$
- $k_A = 4$ and $k_I = 2$

Total collected premiums

High reaction





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Total collected premiums

Business line I

- Jointly rather than separately, if:

$$q_I \geq \frac{k_I}{k_I - 1} \frac{P_I^{sa}}{\pi_I + \Psi^{min}}.$$

- Separately rather than jointly, if:

$$q_I \leq \frac{k_I}{k_I - 1}.$$

Total collected premiums

Business line A

- Jointly rather than separately, if:

$$\begin{cases} q_A \geq \frac{k_A}{k_A-1}\frac{P_A^{sa}}{\pi_A+\Psi^{min}}, & w^d < w^d_{ct}, \\ q_A \leq \frac{k_A}{k_A-1}\frac{P_A^{sa}}{\pi_A+\Psi^{max}}, & w^d > w^d_{ct}. \end{cases}$$

- Separately rather than jointly, if:

$$\begin{cases} q_A \leq \frac{k_A}{k_A-1} & w^d < w^d_{ct}, \\ q_A \geq \frac{k_A}{k_A-1} & w^d > w^d_{ct}. \end{cases}$$

Thank you.