Modeling Mortality of Multiple Populations with Vector Error Correction Models: Applications to Solvency II

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Introduction

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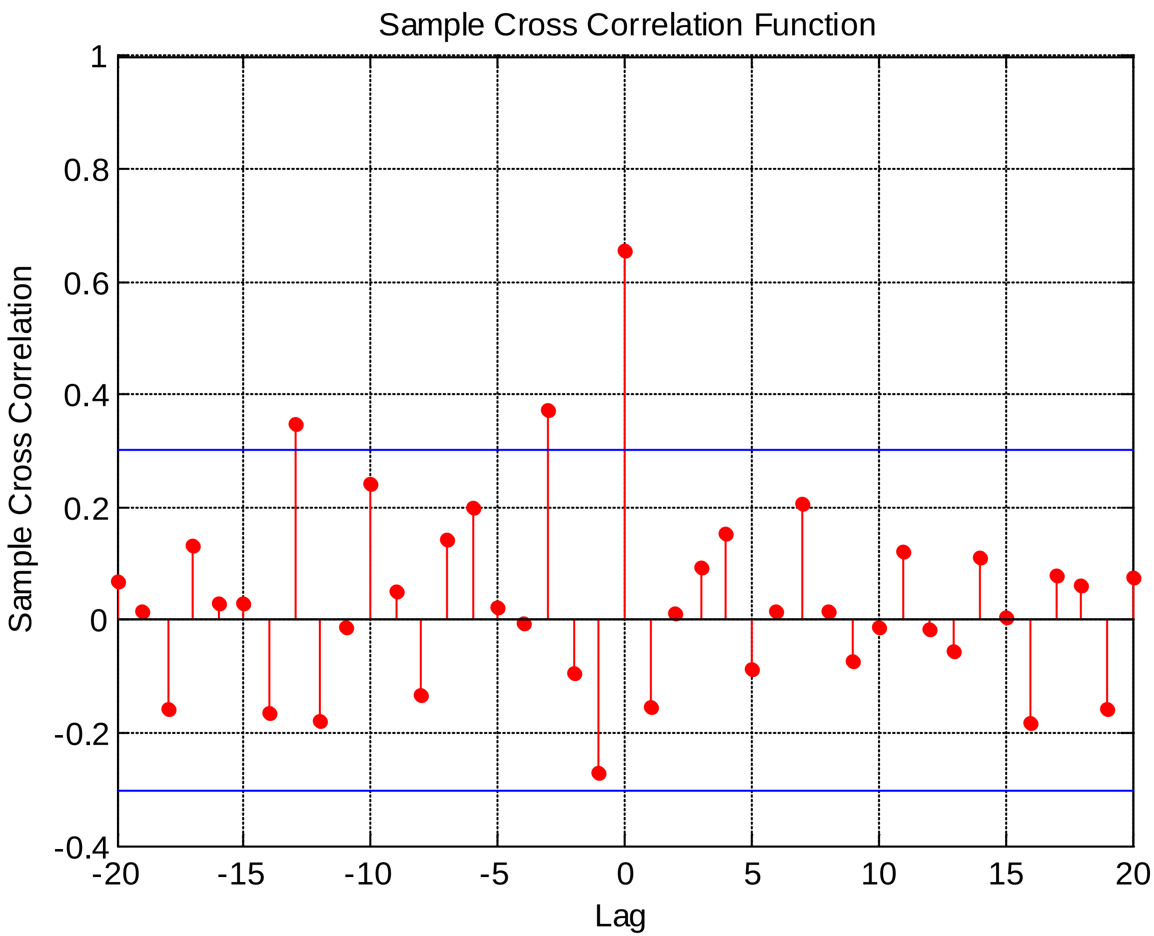
Multi-population mortality model

- Captures the interdependence of mortality dynamics between different populations
- Can be used for modeling
  - Males and females
  - Different national populations
  - A population and its subpopulation
- Necessary for standardized longevity risk hedge with the presence of population basis risk

EW & CMI males, year 1961-2005, age 60-85



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Our objectives

- Develop a model with a symmetric structure
- Evaluation and comparison
- Application to Solvency II capital requirements

Models and Fitting

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The Cairns' Framework

- Based on the Lee-Carter structure

$$ln(m_{x,t}^{(1)}) = \alpha_x^{(1)} + \beta_x \kappa_t^{(1)}$$
$$ln(m_{x,t}^{(2)}) = \alpha_x^{(2)} + \beta_x \kappa_t^{(2)}$$

- $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ do not diverge

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Modeling $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$

- RW+AR1 model (Cairns et al. 2011)
  - $\kappa_t^{(1)} = \kappa_{t-1}^{(1)} + \mu + \epsilon_t^{(1)}$
  - $\kappa_t^{(1)} - \kappa_t^{(2)} = \mu_\Delta + \phi(\kappa_{t-1}^{(1)} - \kappa_{t-1}^{(2)}) + \epsilon_t^{(2)}$

Modeling $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$

- Vector Autoregressive (VAR) model

$$\Delta\kappa_t^{(1)} = \phi_0 + \phi_1 \Delta\kappa_{t-1}^{(1)} + \phi_2 \Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(1)}$$

$$\Delta\kappa_t^{(2)} = \theta_0 + \theta_1 \Delta\kappa_{t-1}^{(1)} + \theta_2 \Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(2)}$$

- Constraint:
  - $\Delta\kappa_t^{(1)}$ and $\Delta\kappa_t^{(2)}$ converge to the same value over the long run, i.e.
  - $\frac{\phi_0}{1-\phi_1-\phi_2} = \frac{\theta_0}{1-\theta_1-\theta_2}$

Modeling $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$

- Vector Error Correction Model (VECM) with non-divergence constraint
  - $\Delta\kappa_t^{(1)} = \rho^{(1)}\left(a\kappa_{t-1}^{(1)} + b\kappa_{t-1}^{(2)} + c\right) + \phi_0 + \phi_1\Delta\kappa_{t-1}^{(1)} + \phi_2\Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(1)}$
  - $\Delta\kappa_t^{(2)} = \rho^{(2)}\left(a\kappa_{t-1}^{(1)} + b\kappa_{t-1}^{(2)} + c\right) + \theta_0 + \theta_1\Delta\kappa_{t-1}^{(1)} + \theta_2\Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(2)}$
  - Non-divergence constraint:
    - $a = -b$

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Model fitting

- A two-stage fitting method
  - Stage 1: Use maximum likelihood method to estimate Lee-Carter parameters.

    i.e. $\alpha_x^{(i)},\ \beta_x,\ \kappa_t^{(i)}$
  - Stage 2: Estimate the parameters in the stochastic process of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ using maximum likelihood method.

Model Comparison

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RW+AR1

VAR

VECM

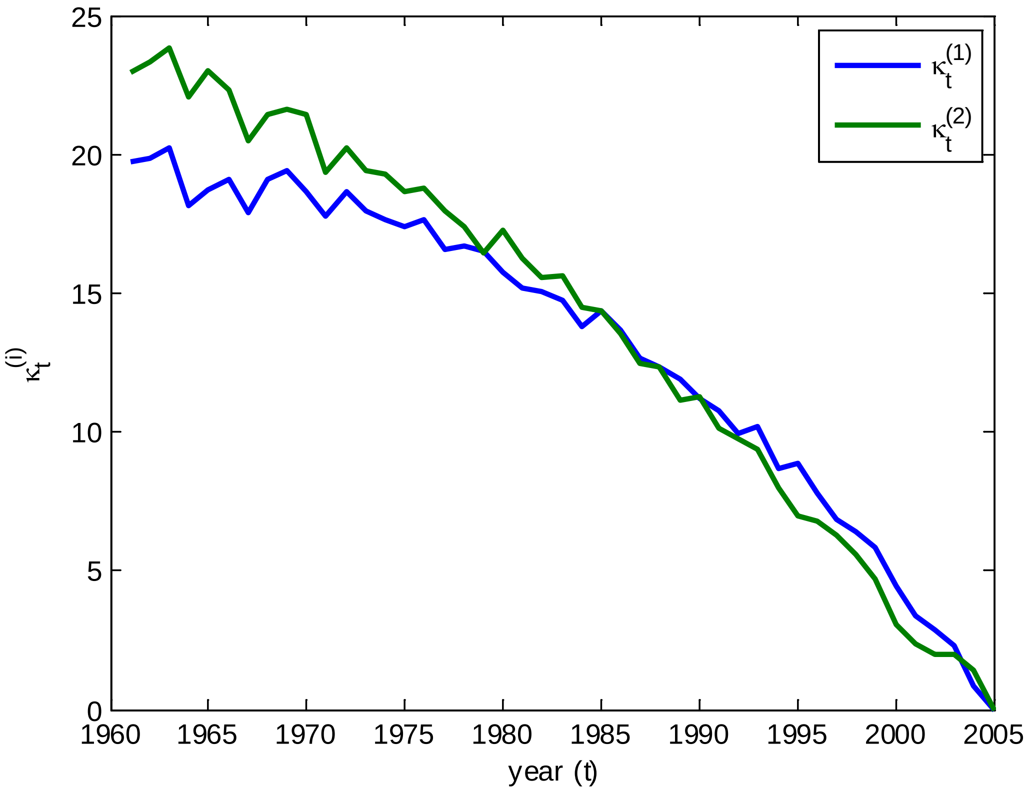
Comparison

- Comparison criteria:
  - Forecasting
  - Robustness
  - Goodness of fit

Data

- Population 1: EW males
- Population 2: CMI males
- Year: 1961 – 2005
- Age: 60 – 85

Kappas for two populations



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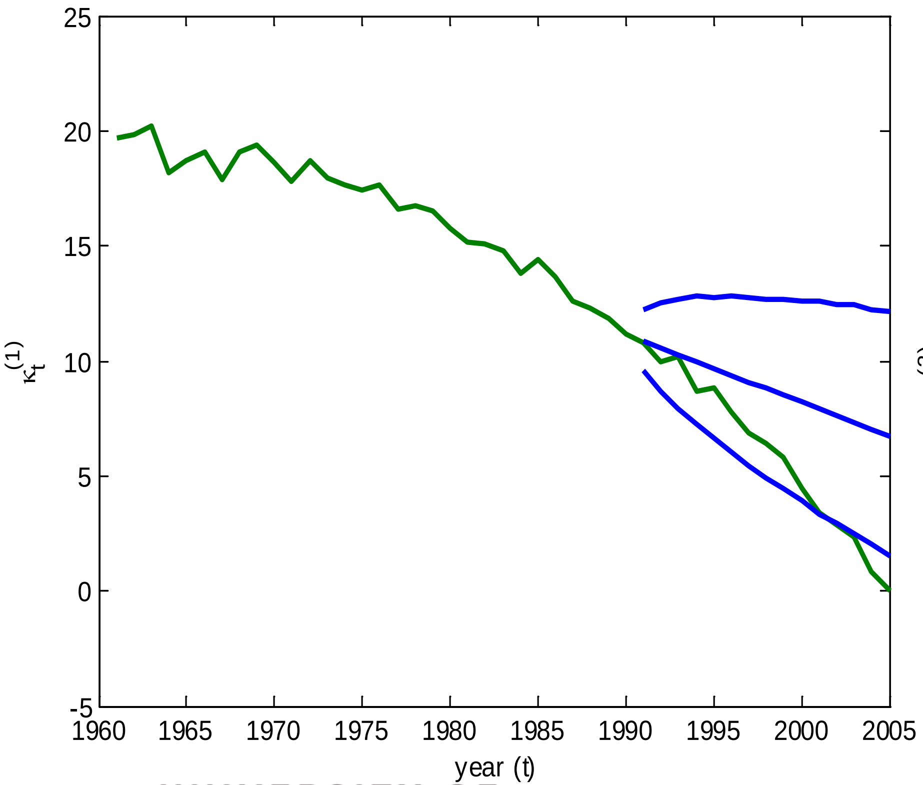
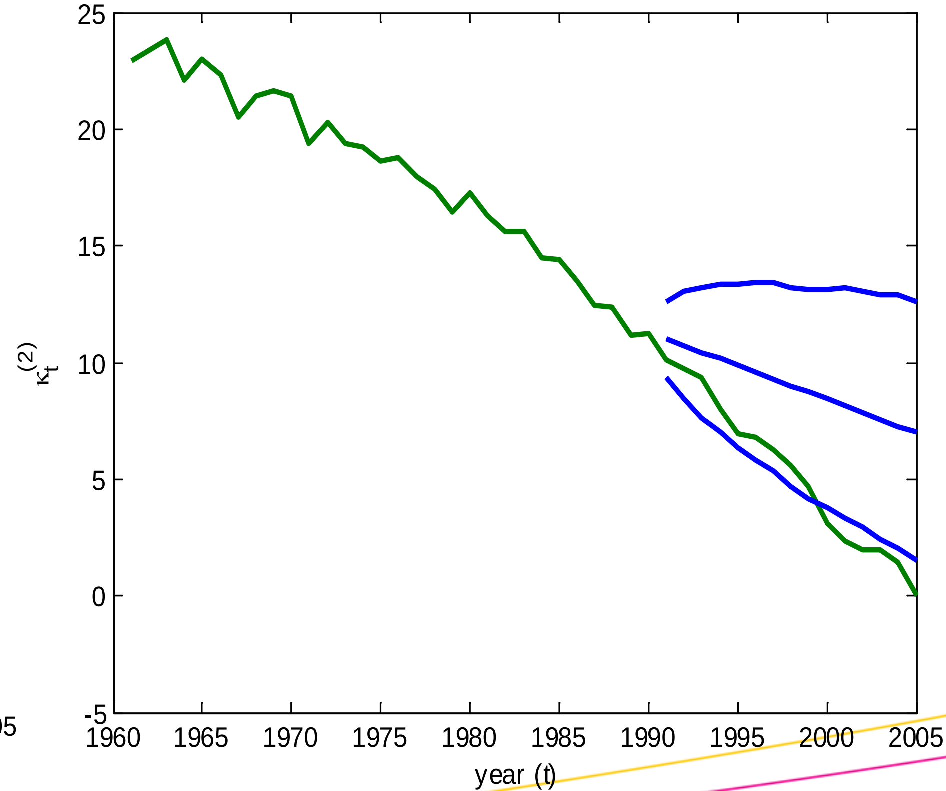
In-sample forecast RW+AR1

- Mean forecast and 95% confidence interval





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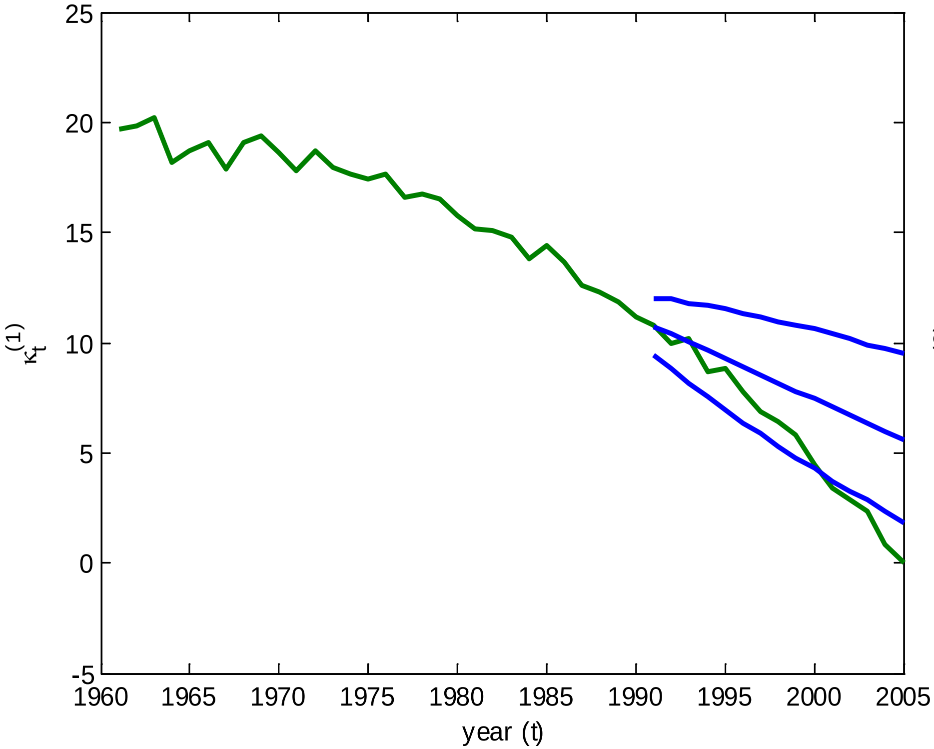
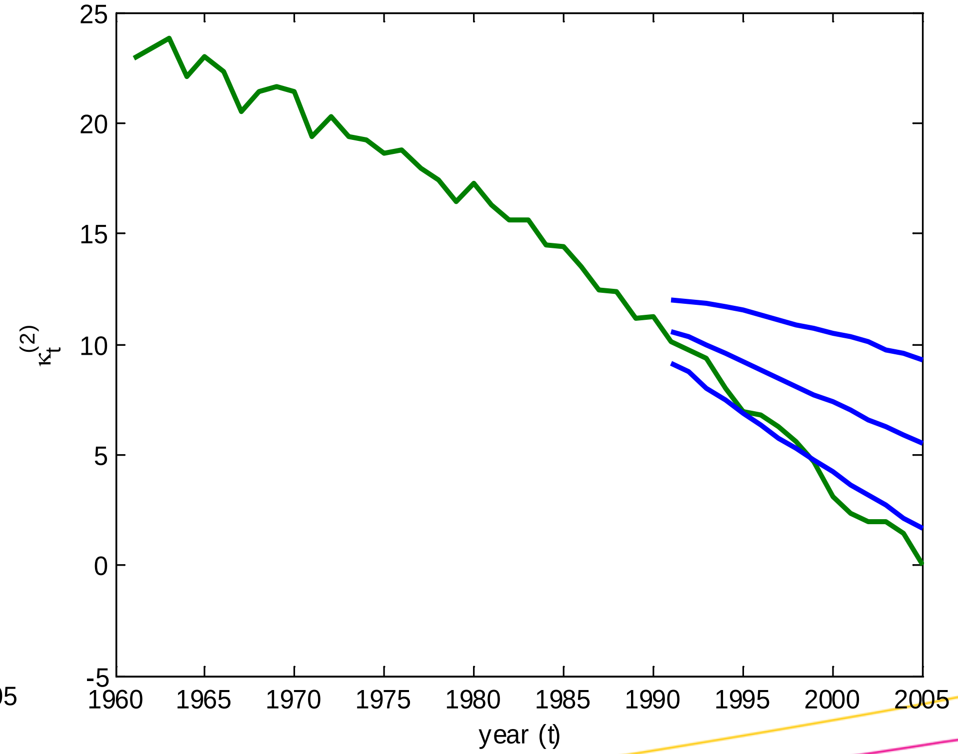
In-sample forecast
VAR

- Mean forecast and 95% confidence interval





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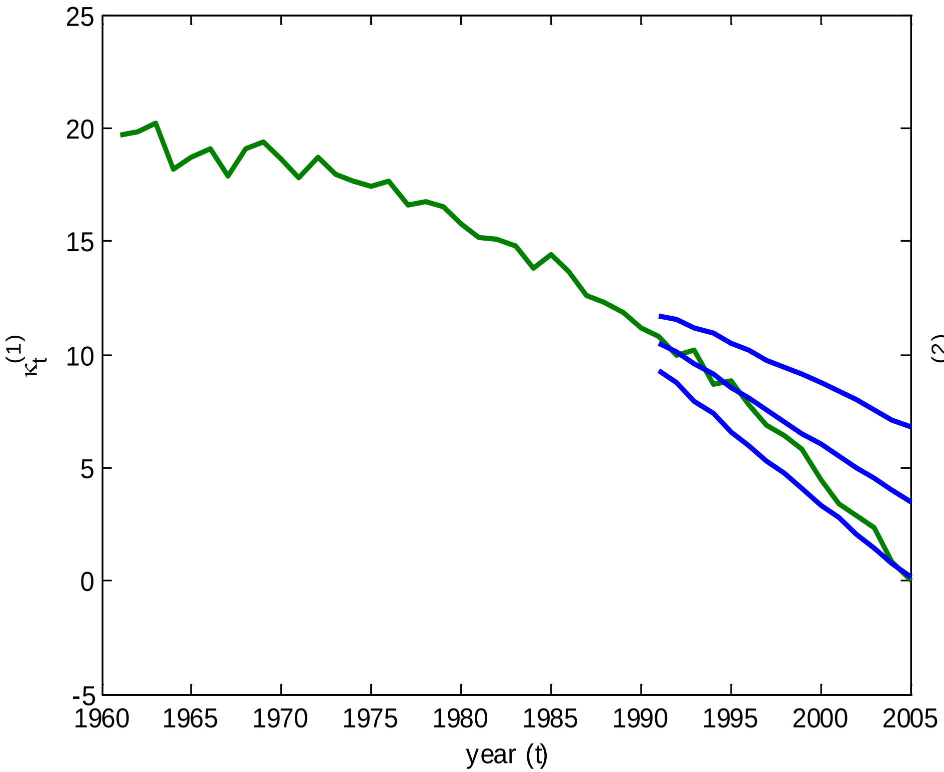
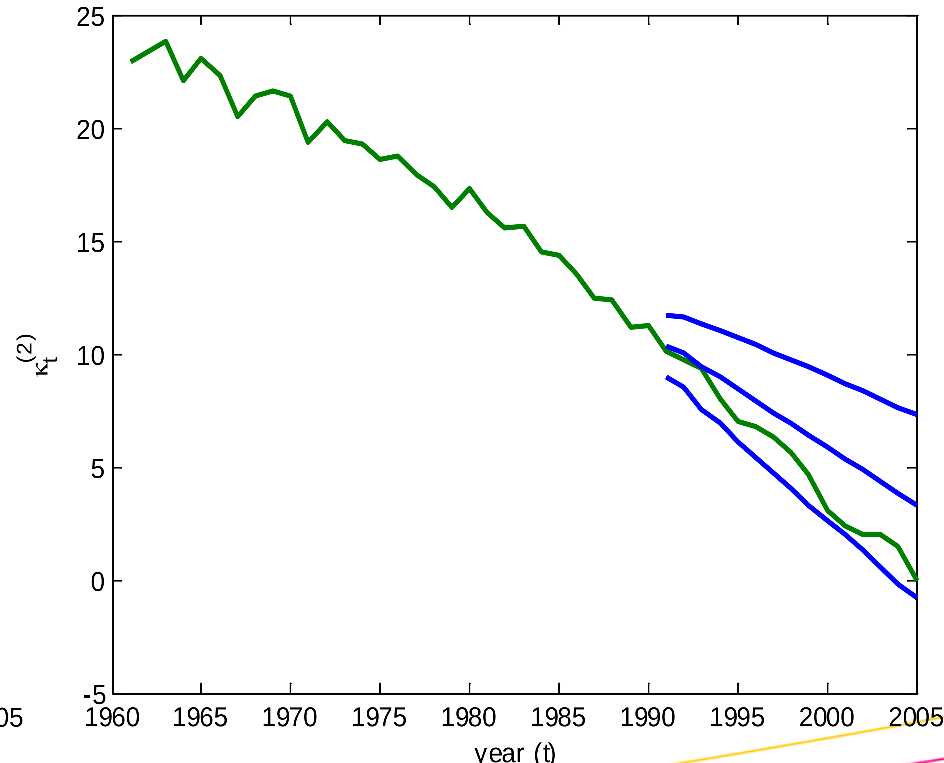
In-sample forecast VECM

- Mean forecast and 95% confidence interval





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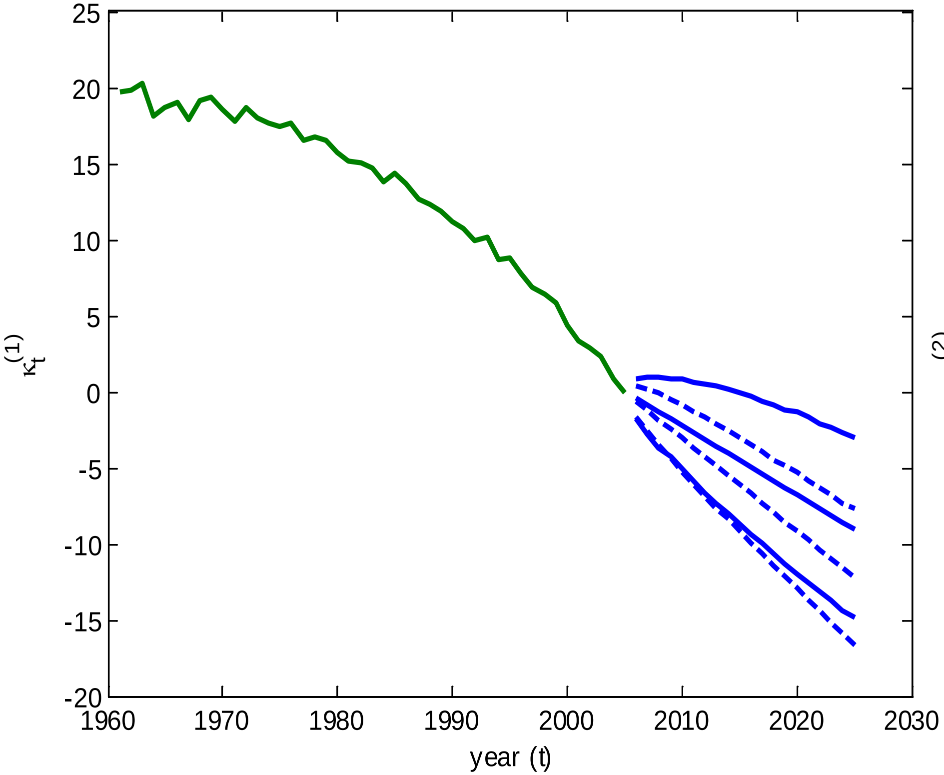
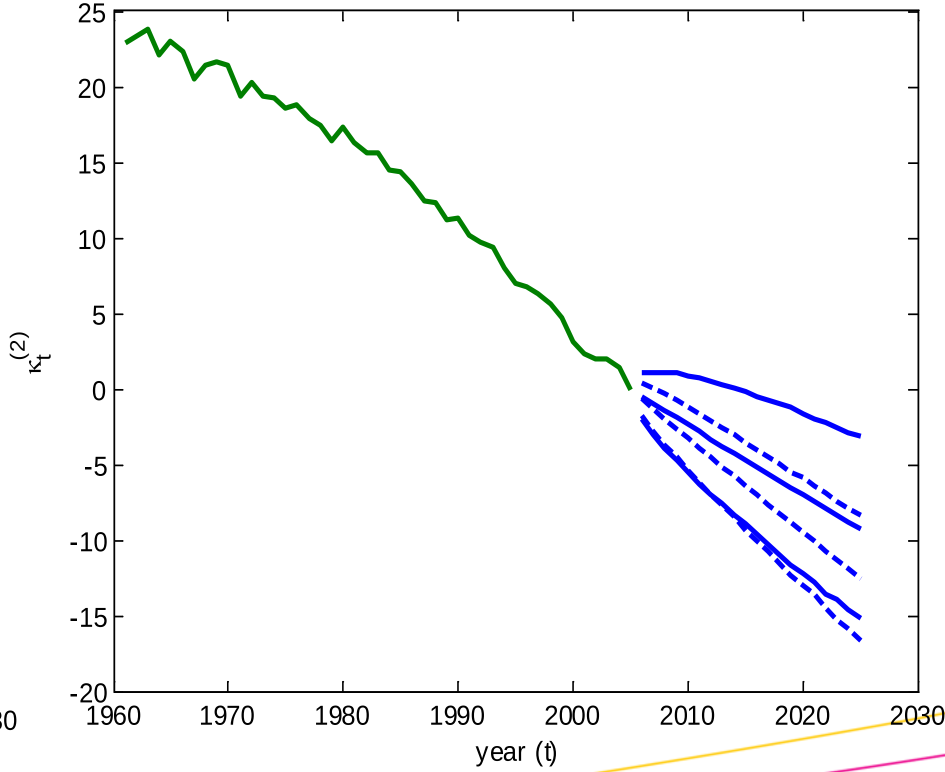
Robustness: Lookback window RW+AR1

- Solid line: 45 year window; Dashed line: 30 year window





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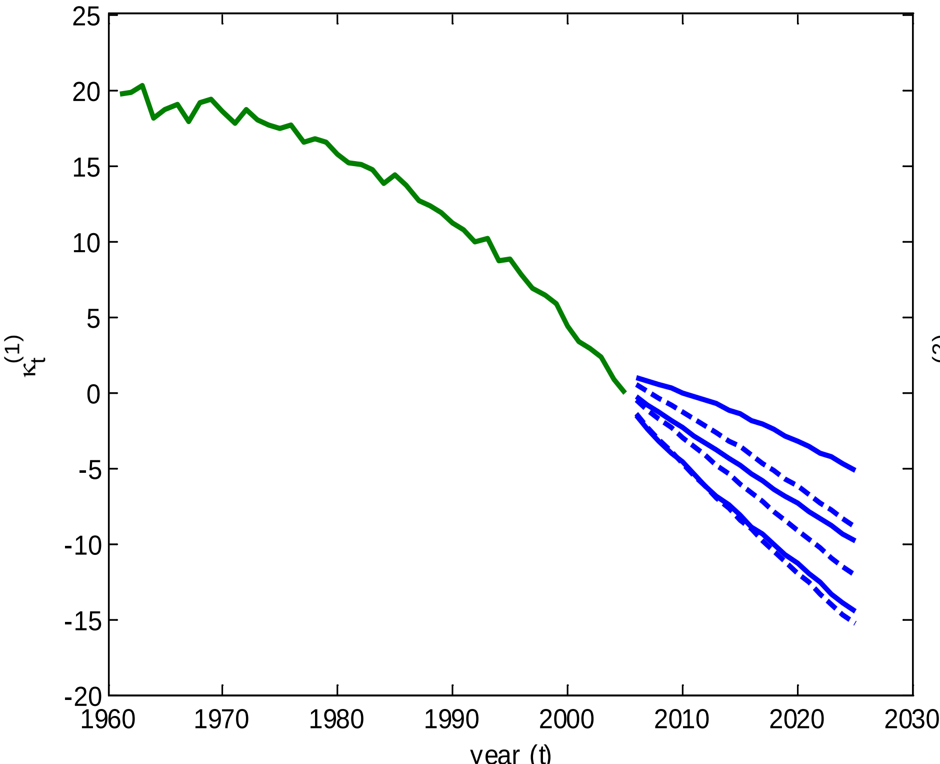
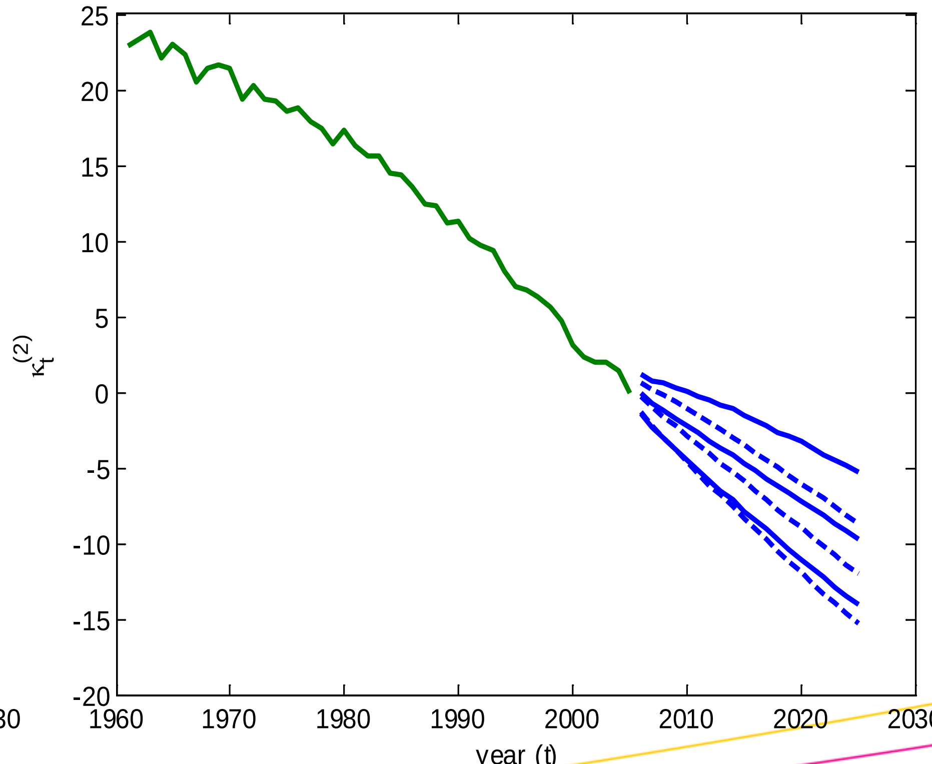
Robustness: Lookback window VAR

- Solid line: 45 year window; Dashed line: 30 year window





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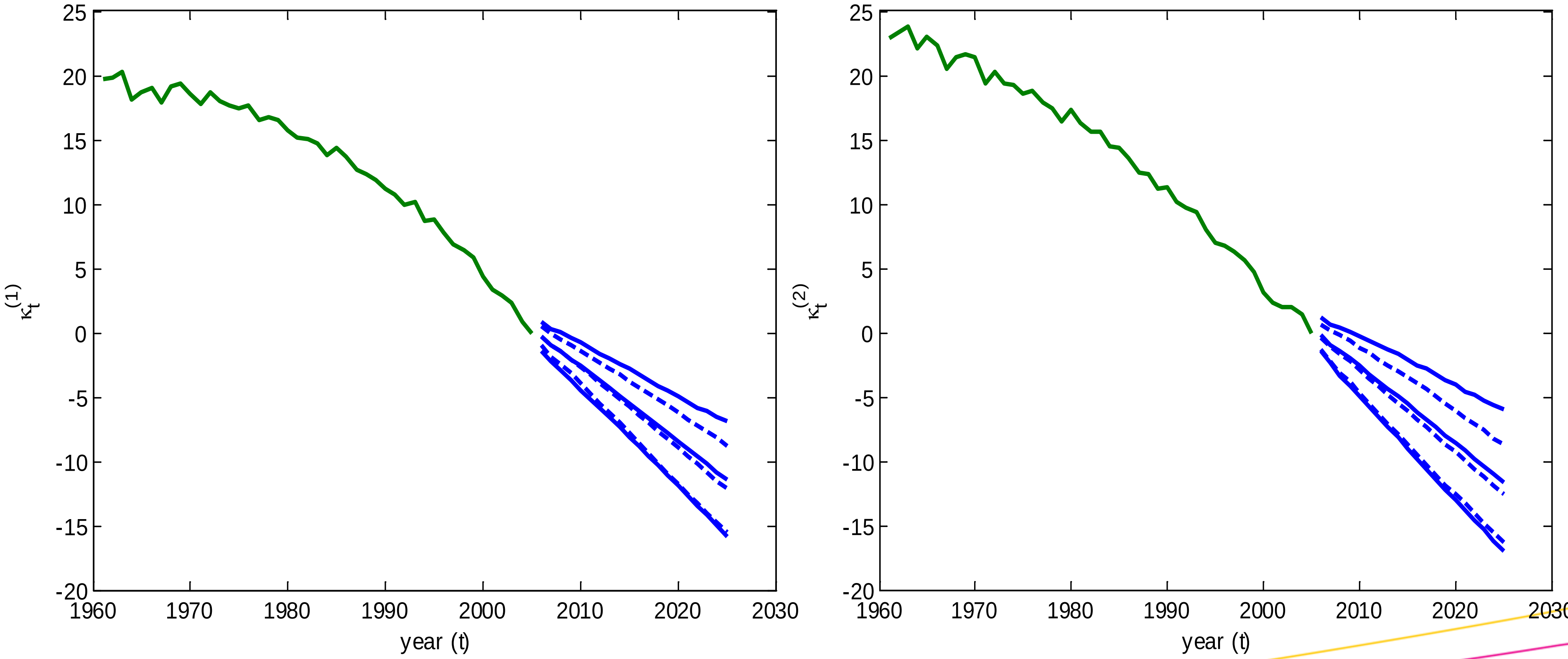
Robustness: Lookback window VECM

- Solid line: 45 year window; Dashed line: 30 year window



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Goodness of Fit

	RW+AR1	VAR	VECM
BIC	184.8614	181.5060	179.8904

Application of Multi-Population Models to a Longevity Swap

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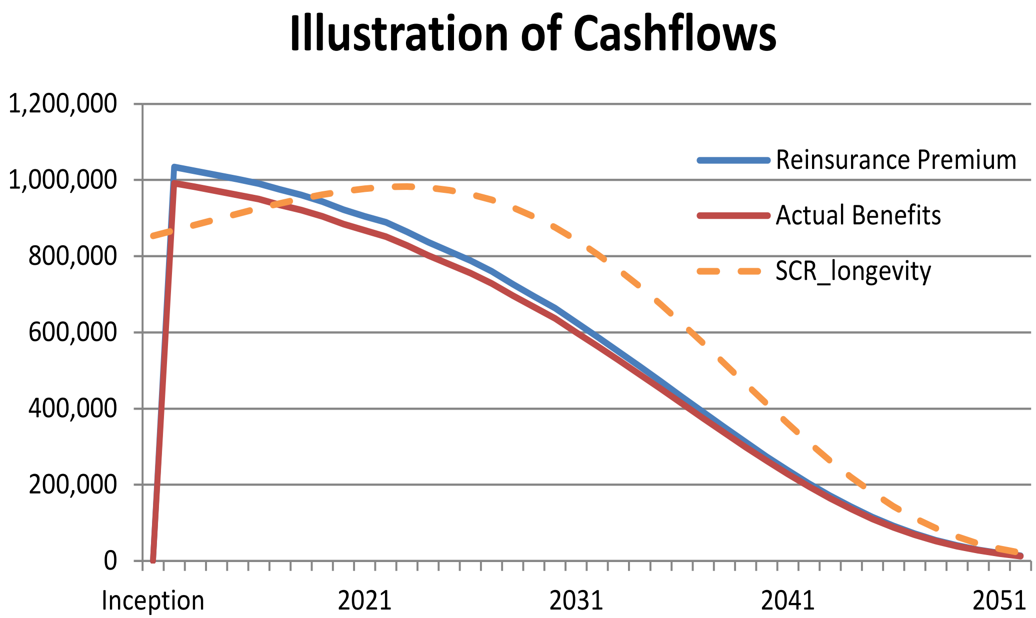
Multi-Population Applications

- Longevity Basis Risk
  - Apply the wrong trend:
    Pop2 improves faster than Pop1
  - Under-estimate uncertainty for Pop2
- Longevity Swap Reinsurance
  - Trade floating pensions/annuity benefits for fixed stream of reinsurance premiums
  - UK: reinsurance market lively
    ~ £10 bn transferred p.a.

Longevity Swap Reinsurance

- Reinsurer Risk Charge
- Solvency Capital
  - Creates frictional cost



Simplifications:

- BE assumptions
- Males aged 65 years
- No operational risk
- No diversification
- Interest rate 1.75%

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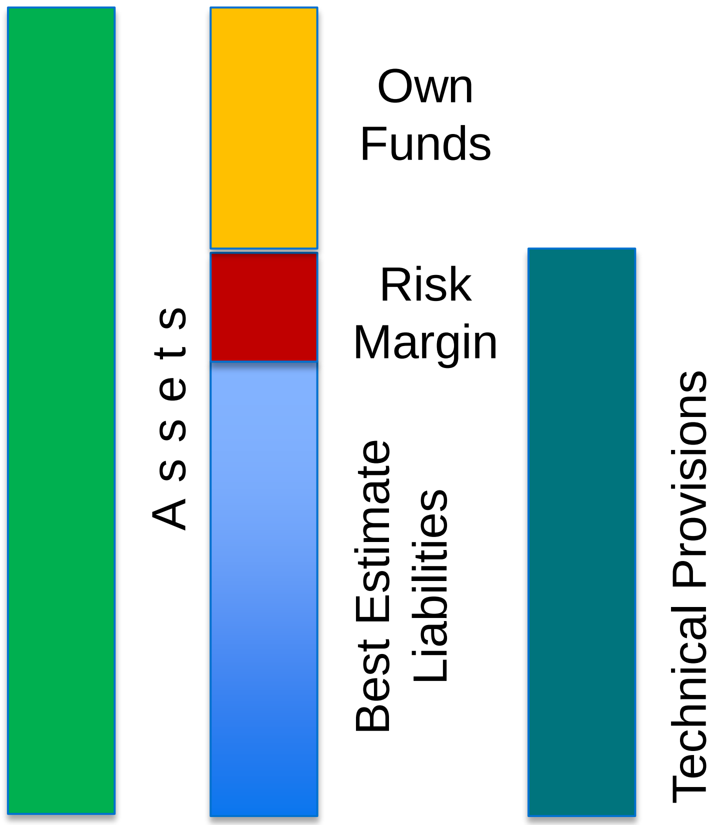
Solvency 2



Own Funds > SCR

TP = BEL + Risk Margin
= Exit Value
= Tradable Price

Risk Margin for comparison

$$RM = 6\% \times \sum_t SCR_t (1+i)^{-t}$$

$$SCR_0 = \Delta NAV_1$$

$$SCR_{t>0} \propto BEL_{t>0}$$

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Model Comparison

Standard Model (-20%q_x)

Model	Population	PV Benefits	Risk Margin	Risk Charge
RW + AR1	Pop1	14,550,749	1,436,721	9.87%
VAR	Pop1	14,626,814	1,467,333	10.03%
VECM	Pop1	14,849,150	1,505,363	10.14%
RW + AR1	Pop2*	16,884,199	1,728,923	10.24%
VAR	Pop2*	16,968,380	1,773,930	10.45%
VECM	Pop2*	17,229,135	1,834,734	10.65%
RW + AR1	Pop2	17,251,537	2,049,079	11.88%
VAR	Pop2	16,958,946	1,747,221	10.30%
VECM	Pop2	17,286,347	1,870,938	10.82%

Basis Risk!

164bps

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Model Comparison

Internal Model (99.5% VAR)

Model	Population	PV Benefits	Risk Margin	Risk Charge
RW + AR1	Pop1	14,550,749	1,078,475	7.41%
VAR	Pop1	14,626,814	779,691	5.33%
VECM	Pop1	14,849,150	690,203	4.65%
RW + AR1	Pop2*	16,884,199	1,252,725	7.42%
VAR	Pop2*	16,968,380	897,747	5.29%
VECM	Pop2*	17,229,135	881,035	**5.11%**
RW + AR1	Pop2	17,251,537	578,957	3.36%
VAR	Pop2	16,958,946	1,017,929	6.00%
VECM	Pop2	17,286,347	1,270,306	**7.35%**

Basis Risk!

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224bps

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