Mortality Risk Pooling Arrangements Using Multi-state Models of Functional Disability and Health Status

Doreen Kabuche[1,2], Michael Sherris[1,2], Andrés M. Villegas[1,2] and Jonathan Ziveyi[1,2]

[1]School of Risk and Actuarial Studies, UNSW Sydney
[2]ARC Centre of Excellence in Population Ageing Research (CEPAR)

September 13, 2022

Outline

Outline

Background

- Multi-state models for pricing LTC policies and annuity products.
- Individual's (healthy) life expectancy is determined by many variables: age, gender, health and functional disability statuses etc.
- For example, mortality and disability risks increase with age.
- The transition rates to and from disability depend on an individual's health status, such as chronic illness [Sherris and Wei, 2021].

Previous Literature

- Many studies propose multi-state models for disability: [Olivieri and Pitacco, 2001, Rickayzen and Walsh, 2002, Leung, 2006, Fong et al., 2015] etc.
- Some models analyse the dynamics of functional disability and health status [Sherris and Wei, 2021].
- Other models incorporate systematic trends and uncertainties [Li et al., 2017, Sherris and Wei, 2021, Fu et al., 2021].

Literature Gaps

- No previous literature on mortality sharing (mutual insurance) designs across functional disability states and chronic illness status.
- Prior research mostly focuses on functional disability, do not fully separate health status from disability [Hieber and Lucas, 2020, Chen et al., 2021].
- Disability models ignore recovery probabilities, trends and uncertainties in LTC and mortality risks.

Motivation

Research Goal

- Design a framework for pooling different health and mortality risks: 'pooled health care annuity product'.

Why mortality risk sharing in a multi-state setting?

- Pooling heterogeneous lives is attractive to individuals in good and poor health, less adverse selection costs [Valdez et al., 2006].
- Mutual risk sharing products e.g pooled annuities and tontines provide enhanced annuity benefits through mortality and morbidity credits [Piggott et al., 2005, Qiao and Sherris, 2013].

Outline

Multi-state Model Setup

- Present a framework for pooling mortality risk using multi-state models of functional disability and health status [Fu et al., 2021, Sherris and Wei, 2021].
- Simulate transition rates for calculating healthy (life) expectancy and the annuity values for the elderly.
- We use a matrix approach to provide a general structure for setting up a pooled annuity product even for complex multi-state models.
- We retrieve the fund dynamics using the matrix-oriented technique based on Thiele type theorems [Hoem, 1969, Bladt et al., 2020].

Multi-state Model Setup

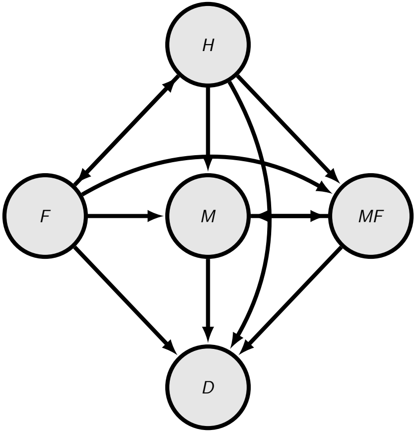

Figure 1: *A Five-State Transition Model*

Multi-state Pooling

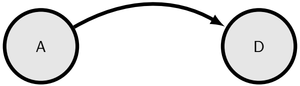

Figure 2: *A Two-State Transition Model*

$$F_0 = l_x^* \ddot{a}_x B_0,$$

$$F_1 = (F_0 - l_x^* B_0) * (1 + R_1),$$

$$B_1 = \frac{F_1}{l_{x+1}^* \ddot{a}_{x+1}},$$

$$B_{t+1} = B_t * \left(\frac{1 + R_t}{1 + r}\right) \left(\frac{p_{x+t}}{p_{x+t}^*}\right).$$

Multi-state Pooling

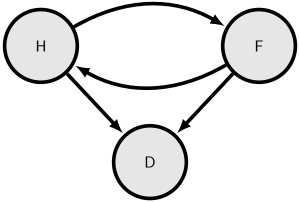

Figure 3: *A Three-State Model with Recovery*

$$F_0^h = l_x^{h^*} \left(a_x^{hh} B_0^h + a_x^{hf} B_0^f \right),$$
$$F_0^f = l_x^{f^*} \left(a_x^{fh} B_0^h + a_x^{ff} B_0^f \right),$$

$$\begin{bmatrix} F_0^h \\ F_0^f \end{bmatrix} = \begin{bmatrix} l_x^{h^*} \\ l_x^{f^*} \end{bmatrix} \otimes \left(\begin{bmatrix} a_x^{hh} & a_x^{hf} \\ a_x^{fh} & a_x^{ff} \end{bmatrix} \begin{bmatrix} B_0^h \\ B_0^f \end{bmatrix} \right),$$

$$\mathbf{F}_0 = \mathbf{L}_x \otimes (\mathbf{A}_x \mathbf{B}_0),$$

$$\mathbf{F}_t = \mathbf{L}_{x+t} \otimes (\mathbf{A}_{x+t} \mathbf{B}_t).$$

Multi-state Pooling

$$\mathbf{F}_{t+1} = \mathbf{L}_{x+t+1} \otimes \left(\overset{\text{Mortality/morbidity credits}}{\underset{\uparrow}{}} (\mathbf{P}^*_{x+t})^{-1} \left(\frac{\mathbf{F}_t - \mathbf{L}_{x+t} \otimes \mathbf{B}_t}{\mathbf{L}_{x+t}} \right) \overset{\text{Financial credits}}{\underset{\uparrow}{}} (1 + R_t) \right),$$

$$\mathbf{F}_{t+1} = \mathbf{L}_{x+t+1} \otimes (\mathbf{A}_{x+t+1} \mathbf{B}_{t+1}), \tag{1}$$

$$\mathbf{P}^*_{x+t} = \begin{bmatrix} p^{hh*}_{x+t} & p^{hf*}_{x+t} \\ p^{fh*}_{x+t} & p^{ff*}_{x+t} \end{bmatrix},$$

$$\mathbf{L}_{x+t+1} = \begin{bmatrix} l^{h*}_{x+t+1} \\ l^{f*}_{x+t+1} \end{bmatrix}.$$

Outline

Numerical Illustration

- Estimate the models based on the HRS survey data.
- The initial pool size is set to 1000 and according to the HRS interview respondents in 2017 we divide
  - Three state - 920 healthy males and 80 disabled 65-old males.
  - Five state - 650 good health not functionally disabled, 250 ill health not functionally disabled, 40 good health and functionally disabled and 60 ill health and functionally disabled.
- Set $B_0^h = \$12,000$ and $B_0^f = \$36,000$ per year at the start for the three-state model.
- Set $B_0^h = \$12,000$, $B_0^m = \$24,000$, $B_0^f = \$36,000$ and $B_0^{mf} = \$48,000$ per year at the start for the five-state model.
- Idiosyncratic risk: Multinomial distribution 1000 paths.
- Annual pricing rate: $r = 3\%$.

Estimated Transition Probabilities

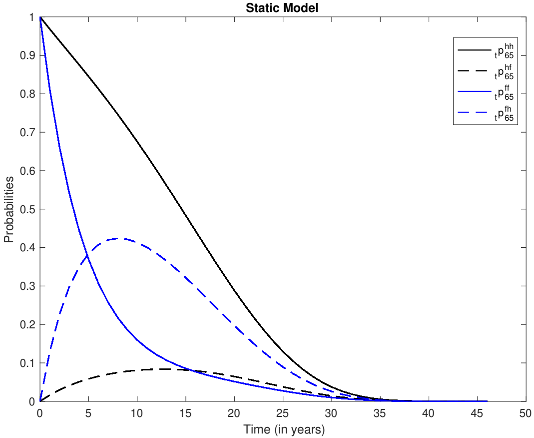

Figure 4: t-year fitted transition probabilities of a 65 year old male from the static model based on the three-state functional disability model.

Estimated Transition Probabilities

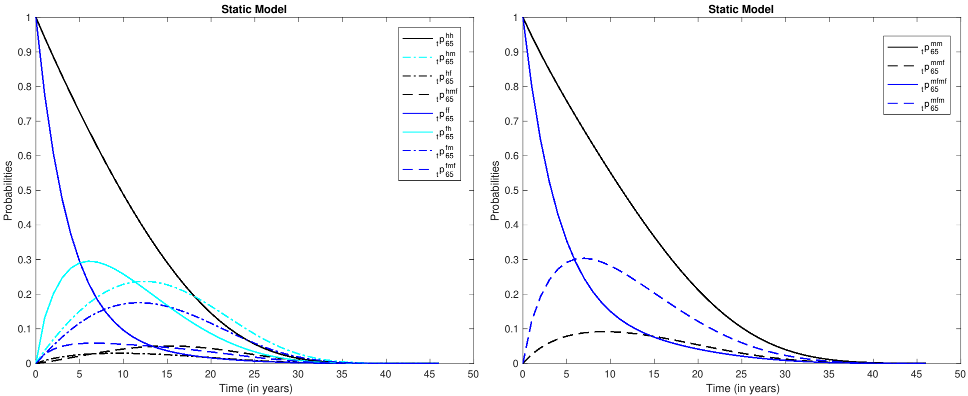

Figure 5: t-year fitted transition probabilities of a 65 year old male from the static model based on the five-state functional disability and health status model. The left panel for individuals in good health, while the right panel for individuals in ill health.

Main Results

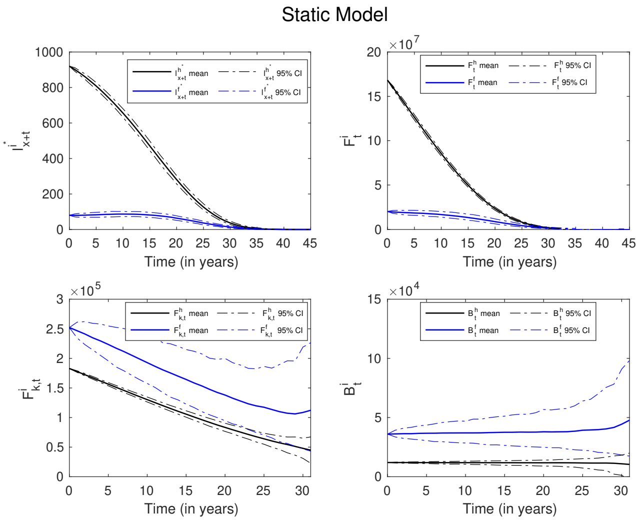

Figure 6: The static model's number of survivors, pool fund values, individual fund value and annuity benefits for healthy and disabled pool participants.

Main Results

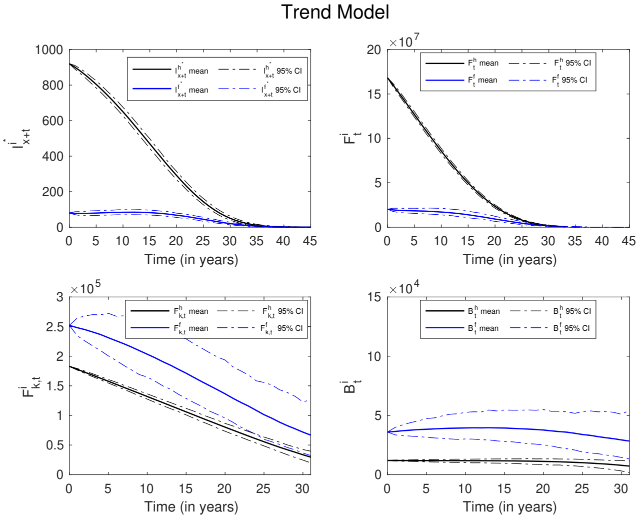

Figure 7: The trend model's number of survivors, pool fund value, individual fund values and annuity benefits for healthy and disabled pool participants.

Main Results

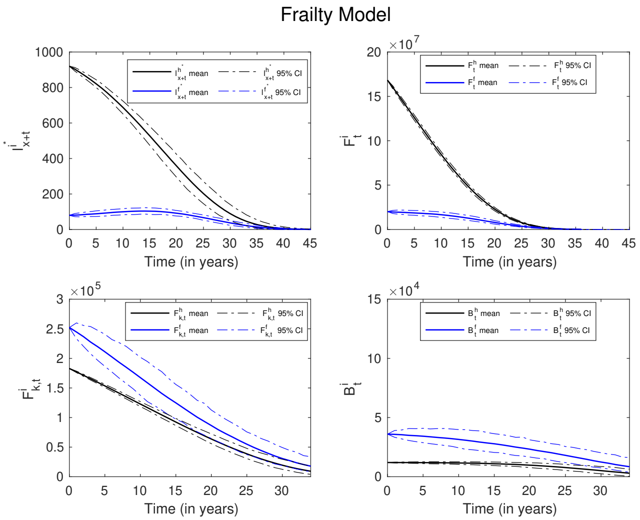

Figure 8: The frailty model's number of survivors, pool fund value, individual fund values and annuity benefits for healthy and disabled pool participants.

Main Results

Table 1: Pooled Annuity Payouts without Systematic Trends and Uncertainty using the Functional Disability Model.

		Age 75			Age 95		
Annuity Payout ($)		5%	Mean	95%	5%	Mean	95%
Static	Healthy	10,508	11,921	13,094	2,925	11,208	18,648
	Disabled	28,258	36,710	47,172	18,400	42,563	88,704
Trend	Healthy	10,155	11,802	13,066	3,094	7,870	11,945
	Disabled	30,153	39,440	52,618	14,496	29,296	50,915
Frailty	Healthy	10,337	11,619	12,699	2,442	5,189	8,764
	Disabled	24,502	31,463	40,253	7,203	12,219	19,475

Main Results

Table 2: Pooled Annuity Payouts with Systematic Trends and Uncertainty using the Functional Disability Model.

		Age 75			Age 85		
Annuity Payout ($)		5%	Mean	95%	5%	Mean	95%
Static	Healthy	10,104	11,870	13,780	6,127	10,993	15,441
	Ill	20,636	23,818	26,866	17,884	23,563	28,591
	Disabled	18,819	38,565	63,829	9,022	45,750	96,764
	Ill and Disabled	35,054	49,032	66,469	31,556	51,232	83,253
Trend	Healthy	9,905	11,858	13,459	5,199	10,274	14,401
	Ill	20,447	24,143	27,656	16,448	22,060	26,967
	Disabled	19,198	37,894	63,559	7,807	48,257	108,647
	Ill and Disabled	35,899	54,268	76,040	30,307	50,129	79,339

Main Results

Table 3: Pooled Annuity Payouts without Systematic Trend and Uncertainty using the Functional Disability and Health Status Model.

		Age 75			Age 85		
Annuity Payout ($)		5%	Mean	95%	5%	Mean	95%
Static	Healthy	10,104	11,870	13,780	6,127	10,993	15,441
	Ill	20,636	23,818	26,866	17,884	23,563	28,591
	Disabled	18,819	38,565	63,829	9,022	45,750	96,764
	Ill and Disabled	35,054	49,032	66,469	31,556	51,232	83,253
Trend	Healthy	9,905	11,858	13,459	5,199	10,274	14,401
	Ill	20,447	24,143	27,656	16,448	22,060	26,967
	Disabled	19,198	37,894	63,559	7,807	48,257	108,647
	Ill and Disabled	35,899	54,268	76,040	30,307	50,129	79,339

Main Results

Table 4: Pooled Annuity Payouts without Systematic Trends and Uncertainty using the Functional Disability and Health Status Model.

		Age 75			Age 80		
Annuity Payout ($)		5%	Mean	95%	5%	Mean	95%
Frailty	Healthy	8,288	10,572	12,555	4,944	9,243	12,734
	Ill	20,265	22,368	24,292	17,210	20,163	22,580
	Disabled	23,487	40,525	61,469	20,791	42,676	73,551
	Ill and Disabled	29,115	39,688	50,545	21,958	33,643	49,840

Main Results

Table 5: Pooled Annuity Payouts with Systematic Trend and Uncertainty using the Functional Disability and Health Status Model.

Annuity Payout ($)		Age 75 5%	Age 75 Mean	Age 75 95%	Age 85 5%	Age 85 Mean	Age 85 95%
Trend	Healthy	9,823	11,728	13,374	5,861	11,082	16,048
	Ill	20,358	23,881	27,253	16,809	23,453	29,056
	Disabled	19,140	38,380	63,526	3,085	45,167	105,342
	Ill and Disabled	32,917	50,190	70,841	31,449	53,018	83,956
Frailty	Healthy	9,008	11,786	14,165	3,577	10,848	17,615
	Ill	21146	23914	26488	19,548	23,906	27,586
	Disabled	19,164	37,300	62,011	7,118	44,600	104,832
	Ill and Disabled	36,605	49,191	65,939	31,605	50,064	76,340

Outline

Conclusion

- Pooling across heterogeneous individuals improves the expected retirement income, especially for those in poor health due to increased mortality and morbidity credits.
- Ignoring systematic trends and uncertainty reduces both groups' annuity benefits at older ages, a significant decreasing trend for healthy.
- Changes in the pool size is another main factor affecting the pooled annuity benefits.
- For smaller pool sizes, the variation in the benefit payments at future ages is very wide.

Research Contribution and Future Work

- Theoretical contribution: we propose a framework for sharing mortality risk across multiple states (multi-state pooling).
- Practical contribution: we propose a pooled health care annuity, a product that combines pooled life annuity and LTC insurance, considering functional disability and health status.
- Future work: incorporate equity funding.

Thanks! Questions/comments?

References I

Bladt, M., Asmussen, S., and Steffensen, M. (2020).
Matrix representations of life insurance payments.
European Actuarial Journal, 10(1):29–67.

Chen, A., Chen, Y., and Xu, X. (2021).
Care-dependent tontines.
Available at SSRN 3880661.

Fong, J. H., Shao, A. W., and Sherris, M. (2015).
Multistate actuarial models of functional disability.
North American Actuarial Journal, 19(1):41–59.

References II

Fu, Y., Sherris, M., and Xu, M. (2021).
Functional disability with systematic trends and uncertainty: A comparison between china and the us.
Available at SSRN 3785743.

Hieber, P. and Lucas, N. (2020).
Life-care tontines.
Available at SSRN 3688386.

Hoem, J. M. (1969).
Markov chain models in life insurance.
Blätter der DGVFM, 9(2):91–107.

References III

Leung, E. (2006).
A multiple state model for pricing and reserving private long term care insurance contracts in australia.
Australian Actuarial Journal, 12(2):187–247.

Li, Z., Shao, A. W., and Sherris, M. (2017).
The impact of systematic trend and uncertainty on mortality and disability in a multistate latent factor model for transition rates.
North American Actuarial Journal, 21(4):594–610.

Olivieri, A. and Pitacco, E. (2001).
Facing ltc risks.
In *Proceedings of the 32nd astin colloquium, Washington*.

References IV

- Piggott, J., Valdez, E. A., and Detzel, B. (2005).
  The simple analytics of a pooled annuity fund.
  Journal of Risk and Insurance, 72(3):497–520.

- Qiao, C. and Sherris, M. (2013).
  Managing systematic mortality risk with group self-pooling and annuitization schemes.
  Journal of Risk and Insurance, 80(4):949–974.

- Rickayzen, B. D. and Walsh, D. E. (2002).
  A multi-state model of disability for the united kingdom: implications for future need for long-term care for the elderly.
  British Actuarial Journal, pages 341–393.

References V

Sherris, M. and Wei, P. (2021).
A multi-state model of functional disability and health status in the presence of systematic trend and uncertainty.
North American Actuarial Journal, 25(1):17–39.

Valdez, E. A., Piggott, J., and Wang, L. (2006).
Demand and adverse selection in a pooled annuity fund.
Insurance: Mathematics and Economics, 39(2):251–266.

Appendix

Model Estimation

- The multi-state latent factor intensity model proposed in Li et al. (2017) to estimate the transition rates.
- The transition intensity for transition type s for an individual j at time t is assumed to be of the form

$$\lambda_{j,s}(t) = \exp\left(\beta_s + \gamma_s^{'} w_j(t) + \alpha_s \psi(t)\right). \tag{2}$$

- β_s: baseline log-intensity for transition type s
- $w_j(t)$: vector with the observed predictors for individual j
- $\psi(t)$: stochastic latent process for systematic uncertainties
- γ_s and α_s: measure sensitivities of logarithm of $\lambda_{j,s}(t)$ w.r.t $w_j(t)$, $\psi(t)$

Appendix

1. Static model: the transition rate $\lambda_{j,s}(t)$ is assumed to be dependent on age and gender only

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_s^{\text{age}} x_j(t) + \gamma_s^{\text{female}} G_j. \tag{3}$$

2. Trend model: the systematic time trend/ linear time index is included

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_s^{\text{age}} x_j(t) + \gamma_s^{\text{female}} G_j + \gamma_s^{\text{time}} t. \tag{4}$$

3. Frailty model: time trend and the latent factor $\psi(t)$ are included, to account for systematic uncertainty

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_s^{\text{age}} x_j(t) + \gamma_s^{\text{female}} G_j + \gamma_s^{\text{time}} t + \alpha_s \psi(t), \tag{5}$$

α_s measures the the sensitivity of $\ln(\lambda_{j,s}(t))$ w.r.t the latent factor.