On the Welfare of Pension Decumulation Strategies

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Introduction & Motivation

- The decumulation strategy requires the individual to decide upon:
  - how much to withdrawn periodically from the pension pot to live on
  - how to allocate the remaining funds during retirement
- The search for the appropriate decumulation option has to take account of a number of particularities in individual preferences and the enabling environment (consumption vs bequest, exposure to uninsurable risks (e.g., divorce, death of a partner, health shocks)
- Conventional life annuities, lump sum payments and income drawdowns are the classical pension decumulation strategies
- Recent alternatives include Participating & non-participating Longevity-linked life annuities (PLLA), Advanced Life Deferred Annuities (ALDA), Ruin Contingency Life Annuities (RCLA), Tontines,...
- Explicitly model consumers' behaviour and objectives though an objective function based on utility theory

Typical Retirement Income Goals

- Maximize the expected retirement income (& consumption) over one's lifetime
- Consumption smoothing
- Generate a lifetime retirement income that cannot be outlived
- Preserve the ability to bequest unused wealth
- Liquidity concerns in case of unforeseen expenses (e.g., long-term care)
- Protect against common financial and biometric risks (e.g., longevity, inflation, investment, life events, fraud)
- Preserve the purchasing power of income
- Preserve the chance to profit from upward trends in financial markets
- Keep the investment strategy simple

Welfare analysis: Utility function

- Time-separable utility function that considers lifetime consumption and bequest motive
- Adjusted MDUF considering the following preferences (Bell et al., 2017):
  - Higher income
  - Smooth income
  - Outliving savings is a poor outcome
  - Residual benefit has value
  - People are risk averse
  - bequest may be a luxury good
  - Time preference

Welfare analysis: Expected utility

We assume individuals want to maximize the expected present value of utility derived from consumption through their remaining lifetime,

$$U_t = \max_{c_t, c_{t+1}, \ldots, c_T} \mathbb{E}_t \left[\sum_{t=0}^{\omega - x} \beta^t \left\{ {}_t p_x u\left(c_t\right) + {}_{t-1|} q_x \upsilon\left(W_t\right) \right\} \right]$$

with

$$u\left(c_t\right) = \frac{c_t^{1-\rho}}{1-\rho}, \quad c_t \geq 0$$

$$\upsilon\left(W_t\right) = \left(\frac{\phi}{1-\phi}\right)^{\rho} \frac{\left(\frac{\phi}{1-\phi} c_b + W_t\right)^{1-\rho}}{1-\rho}, \quad W_t \geq 0$$

where β is the subjective utility discount factor, $\rho > 0$ the level of risk aversion, c_b measures the degree to which bequests are considered as luxury goods, $\phi \in [0, 1)$ measures the strength of the member's bequest motive when bequest has kicked in $(W_t > c_b)$ (see, e.g., Lockwood 2018)

Welfare analysis: Expected Utility

The dynamics of wealth is

$$W_{t+1} = (W_t - c_t + P_t)\left(1 + \tilde{R}_{t+1}\right)$$

where $\tilde{R}_{t+1}$ is the stochastic post-tax return with marginal tax rate s:

$$\tilde{R}_{t+1} = (1 - s)\, R_{t+1}$$

and P_t is the amount of old-age pension received at time t

The pre-tax nominal return at time t is a function of the asset allocation between stocks w_E and bonds w_B

$$R_{t+1} = f\left(w_E, w_B, R^E_{t+1}, R^B_{t+1}\right)$$

Welfare analysis

Utility Score & Risk-Adjusted Income

Utility Score S_0: constant level of income which delivers an equivalent level of expected utility

$$S_0 = \left[U_0 \times \frac{1-\rho}{\left(\sum_{t=0}^{\omega - x} \beta^t \left\{ {}_t p_x + {}_{t-1|} q_x \left(\frac{\phi}{1-\phi} \right) \right\} \right)} \right]^{\frac{1}{1-\rho}}$$

Risk-Adjusted Income S_c: constant level of income which delivers an equivalent level of income utility

$$S_C = \left[U_C \times \frac{1-\rho}{\left(\sum_{t=0}^{\omega - x} \beta^t \, {}_t p_x \right)} \right]^{\frac{1}{1-\rho}}$$

Welfare analysis

Utility Score & Risk-Adjusted Income

- Certainty Equivalent Consumption CEC_0: consumption level in the one-period CRRA utility function that matches the lifetime utility

$$CEC_0 = \left[U_C \times (1-\rho)\right]^{\frac{1}{1-\rho}}$$

- Welfare gain G_0 (Bell et al., 2017):

$$G_0 = (S_0 - S_0^*)\left[\sum_{t=0}^{\omega-x} \beta^t \left\{ {}_t p_x + {}_{t-1|}\, q_x \left(\frac{\phi}{1-\phi}\right)\right\}\right]$$

Decumulation strategies tested

- Strategies

1. $(1/e_{x,t=0})$ rule
2. $(1/\ddot{a}_{x,t=0})$ rule without annuitization
3. $(1/e_{x,t})$ rule
4. "4%" SWR rule (Bengen, 2004)
5. Participating Longevity-linked life annuity (PLLA) (Bravo & El Mekkaoui, 2018)
6. Nominal life annuity $(1/\ddot{a}_{x,t=0})$ (Alonso-García & Sherris, 2019)
7. $80\% W_0$ (SWR rule) $+ 20\% W_0$ ALDA
8. Inflation-Protected Annuity (IPA)

Longevity-linked life annuity

The structure of the contract

We consider a participating longevity-linked life annuity (PLLA) as proposed in Bravo & El Mekkaoui (2018); The annual benefit dynamics is given by

$$b_{t_0+k} = b_{t_0} \times \mathcal{I}_{t_0+k} \times \mathcal{R}_{t_0+k}, \qquad k = 1, \ldots, \omega - x_0 \tag{1}$$

where $\mathcal{I}_{t_0+k}$ is a longevity index

$$\mathcal{I}_{t_0+k} = \frac{{}_k p_{x_0}^{[\mathcal{F}_0]}(t_0)}{{}_k p_{x_0}^{[\mathcal{F}_k]}(t_0)} = \prod_{j=0}^{k-1} \frac{p_{x_0+j}^{[\mathcal{F}_0]}(t_0+j)}{p_{x_0+j}^{[\mathcal{F}_k]}(t_0+j)}, \quad k = 1, \ldots, \omega - x_0 \tag{2}$$

and $\mathcal{R}_{t_0+k}$ is interest rate adjustment index (IRA)

$$\mathcal{R}_{t_0+k} = \prod_{t=1}^{k} \left(\frac{1+R_t}{1+i} \right), \quad k = 1, \ldots, \omega - x_0 \tag{3}$$

where R_t denotes the net investment return observed in year t and i is the guaranteed interest rate

Capped Longevity-linked life annuity

Pricing

Similar to Bravo & El Mekkaoui (2018) and Denuit *et al.* (2011), we delimitate the systematic longevity risk that can be transferred to annuitants by assuming the annuity benefit is capped by $\mathcal{I}^{\max}_{t_0+k}$, with $\mathcal{I}^{\max}_{t_0+k} > 1$, and there is also a minimum guarantee, $\mathcal{I}^{\min}_{t_0+k}$, with $0 < \mathcal{I}^{\min}_{t_0+k} < 1 < \mathcal{I}^{\max}_{t_0+k}$.
The Capped longevity-linked life annuity (CLLLA) price can be written as

$$\ddot{a}^{CPLLA}_{x_0} = \sum_{k=0}^{\omega-x_0} Z\left(0,k\right)_k p^{[\mathcal{F}_k]}_{x_0}\left(t_0\right) \max\left\{\min\left(\mathcal{I}_{t_0+k}; \mathcal{I}^{\max}_{t_0+k}\right); \mathcal{I}^{\min}_{t_0+k}\right\} \quad (4)$$

For simplicity, we assume the upper and lower barriers are constant during the whole contract, i.e., $\mathcal{I}^{\max}_{t_0+k} = \mathcal{I}^{\max}$ and $\mathcal{I}^{\min}_{t_0+k} = \mathcal{I}^{\min}$ for $k = 1, ..., \omega - x_0$.

Stochastic Modeling and Calibration

The mortality model

Log bilinear Lee-Carter model under a Poisson setting (Brouhns et al., 2002; Renshaw and Haberman, 2003)

$$D_{x,t} \sim \mathcal{P}oisson\left(\mu_x\left(t\right) E_{x,t}\right) \tag{5}$$

with

$$\mu_x\left(t\right) = \exp\left(\alpha_x + \beta_x \kappa_t\right), \tag{6}$$

s.t. constraints

$$\sum_{t=t_{\min}}^{t_{\max}} \kappa_t = 0 \quad \text{and} \quad \sum_{x=x_{\min}}^{x_{\max}} \beta_x = 1.$$

Denuit and Goderniaux (2005) life table closing method was applied, with $\omega = 120$

Mortality Data: U.S. Male population by individual age and calendar year; Source: HMD (2019)

Lookback window: 1970-2017; ages 60-95

Stochastic Modeling and Calibration

The financial market model

Retiree's assets are invested in a fixed target portfolio: bonds (70%), stocks (30%)

Bond prices are based on a two-factor Vasicek (1977) model; The model assumes the r_t process is a sum of two independent O-U processes x and y,

$$\begin{aligned} r_t &= x_t + y_t, \quad r(0) = x_0 + y_0 \\ dx_t &= \beta_x (\mu_x - x_t) dt + \sigma_x dW_1(t), \quad x(0) = x_0 \\ dy_t &= \beta_y \left(\mu_y - y_t\right) dt + \sigma_y dW_2(t), \quad y(0) = y_0 \end{aligned} \tag{7}$$

where (W_1, W_2) is a two-dimensional Brownian motion with instantaneous correlation ρ as from

$$dW_1(t)\, dW_2(t) = \rho$$

where x_0, y_0, β_x, β_y, μ_x, μ_y, σ_x, σ_y are positive constants, and where $-1 \leq \rho \leq 1$.

Stochastic Modeling and Calibration

The financial market model

We assume the retiree's assets are invested in a portfolio consisting of two risky assets: coupon bonds and stocks

Calibration: historical (monthly) data on US 3-month and 10-year maturity bond yields from January 1, 2010 to 2019-09-01

Vasicek model ML estimates

	β_i	μ_i	σ_i
Short rate x	0.1742760760	0.0050299138	0.0003765062
Long rate y	2.776074096	0.023436466	0.004193339

Table: Note: Estimates of the 2-factor Vasicek model based on data provided by Datastream. Source: own calculation

with $\hat{\rho} = 0.1724159$.

Stochastic Modeling and Calibration

The financial market model

We assume stock prices follow a geometric Brownian motion described by

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \tag{8}$$

where W_t is a standard Wiener process, and μ and σ are constants

Calibration: S&P 500 Price Index daily data from January 1, 2010 to 2019-09-01 considering the index values adjusted for dividends and splits
ML parameter estimates:

$$(\hat{\mu}, \hat{\sigma}) = (0.1095359; 0.1494558)$$

We assume CPI rate follows an *i.i.d* Normal distribution
$\mathcal{N}\left(\mu_{CPI} = 0.0343; \sigma_{CPI} = 0.015\right)$

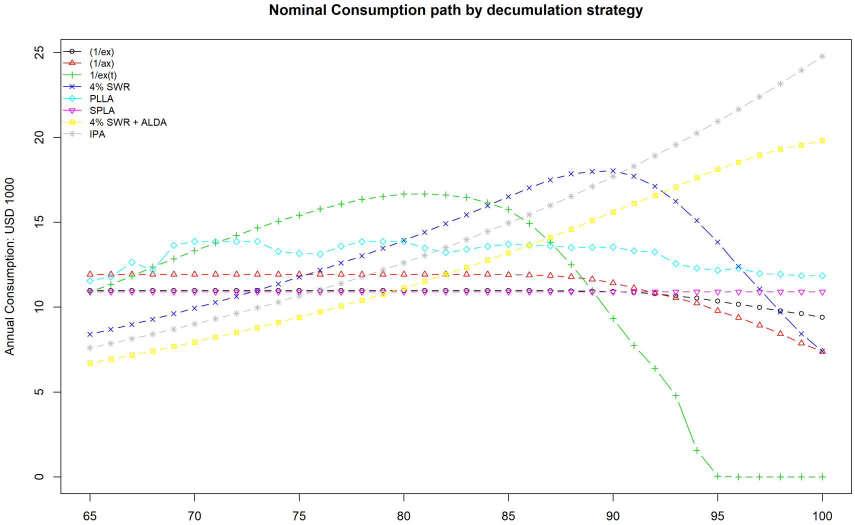
Stochastic Modeling and Calibration

Baseline parameters

Parameter	Value	Notes
Subjective utility discount factor: β	0.975	
Risk aversion coef.: ρ	5	
Strength of bequest motive: ϕ	0.83	
Bequests as luxury goods level: b_c	25.8	$\times 10^3$ USD
Initial wealth (1000s): W_0	210	$\times 10^3$ USD
Tax rate: s	20%	
ALDA Deferment period: k	20	years
Pension benefits: P_t	0	
Annuity loading factor $\ddot{a}_x$	10%	
PLLA bounds: $\left(\mathcal{I}_t^{\min}\mathcal{R}_{t_0+k}; \mathcal{I}_t^{\max}\mathcal{R}_{t_0+k}\right)$	(80%; 120%)	
Number of simulations.	1000	
Investment horizon, starting age 65	55	years
Guaranteed interest rate (GIR): $\ddot{a}_x$	1%	
Minimum consumption level $C_t^{\min}$	6	$\times 10^3$ USD

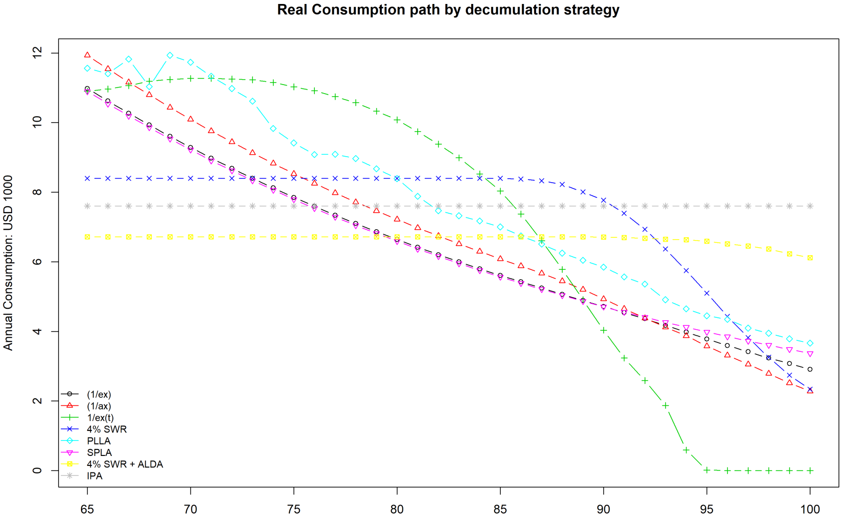
Simulation results: baseline scenario

Consumption path (nominal) by decumulation strategy



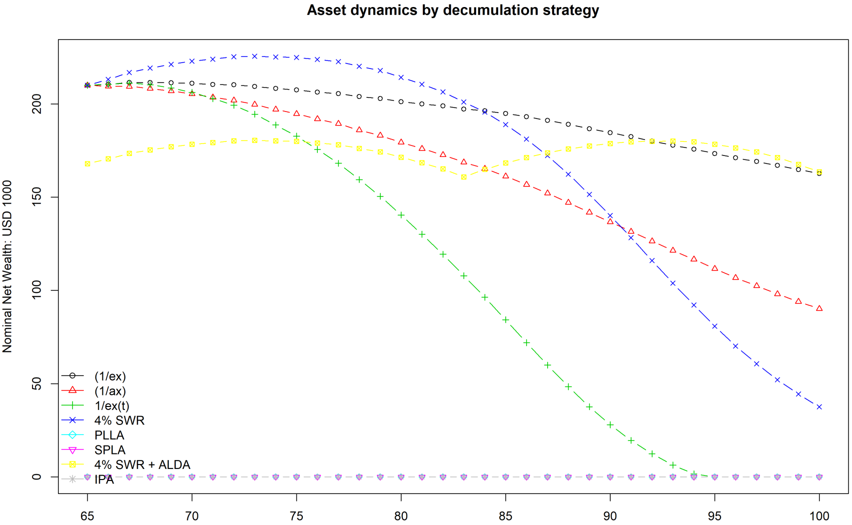
Simulation results: baseline scenario

Consumption path (real) by decumulation strategy



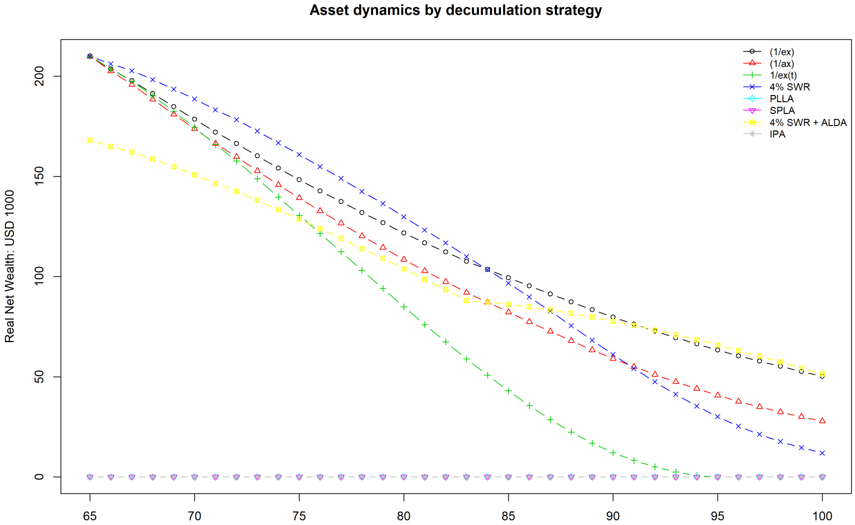
Simulation results: baseline scenario

Asset path by decumulation strategy (nominal)

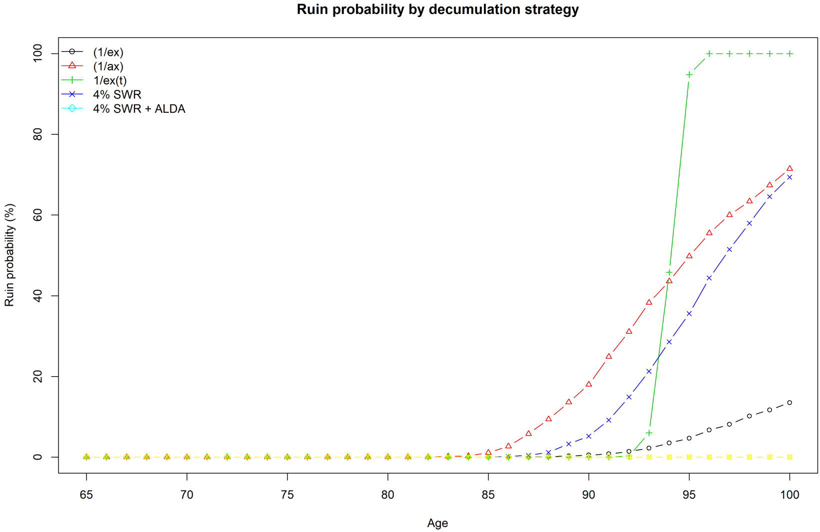


Simulation results: baseline scenario

Asset path by decumulation strategy (real)



Ruin probability



Simulation results: baseline scenario

Baseline scenario

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	9,38	8,77	6,98	9,06	12,53	10,90	14,57	22,24
Ct mean (real)	4,68	4,73	4,59	4,92	5,96	5,00	5,74	7,61
Ct volat. (sd)	2,43	4,13	7,19	8,23	1,30	0,00	6,80	11,92
Ctr volat. (sd)	3,06	3,65	4,97	4,03	3,33	2,66	1,64	0,00
Wt mean	180,98	129,72	68,62	112,77	0,00	0,00	150,22	0,00
Wt mean (real)	87,89	72,11	50,81	72,36	0,00	0,00	75,62	0,00
Wt (age 100)	162,70	90,18	0,00	37,54	0,00	0,00	163,54	0,00
Wtr (age 100)	50,25	27,84	0,00	11,95	0,00	0,00	51,41	0,00
EUtil	-4,0	-263,7	-744078,0	-318,4	-0,00012	-0,00025	-0,00071	-0,00043
EUtil (real)	-336,0	-7004,4	-39632132	-20847,4	-0,00080	-0,00159	-0,00176	-0,00105
S0	11,29	11,78	9,29	10,47	13,69	11,45	8,82	9,99
S0 (real)	7,01	7,16	4,30	7,88	8,73	7,25	7,03	7,99
RAI	10,75	11,22	8,85	9,97	13,03	10,90	8,40	9,51
RAI (real)	6,67	6,82	4,09	7,50	8,31	6,91	6,69	7,61
CEC	5,55	5,79	4,57	5,15	6,73	5,63	4,34	4,91
CEC (real)	3,45	3,52	2,11	3,87	4,29	3,57	3,45	3,93
G0	-	8,44	-34,16	-13,91	41,05	2,71	-42,15	-22,21
G0 (real)	-	2,64	-46,33	14,86	29,40	4,24	0,34	16,79

Lifecycling Strategies in the Decumulation Phase

- Several studies highlighted the importance of asset allocation approaches in maximizing the retirement income a retiree is able to generate from his/her terminal wealth with systematic withdrawal plans (e.g., Milevsky and Kyrychenko, 2008; Cooley et al., 2001; Hubbard, 2006; Wiafe, 2015).
- Similar to Wiafe (2015), we tested four static Strategic Asset Allocation strategies (SAA):
    1. **Conservative**: The conservative plan allocates the retirees entire wealth to conservative assets (bonds)
    2. **Balanced**: allocates the retirees wealth equally between growth (stocks) and conservative assets (bonds)
    3. **Growth**: allocates 70% of the retiree's wealth to stocks and the remaining to bonds
    4. **Aggressive**: allocates all the retiree's wealth to stocks at retirement

Static Strategic Asset Allocation strategies

Conservative strategy

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	5,70	5,46	5,43	5,68	12,40	10,90	10,12	22,24
Ct mean (real)	3,70	3,72	3,73	3,71	5,91	5,00	4,62	7,61
Ct volat. (sd)	5,50	5,96	5,77	6,70	1,30	0,00	3,12	11,92
Ctr volat. (sd)	3,87	4,31	4,31	4,18	3,32	2,66	2,38	0,00
Wt mean	66,19	57,54	56,74	65,09	0,00	0,00	63,57	0,00
Wt mean (real)	48,47	43,76	43,31	48,97	0,00	0,00	43,49	0,00
Wt (age 100)	0,00	0,00	0,00	0,00	0,00	0,00	8,98	0,00
Wtr (age 100)	0,00	0,00	0,00	0,00	0,00	0,00	2,82	0,00
EUtil	########	########	########	########	-0,00013	-0,00025	-0,00071	-0,00043
EUtil (real)	########	########	########	########	-0,00091	-0,00159	-0,00204	-0,00105
S0	9,79	10,29	8,92	10,00	13,58	11,45	8,81	9,99
S0 (real)	5,60	6,07	4,22	7,32	8,54	7,25	6,80	7,99
RAI	9,33	9,80	8,50	9,52	12,93	10,90	8,39	9,51
RAI (real)	5,33	5,78	4,02	6,97	8,13	6,91	6,47	7,61
CEC	4,82	5,06	4,39	4,91	6,68	5,63	4,33	4,91
CEC (real)	2,75	2,99	2,08	3,60	4,20	3,57	3,34	3,93
G0	0,00	8,55	-14,91	3,46	64,75	28,27	-16,78	3,34
G0 (real)	0,00	8,16	-23,51	29,43	50,28	28,36	20,51	40,91

Static Strategic Asset Allocation strategies

Balanced strategy

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	10,22	10,40	8,36	14,73	12,47	10,90	17,34	22,24
Ct mean (real)	4,86	5,10	5,34	6,26	5,92	5,00	6,27	7,61
Ct volat. (sd)	1,00	1,91	8,68	10,08	1,33	0,00	8,82	11,92
Ctr volat. (sd)	2,86	3,26	5,65	2,75	3,29	2,66	0,75	0,00
Wt mean	476,25	372,34	78,61	285,10	0,00	0,00	387,87	0,00
Wt mean (real)	164,10	136,67	56,96	124,79	0,00	0,00	138,48	0,00
Wt (age 100)	474,47	362,12	0,00	288,43	0,00	0,00	433,50	0,00
Wtr (age 100)	146,52	111,79	0,00	90,03	0,00	0,00	135,00	0,00
EUtil	-0,05425	-0,02962	########	-0,97568	-0,00013	-0,00025	-0,00071	-0,00043
EUtil (real)	-4,23231	-2,95732	########	-90,78763	-0,00081	-0,00159	-0,00177	-0,00105
S0	11,37	12,19	9,95	10,70	13,53	11,45	8,82	9,99
S0 (real)	7,15	7,59	4,59	8,29	8,67	7,25	7,03	7,99
RAI	10,83	11,61	9,47	10,19	12,89	10,90	8,40	9,51
RAI (real)	6,81	7,22	4,37	7,90	8,25	6,91	6,69	7,61
CEC	5,59	6,00	4,89	5,26	6,66	5,63	4,34	4,91
CEC (real)	3,51	3,73	2,26	4,08	4,26	3,57	3,46	3,93
G0	0,00	14,13	-24,39	-11,41	37,03	1,31	-43,58	-23,62
G0 (real)	0,00	7,49	-43,79	19,58	25,95	1,82	-2,05	14,37

Static Strategic Asset Allocation strategies

Growth strategy

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	10,37	10,86	10,12	18,37	12,37	10,90	18,18	22,24
Ct mean (real)	4,88	5,20	6,27	7,04	5,86	5,00	6,42	7,61
Ct volat. (sd)	0,70	1,18	10,80	11,21	1,40	0,00	9,44	11,92
Ctr volat. (sd)	2,82	3,15	6,63	1,72	3,25	2,66	0,43	0,00
Wt mean	1081,98	913,05	90,78	804,77	0,00	0,00	902,22	0,00
Wt mean (real)	307,94	266,28	64,31	253,43	0,00	0,00	261,15	0,00
Wt (age 100)	1009,11	849,99	0,00	805,61	0,00	0,00	893,95	0,00
Wtr (age 100)	311,55	262,38	0,00	250,13	0,00	0,00	277,37	0,00
EUtil	-0,03710	-61,26956	########	########	-0,00014	-0,00025	-0,00078	-0,00043
EUtil (real)	-3,61350	########	########	########	-0,00085	-0,00159	-0,00251	-0,00105
S0	11,41	12,22	9,99	10,79	13,30	11,45	8,81	9,99
S0 (real)	7,19	7,69	4,82	8,50	8,54	7,25	7,02	7,99
RAI	10,87	11,64	9,51	10,27	12,67	10,90	8,39	9,51
RAI (real)	6,85	7,32	4,59	8,09	8,13	6,91	6,68	7,61
CEC	5,61	6,01	4,91	5,31	6,54	5,63	4,33	4,91
CEC (real)	3,54	3,78	2,37	4,18	4,20	3,57	3,45	3,93
G0	0,00	13,87	-24,37	-10,63	32,37	0,58	-44,43	-24,34
G0 (real)	0,00	8,48	-40,65	22,40	23,07	1,12	-2,93	13,67

Static Strategic Asset Allocation strategies

Aggressive strategy

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	10,35	10,98	13,72	20,44	12,19	10,90	18,41	22,24
Ct mean (real)	4,87	5,21	8,14	7,44	5,76	5,00	6,44	7,61
Ct volat. (sd)	0,62	0,90	15,68	11,83	1,55	0,00	9,65	11,92
Ctr volat. (sd)	2,82	3,11	8,88	1,05	3,21	2,66	0,32	0,00
Wt mean	3208,49	2869,97	114,41	2868,04	0,00	0,00	2716,61	0,00
Wt mean (real)	778,13	700,53	78,21	714,99	0,00	0,00	662,87	0,00
Wt (age 100)	2524,72	2260,97	0,00	2340,06	0,00	0,00	2190,28	0,00
Wtr (age 100)	779,41	697,93	0,00	724,44	0,00	0,00	678,03	0,00
EUtil	-2,24603	-0,00682	#########	-7,48770	-0,00016	-0,00025	-0,00548	-0,00043
EUtil (real)	-38,37906	-0,26834	#########	#########	-0,00096	-0,00159	-0,06538	-0,00105
S0	11,34	12,29	11,17	10,82	12,85	11,45	8,80	9,99
S0 (real)	7,16	7,76	6,26	8,54	8,27	7,25	7,00	7,99
RAI	10,80	11,70	10,63	10,31	12,24	10,90	8,38	9,51
RAI (real)	6,82	7,39	5,96	8,13	7,88	6,91	6,67	7,61
CEC	5,58	6,04	5,49	5,32	6,32	5,63	4,33	4,91
CEC (real)	3,52	3,82	3,08	4,20	4,07	3,57	3,44	3,93
G0	0,00	16,19	-3,00	-8,85	25,83	1,78	-43,42	-23,15
G0 (real)	0,00	10,28	-15,47	23,58	19,00	1,60	-2,75	14,15

Lifecycling Strategies in the Decumulation Phase

- Additionally, we tested several Lifecycle (dynamic) decumulation strategies:
  - Lifecycle Strategy (LC): This strategy begins with an equal allocation between stocks and bonds, with the growth assets gradually reduced to 0% at the end of the investment horizon
  - Reverse Lifecycle (RLC): This strategy begins with a full allocation to conservative assets at retirement, and gradually increases allocation to stocks till the investment is fully invested in growth assets at the final year in retirement
  - Partial Lifecycle (PLC): the retiree holds a fixed (50%) allocation to stocks throughout the investment horizon. The remaining 50% is invested fully in stocks at the onset and reduces to bonds at the end of the horizon

Lifecycle Strategy (LC)

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	9,75	9,60	7,70	11,19	12,50	10,90	15,52	22,24
Ct mean (real)	4,76	4,92	5,01	5,47	5,94	5,00	5,92	7,61
Ct volat. (sd)	1,80	3,07	8,01	8,97	1,30	0,00	7,52	11,92
Ctr volat. (sd)	2,97	3,45	5,40	3,69	3,31	2,66	1,37	0,00
Wt mean	251,25	195,38	74,59	158,46	0,00	0,00	203,14	0,00
Wt mean (real)	110,95	93,94	54,70	90,67	0,00	0,00	93,80	0,00
Wt (age 100)	260,45	183,48	0,00	116,83	0,00	0,00	243,49	0,00
Wtr (age 100)	80,46	56,66	0,00	36,66	0,00	0,00	76,11	0,00
EUtil	########	-17,76415	########	########	-0,00013	-0,00025	-0,00071	-0,00043
EUtil (real)	########	########	########	########	-0,00081	-0,00159	-0,00177	-0,00105
S0	11,33	12,06	9,43	10,61	13,60	11,45	8,82	9,99
S0 (real)	7,09	7,41	4,33	8,11	8,68	7,25	7,02	7,99
RAI	10,78	11,48	8,98	10,11	12,95	10,90	8,40	9,51
RAI (real)	6,75	7,06	4,13	7,73	8,27	6,91	6,69	7,61
CEC	5,57	5,93	4,64	5,22	6,69	5,63	4,34	4,91
CEC (real)	3,49	3,64	2,13	3,99	4,27	3,57	3,45	3,93
G0	0,00	12,57	-32,44	-12,18	38,89	2,08	-42,80	-22,85
G0 (real)	0,00	5,44	-47,16	17,48	27,20	2,80	-1,18	15,35

Reverse Strategy (LC)

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	8,47	7,07	6,27	7,23	12,53	10,90	13,46	22,24
Ct mean (real)	4,48	4,29	4,15	4,33	5,97	5,00	5,49	7,61
Ct volat. (sd)	3,64	5,66	6,38	7,48	1,36	0,00	5,90	11,92
Ctr volat. (sd)	3,28	4,03	4,52	4,18	3,33	2,66	1,89	0,00
Wt mean	122,11	77,23	62,14	81,24	0,00	0,00	114,64	0,00
Wt mean (real)	66,80	52,62	46,42	57,21	0,00	0,00	60,83	0,00
Wt (age 100)	78,04	15,25	0,00	1,99	0,00	0,00	100,93	0,00
Wtr (age 100)	24,09	4,71	0,00	0,66	0,00	0,00	32,13	0,00
EUtil	-2,14362	-72,61264	#########	-9,83839	-0,00012	-0,00025	-0,00071	-0,00043
EUtil (real)	#########	#########	#########	#########	-0,00078	-0,00159	-0,00177	-0,00105
S0	11,21	11,30	9,03	10,26	13,72	11,45	8,82	9,99
S0 (real)	6,84	6,64	4,22	7,56	8,77	7,25	7,02	7,99
RAI	10,67	10,76	8,60	9,77	13,06	10,90	8,40	9,51
RAI (real)	6,51	6,32	4,02	7,20	8,35	6,91	6,68	7,61
CEC	5,51	5,55	4,44	5,05	6,75	5,63	4,34	4,91
CEC (real)	3,36	3,27	2,07	3,72	4,31	3,57	3,45	3,93
G0	0,00	1,57	-37,23	-16,09	43,00	4,12	-40,74	-20,80
G0 (real)	0,00	-3,34	-44,81	12,38	32,98	7,17	3,14	19,72

Partial Lifecycle (PLC)

Metric	Strategy							
	$1/e_{x,0}$	$1/a_{x,0}$	$1/e_{x,t}$	SWR 4%	PLLA	$a_{x,0}$	ALDA+	IPA
Ct mean	9,75	9,60	7,70	11,19	12,50	10,90	15,52	22,24
Ct mean (real)	4,76	4,92	5,01	5,47	5,94	5,00	5,92	7,61
Ct volat. (sd)	1,80	3,07	8,01	8,97	1,30	0,00	7,52	11,92
Ctr volat. (sd)	2,97	3,45	5,40	3,69	3,31	2,66	1,37	0,00
Wt mean	251,25	195,38	74,59	158,46	0,00	0,00	203,14	0,00
Wt mean (real)	110,95	93,94	54,70	90,67	0,00	0,00	93,80	0,00
Wt (age 100)	260,45	183,48	0,00	116,83	0,00	0,00	243,49	0,00
Wtr (age 100)	80,46	56,66	0,00	36,66	0,00	0,00	76,11	0,00
EUtil	########	-17,76415	########	########	-0,00013	-0,00025	-0,00071	-0,00043
EUtil (real)	########	########	########	########	-0,00081	-0,00159	-0,00177	-0,00105
S0	11,33	12,06	9,43	10,61	13,60	11,45	8,82	9,99
S0 (real)	7,09	7,41	4,33	8,11	8,68	7,25	7,02	7,99
RAI	10,78	11,48	8,98	10,11	12,95	10,90	8,40	9,51
RAI (real)	6,75	7,06	4,13	7,73	8,27	6,91	6,69	7,61
CEC	5,57	5,93	4,64	5,22	6,69	5,63	4,34	4,91
CEC (real)	3,49	3,64	2,13	3,99	4,27	3,57	3,45	3,93
G0	0,00	12,57	-32,44	-12,18	38,89	2,08	-42,80	-22,85
G0 (real)	0,00	5,44	-47,16	17,48	27,20	2,80	-1,18	15,35