Assessing longevity inequality in the U.S.: What can be said about the future?

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Outline of the talk

1. Background
2. Reconciling mortality rates
3. Empirical results
   - Case study on the “oldest-old”
   - Projection into 2027
4. Conclusions

Some revealing facts about longevity

According to ***The Guardian***:

"Life expectancy gap between rich and poor U.S. regions is **more than 20 years**!"

The overall life expectancy is estimated to be 79.1 in 2015:

For full article see: *https://www.theguardian.com/inequality/2017/may/08/life-expectancy-gap-rich-poor-us-regions-more-than-20-years*

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- Certain affluent counties in central Colorado: 87 years
- Several counties of North and South Dakota: 66 years
- The gap is predicted to grow even wider in future!

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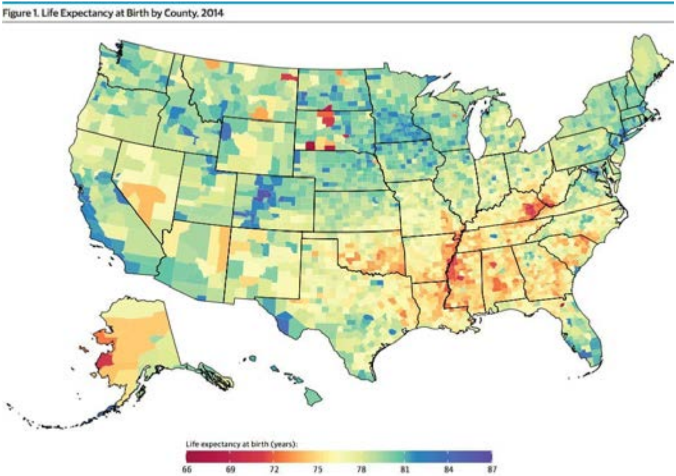
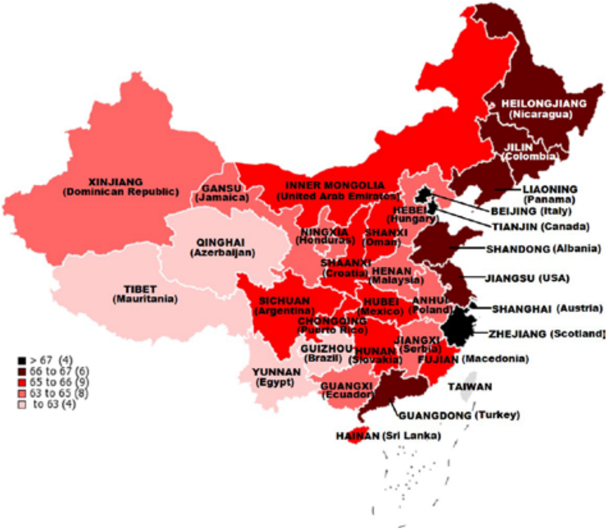
The big picture



Source: JAMA Internal Medicine

Something irrelevant (but interesting!)



Provincial HLE for males in 2015 with international matching

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How do we model mortality rates?

- Central mortality rate $m_{x,t}$, reflects the death probability for age x last birthday in the middle of the calender year t and it is estimated by:

$$m_{x,t} = \frac{D_{x,t}}{E^c_{x,t}}.$$

- Lee-Carter model (1992):

$$\log(m_{x,t}) = a_x + b_x \kappa_t \quad (1)$$

Forecast reconciliation

- Do we need coherent forecasts?
- How to define "coherence" in the context of mortality modeling?
- How do we produce coherent mortality forecasts?

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  Good question! We adopt a forecast reconciliation approach.

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Forecast reconciliation: Baby steps

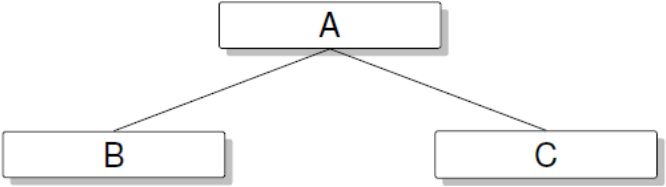

Figure: 2-level simple hierarchy

Forecast reconciliation: Baby steps

We first express the aggregation constraints in a matrix form:

- Define $\boldsymbol{y} = (a, b, c)$ as a vector that contains observations at all levels in the hierarchy;
- Define $\boldsymbol{b} = (b, c)$ as a vector that contains observations at the bottom level only.

The two vectors can then be linked by the equation

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t, \tag{2}$$

where $\boldsymbol{S}$ is a "summing matrix" of dimension 3×2. It is given by

$$\boldsymbol{S} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3}$$

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Forecast reconciliation: Baby steps??

- Let $\hat{\mathbf{y}}_{T+h}$ denote the unreconciled h-step-ahead forecasts at all levels;
- Let $\tilde{\mathbf{y}}_{T+h}$ denote the correspondingly reconciled h-step-ahead forecasts which satisfy all aggregation constraints.

Any linear reconciliation method, according to Wickramasuriya *et al.* (2018), can be expressed as

$$\tilde{\mathbf{y}}_{T+h} = \mathbf{S}\mathbf{P}\hat{\mathbf{y}}_{T+h}, \tag{4}$$

where in our example $\mathbf{P}$ is a 2×3 matrix. The selection of $\mathbf{P}$ is not unique and is a key step in forecast reconciliation. Wickramasuriya *et al.* (2018) propose a closed form estimation of the matrix $\mathbf{P}$ by minimizing the trace of the covariance matrix of the in-sample *reconciled* forecast errors (MinT). The reconciliation matrix $\mathbf{P}$ is given by

$$\mathbf{P} = (\mathbf{S}'\mathbf{W}_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{W}_h^{-1}, \tag{5}$$

where $\mathbf{W}_h$ represents the covariance matrix of the h-step-ahead in-sample unreconciled forecast errors.

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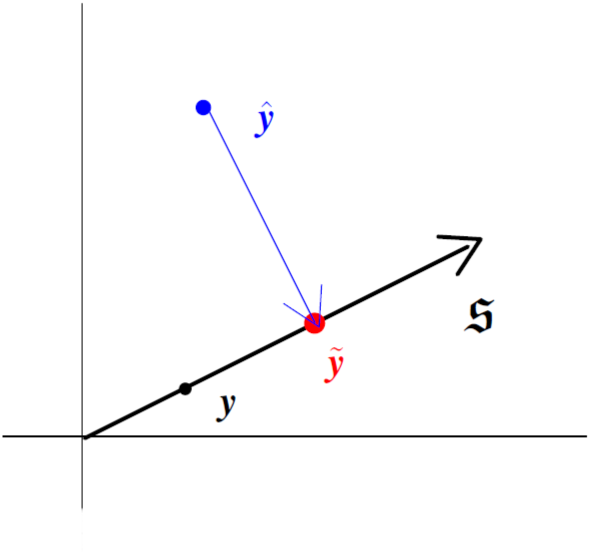
Forecast reconciliation: Understand it visually



Reconciling population exposure

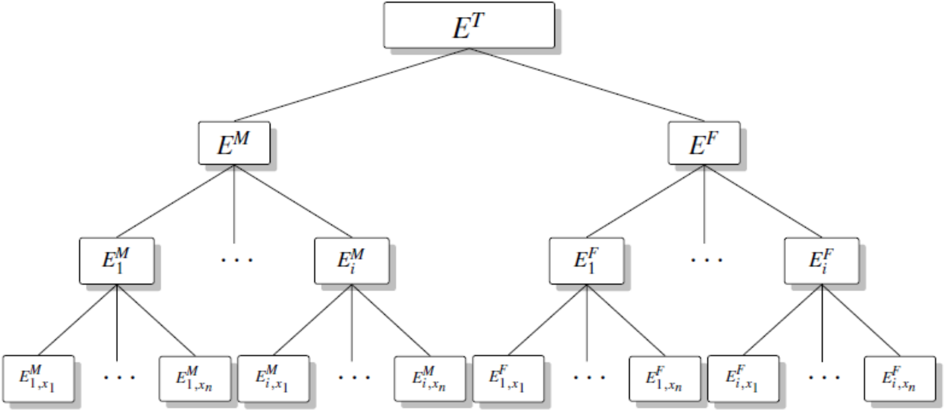


Figure: 4-level hierarchical tree for population exposure

Reconciling population exposure

For the hierarchical structure in the death counts, we have the following aggregation constraints at all times:

$$\sum_{x=x_1}^{x_2} E^j_{i_0,x} = E^j_{i_0}, \quad \forall\, i_0 \in [1,i],\ \ j = M,F \tag{6}$$

$$\sum_{i_0=1}^{i} E^j_{i_0} = E^j,\ \ j = M,F \tag{7}$$

$$E^M + E^F = E^T. \tag{8}$$

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1. Obtain independent forecasts on each level via ARIMA models.
2. Apply MinT approach to obtain reconciled forecasts.

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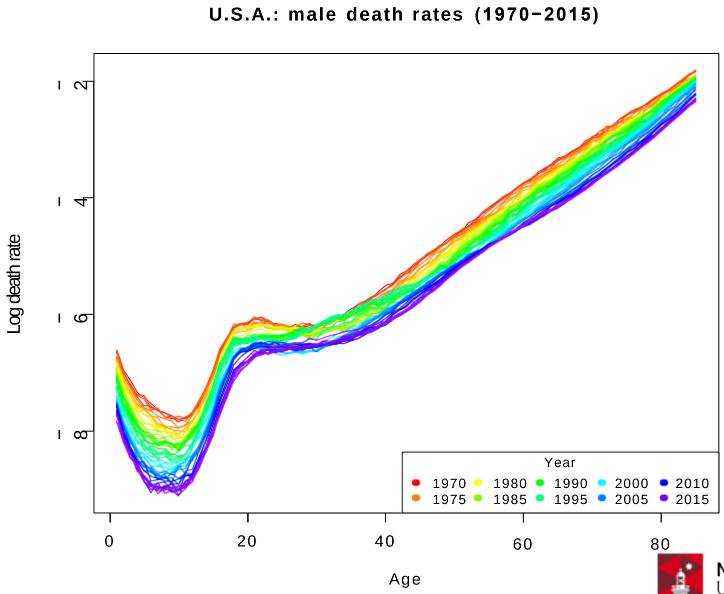
Mortality forecasting



Reconciling death counts

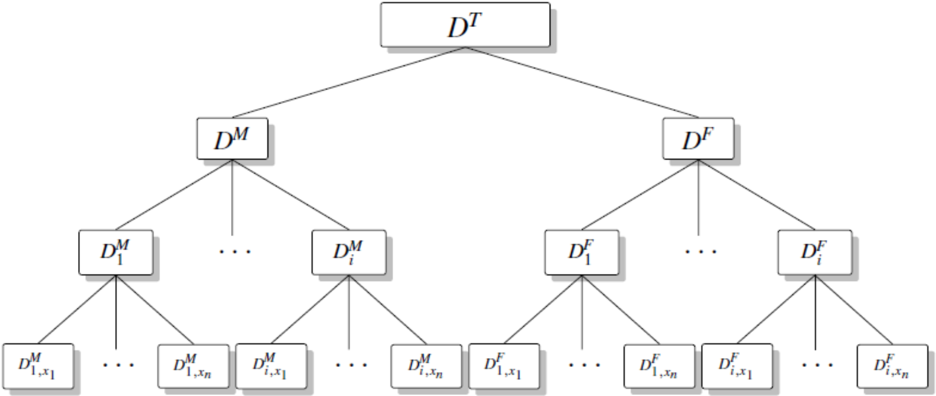


Figure: 4-level hierarchical tree for death counts

Reconciling population exposure

For the hierarchical structure in the death counts, we have the following aggregation constraints at all times:

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1. Obtain independent forecasts on each level by $E \times m$.
2. Apply MinT approach to obtain reconciled forecasts.

Case study

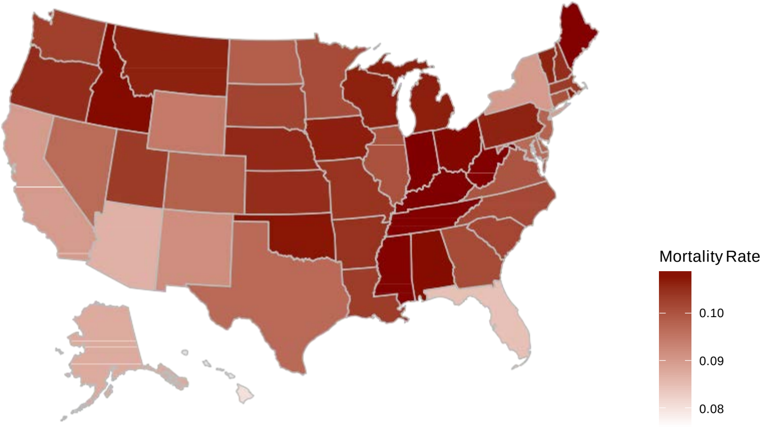
The death and exposure data used to perform this case study are downloaded from the *CDC WONDER online database*.

- Country: the United States.
- Number of states: 50 plus District of Columbia.
- Investigation period: 1968–2017.
- Age range: 80+.

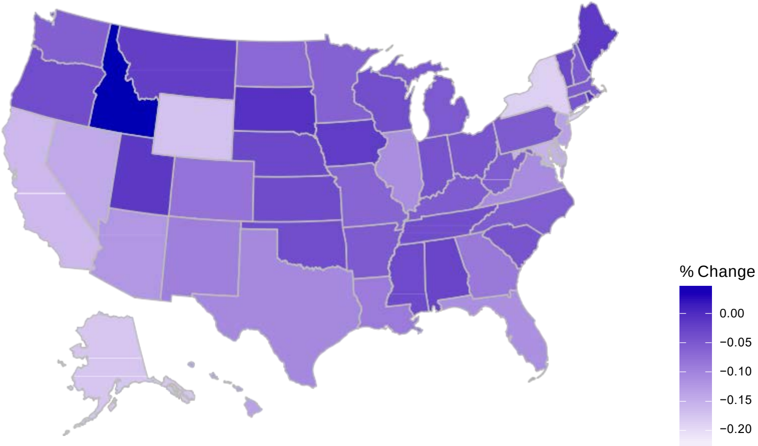
Mortality experience for selected U.S. states in 2017

Rank	State	Crude rate	Rank	State	% change since 1990
1	District of Columbia	7.99%	1	District of Columbia	-24.19%
2	Hawaii	8.03%	2	New York	-18.69%
3	Florida	8.47%	3	Alaska	-17.58%
4	Arizona	8.68%	4	Wyoming	-17.13%
5	Alaska	8.76%	5	California	-16.03%
…	…	…	…	…	…
47	West Virginia	10.91%	47	Maine	-1.23%
48	Maine	10.91%	48	Utah	-1.02%
49	Tennessee	10.92%	49	Rhode Island	-0.39%
50	Kentucky	10.95%	50	South Dakota	-0.38%
51	Indiana	10.98%	51	Idaho	4.91%

U.S. oldest-old mortality rates in 2017



U.S. % change in oldest-old mortality rates: 1990–2017



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Mortality forecasts for selected U.S. states in 2027

Rank	State	Crude rate	Rank	State	% change since 2017
1	District of Columbia	7.38%	1	Georgia	-10.98%
2	Florida	7.64%	2	Nevada	-10.28%
3	Arizona	7.89%	3	South Carolina	-10.16%
4	Hawaii	7.93%	4	Wyoming	-9.97%
5	Alaska	8.06%	5	Florida	-9.74%
…	…	…	…	…	…
47	Iowa	10.65%	47	Ohio	-0.43%
48	Indiana	10.67%	48	Iowa	-0.37%
49	Rhode Island	10.72%	49	Pennsylvania	-0.16%
50	West Virginia	10.74%	50	Connecticut	-0.05%
51	Ohio	10.83%	51	South Dakota	0.06%

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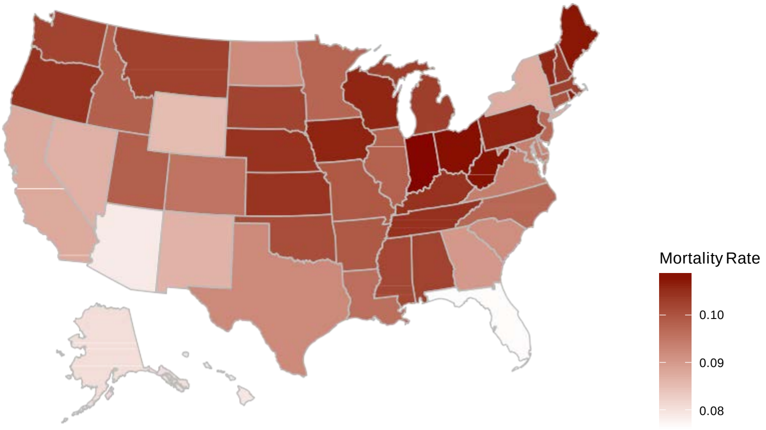

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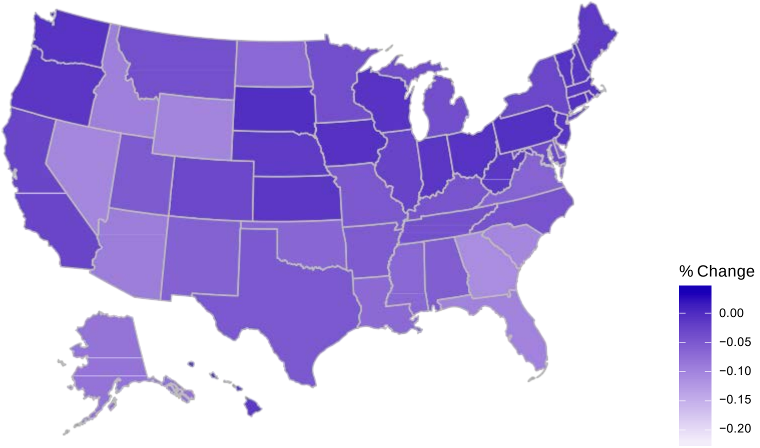
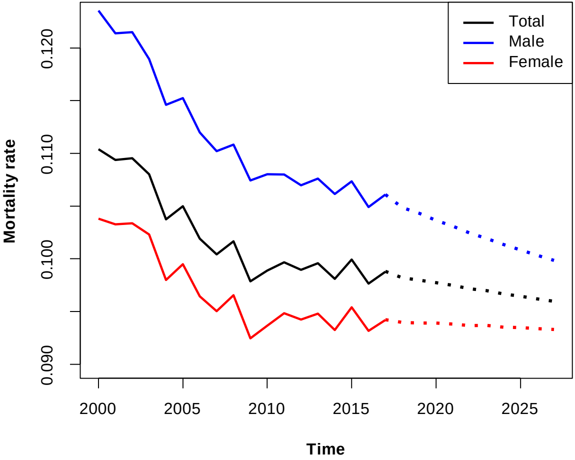


Oldest-old mortality rate forecasts in the U.S.: 2018–2027



Findings

- The worst older-age mortality forecasts are generally observed in "rust belt states" such as Pennsylvania, West Virginia, Ohio and Indiana.
- Older-age males are predicted to experience a more rapid mortality improvement compared to females.
- Future older-age mortality improvement rate will tend to slow down across a majority number of states.

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End of presentation

Thank you!

Any questions/ comments/ suggestions?

Contact email: han.li@mq.edu.au
Research Gate page: *http://www.researchgate.net/profile/Han_Li51*

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References

Wickramasuriya, S. L., Athanasopoulos, G., and Hyndman, R. J. (2018). Optimal forecast reconciliation for hierarchical and grouped time series through trace minimization. *Journal of the American Statistical Association*, (just-accepted), 1–45.