Hotel Park Inn, Amsterdam

# The many uses of VaR for longevity trend risk

Stephen J. Richards

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# Overview

1. About Longevitas

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2. Some questions

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7. Index-based hedges

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5. Deferred annuities
6. VaR v. CTE
7. Index-based hedges
8. Conclusions

# 1 About Longevitas

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- Research partnership with Heriot-Watt.

# 1 Services for actuaries

- Experience analysis and mis-estimation:

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  PROJECTIONS TOOLKIT

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- Rating pension schemes:

  mortalityrating.com

# 2 Some questions

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- How do you put a multi-year trend risk into a one-year view?
- How do different product types behave?
- How do VaR and CTE regimes compare?
- How do you value an index-based hedge?

# 2 Translation table

| | |
|---|---|
| VaR | Value-at-Risk |
| CTE | Conditional Tail Expectation |
| | |
| LC | Model from Lee and Carter [1992] |
| APC | Age-Period-Cohort model |
| M5/CBD | Model from Cairns et al. [2006] |
| M6 | Model from Cairns et al. [2009] |
| (S) | Smoothing as per Eilers and Marx [1996] |
| 2DAC | Model from Richards et al. [2006] |

# 3 Trend risk v. one-year view

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*“Whereas a catastrophe can occur in an instant, longevity risk takes decades to unfold”*

**The Economist [2012]**

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- Insurance regulations have a one-year view of risk.
- How do you reconcile the two?

# 3 Trend risk v. one-year view

Solution from Richards et al. [2014]:

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Solution from Richards et al. [2014]:

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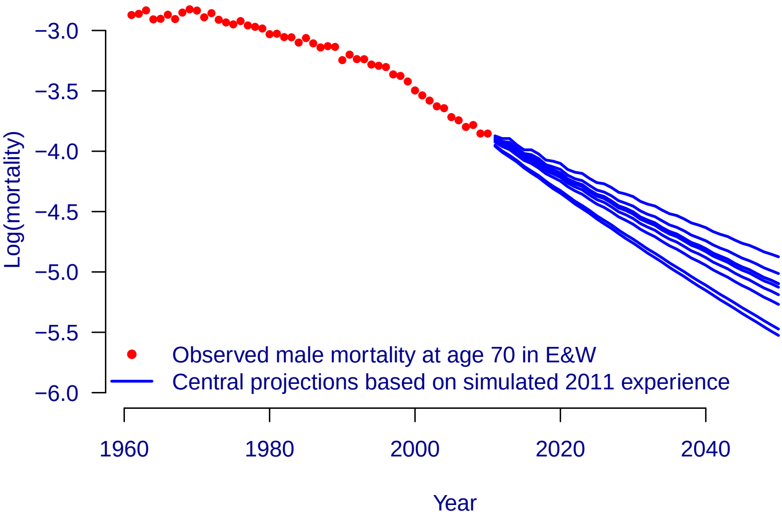
Repeat (1)–(4) a few thousand times...

# 3 Sensitivity of forecast




Source: Lee-Carter example from Richards et al. [2014].

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$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1\right) * 100\%$$

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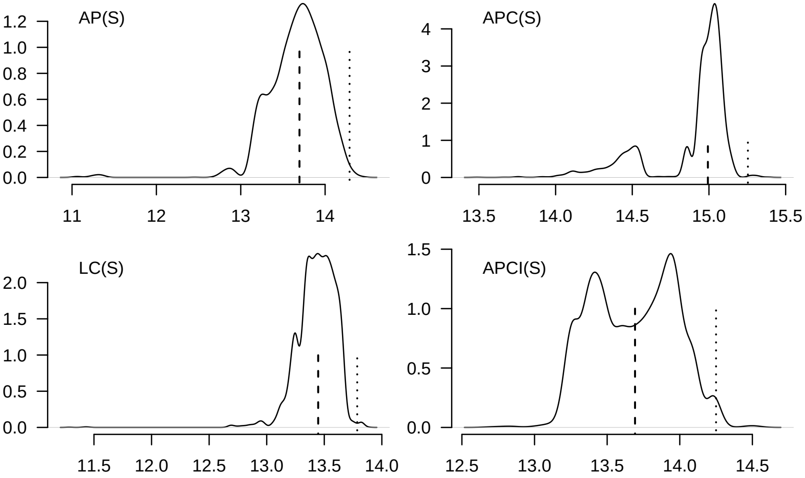
- Estimate $\mathbb{E}[X]$ from mean of sample.
- Estimate $Q_\alpha$ from sample using Harrell and Davis [1982].

# 3 One-year liability densities




Annuities payable to male aged 70. Means marked with dashed line and $Q_{99.5\%}$ marked with dotted line.
Source: Richards et al. [2017, Table 4].

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- The same methodology has other applications. . .

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- 3–5 years for insurer ORSA.

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- 3–5 years for insurer ORSA.
- Ten-year "glide path" to buy-out for pension schemes.

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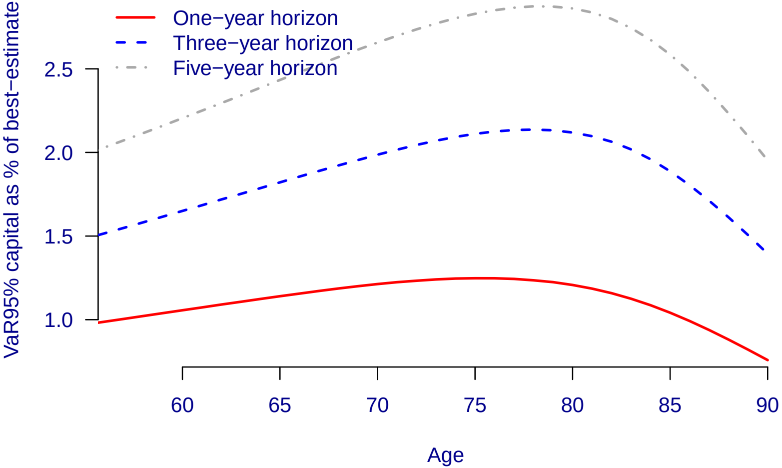
- Extend time horizon to 3–5 years.
- Reduce p-value to, say, 95%...

# 4 Females, LC(S)




Immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

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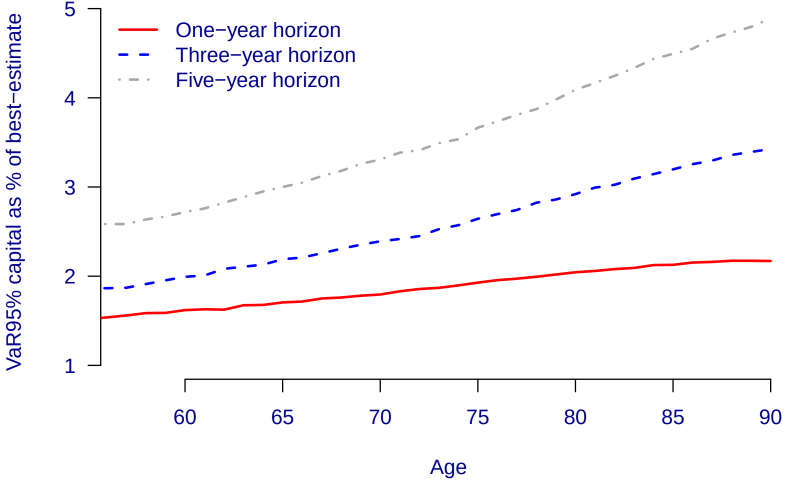


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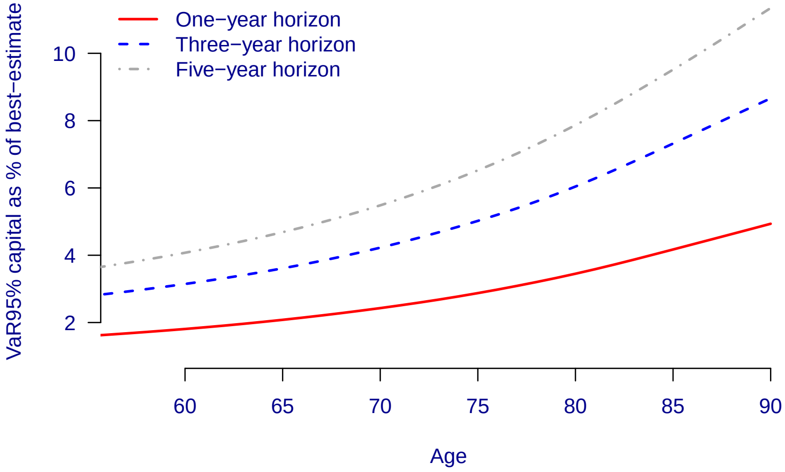


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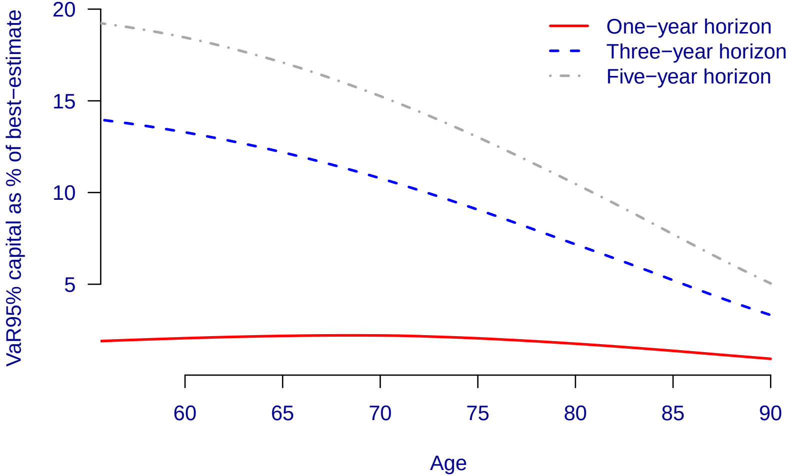


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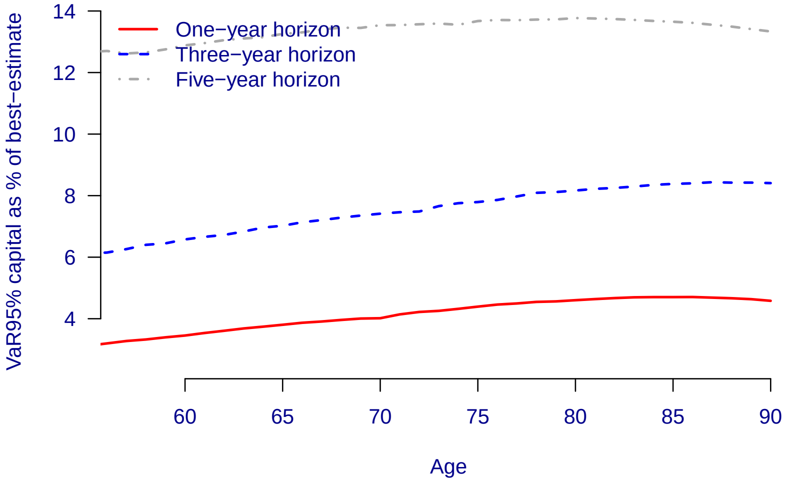


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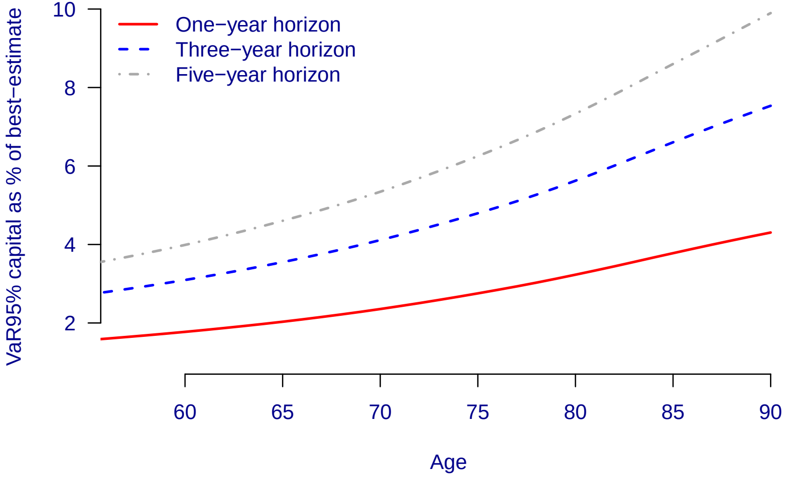


Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

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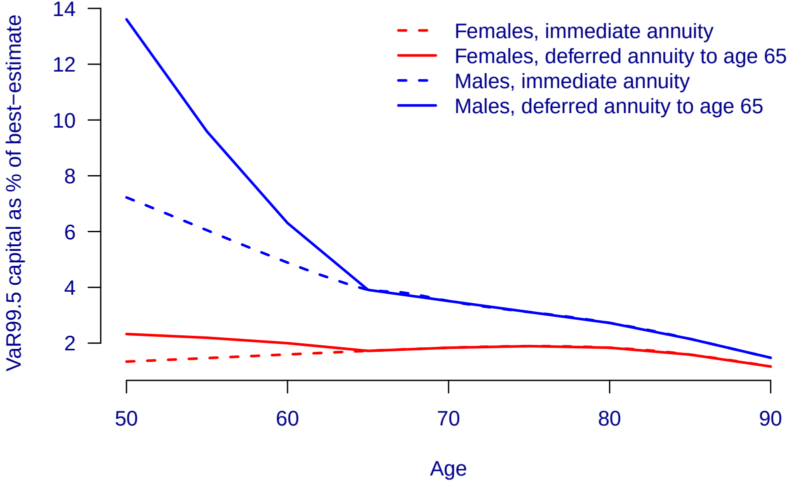
- What about deferred annuities and pensions?
- Assume payment from age 65.
- Compare VaR99.5% solvency capital for immediate and deferred annuities.

# 5 Solvency capital, LC(S)




Deferred and immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

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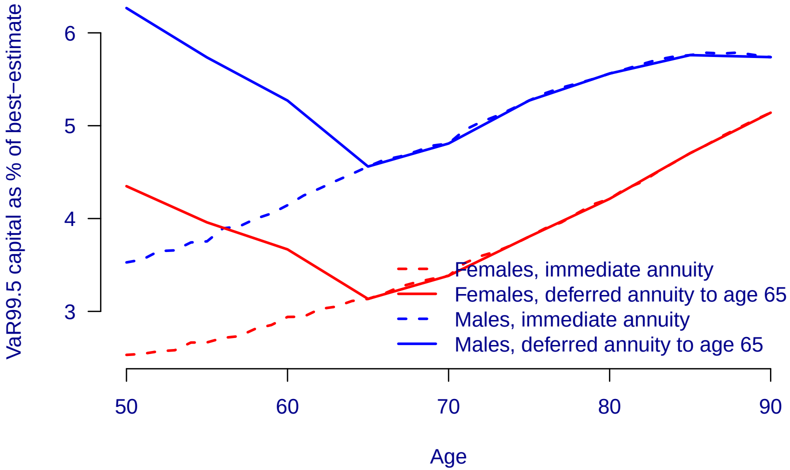
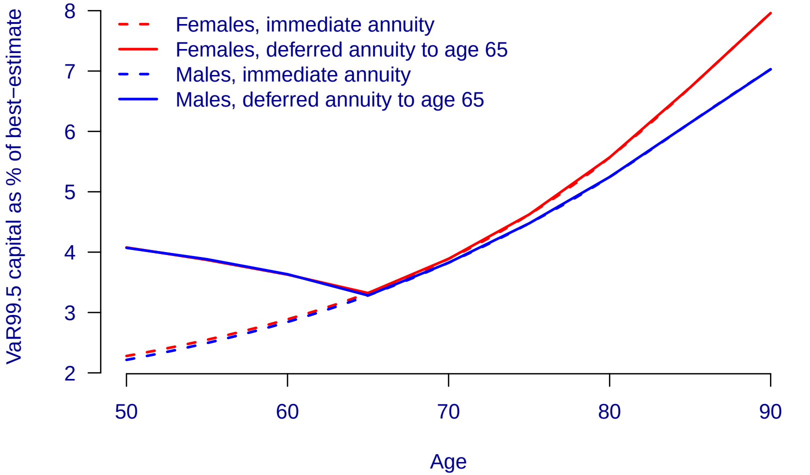



Deferred and immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

# 5 Solvency capital, CBD (M5)

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# 5 Deferred annuities

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- Depending on age, solvency capital for deferred annuities can be double that of annuities in payment.
- Sharp differences in solvency capital by gender.

# 6 VaR v. CTE

# 6 VaR

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- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1\right) * 100\%$$

where $Q_\alpha$ is $\alpha$-quantile of $X$, i.e. $\Pr(X < Q_\alpha) = \alpha$.

# 6 CTE

- Our unknown liability is $X$ (say).

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$$\left(\frac{\mathbb{E}[X|X \geq Q_\alpha]}{\mathbb{E}[X]} - 1\right) * 100\%$$

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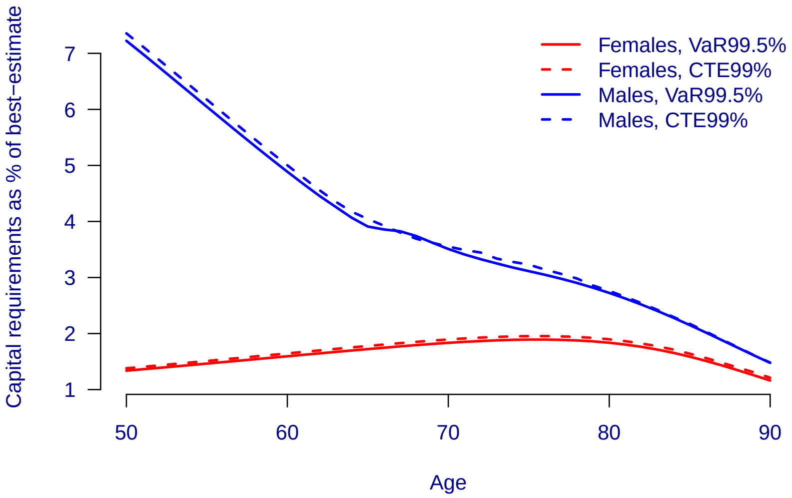
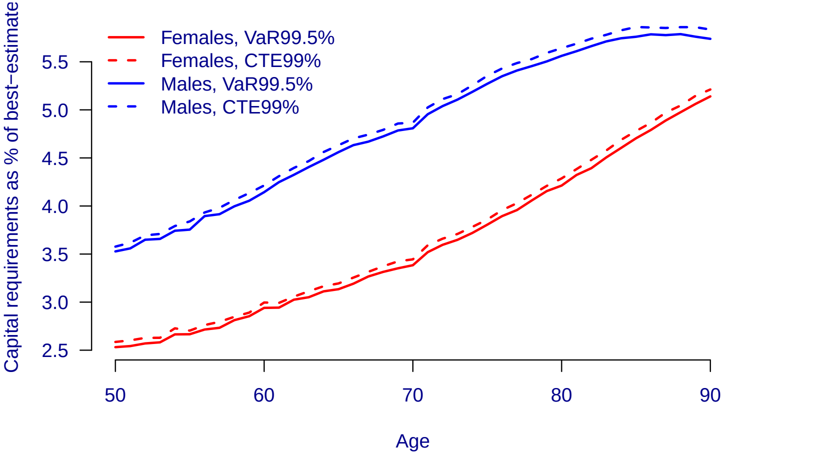
- How does VaR capital compare to CTE capital?
- $\text{CTE}_\alpha > \text{VaR}_\alpha$ (obviously!)
- But how does VaR99.5% compare to CTE99%?
- Can calculate both from same sample...

# 6 VaR v. CTE — LC(S) model



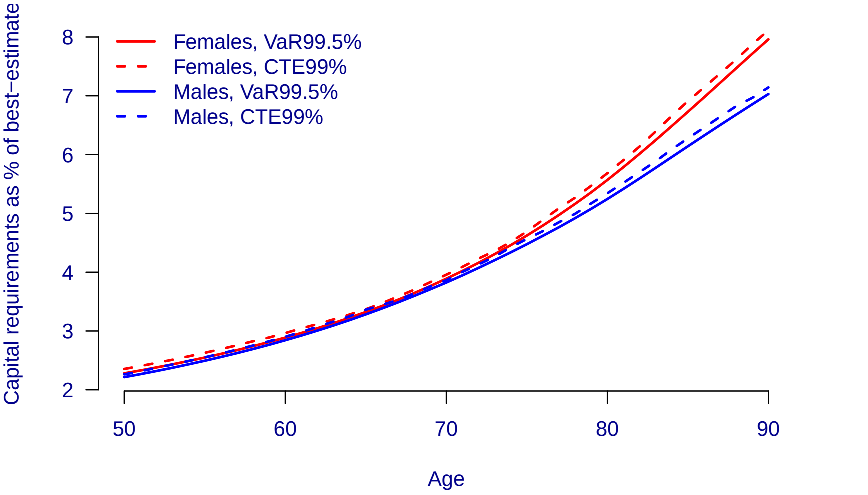

Annuities in payment under Lee-Carter model. UK data ages 50–104, 1971–2016

# 6 VaR v. CTE — APC(S) model



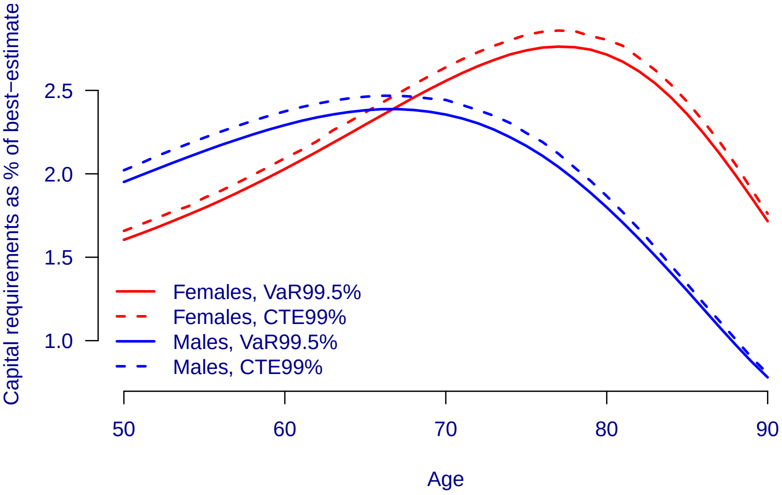

Annuities in payment under APC(S) model. UK data ages 50–104, 1971–2016

# 6 VaR v. CTE — M5 model



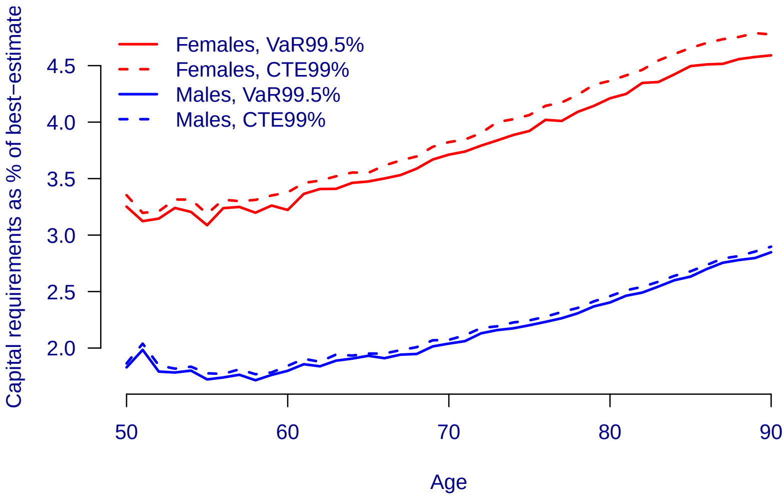

Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

# 6 VaR v. CTE — LC(S)




Annuities in payment under LC(S) model. UK data ages 50–104, 1971–2016

# 6 VaR v. CTE — APC(S)

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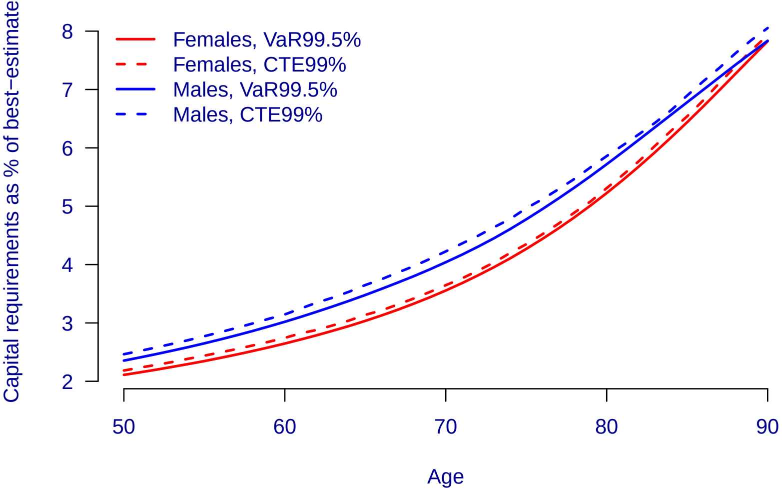
# 6 VaR v. CTE — M5(S)

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- Difference usually under 0.1%.

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  $\Rightarrow$ This is just another multi-year VaR calculation.

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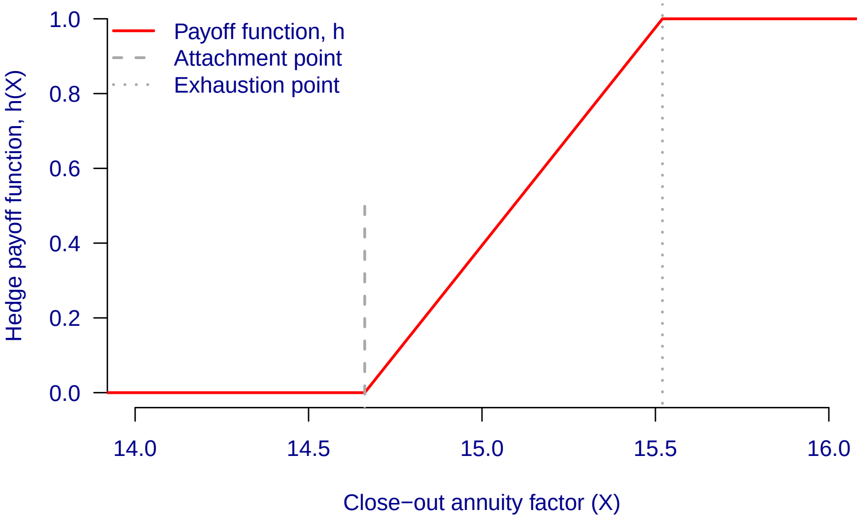
- See Cairns and El Boukfaoui [2017] for detailed discussion.

# 7 Hedge payoff function



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- Probability of payoff is $1 - \alpha_1$.
- Mean payoff can be estimated from VaR results.

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# 7 Example hedge contract

- $n = 15$ years.
- Use Lee-Carter model for close-out calculation.
- Follow Cairns and El Boukfaoui [2017] and set $AP = Q_{60\%}$ and $EP = Q_{95\%}$.
- Probability of a payoff is 0.4.
- Average payoff is 0.375 (from 5,000 simulations).

# 7 Model risk

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- Lee-Carter model used for both sample paths over $n$ years **and** for payoff calculation.
- Assume we keep the Lee-Carter model for payoff calculation and also keep the same attachement and exhaustion points.
- What happens if the sample paths follow a *different* model?

# 7 Hedge assessment

Impact of different sample-path models on payoff:

| Model | Payoff prob. | Mean payoff |
|---|---|---|
| LC(S) | 0.40 | 0.375 |
| M5(S) | 0.53 | 0.592 |
| 2DAC | 0.80 | 0.434 |
| M6 | 0.82 | 0.710 |

Source: own calculations using population data for males in Netherlands, ages 50–104, 1971–2016. Annuity values discounted at 2% p.a.

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$\Rightarrow$ Actuarial judgement required on both counts.

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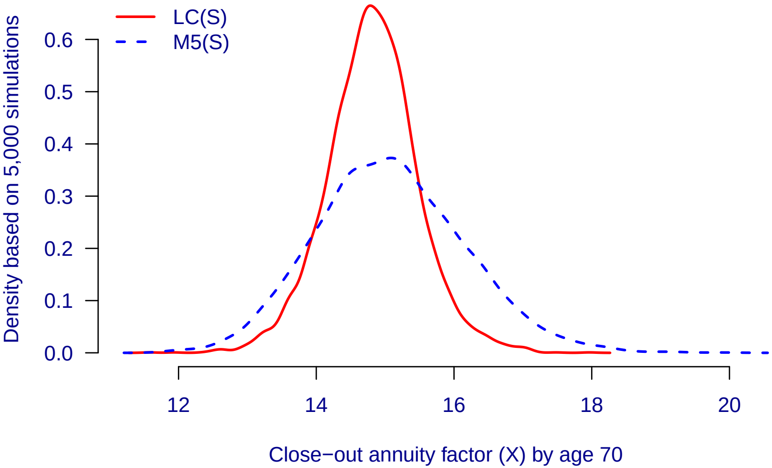
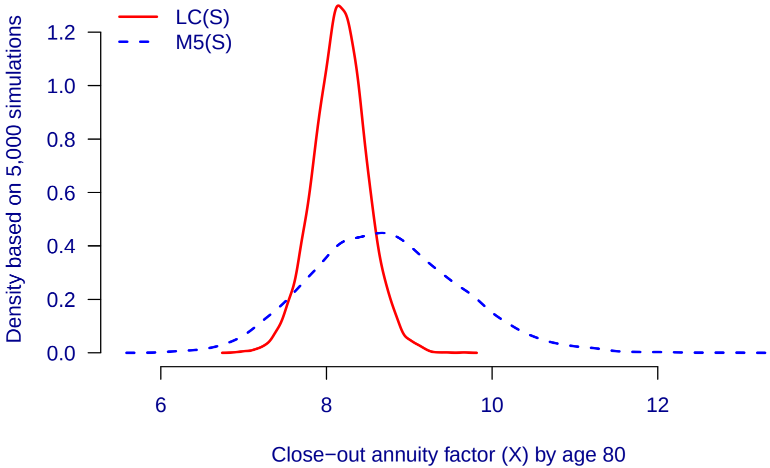
- Consider the spread at various ages under CBD model (M5). . .

# 7 M5 sample paths — age 70




VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

# 7 M5 sample paths — age 80

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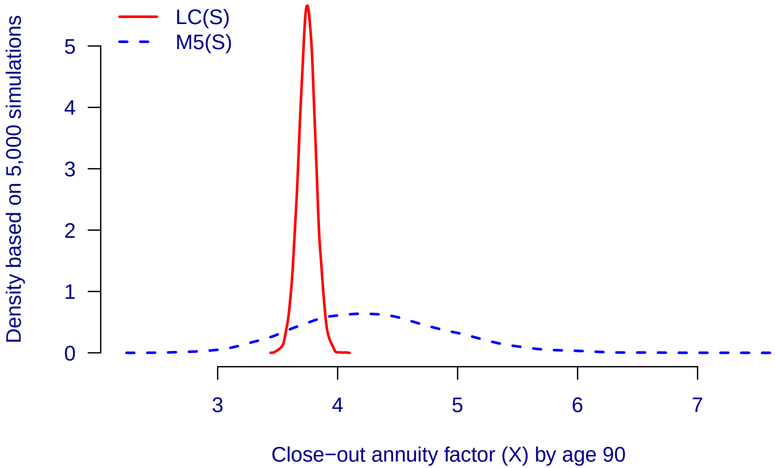
# 7 M5 sample paths — age 90




VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

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If you are interested in the above, let me know!

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More on longevity risk at www.longevitas.co.uk