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Hedging Longevity Risk: Does the Structure Matter?

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Introduction & Motivation

Current Longevity Market

- Theoretical invention of longevity-linked securities:
  - Longevity bonds, Q-forwards, Biomedical RBOs, etc.
- Real-world longevity-linked products:
  - (Unsuccessful) EIB longevity bond (2004), World Bank Chile-based longevity bond (2008, 2009)
  - (Successful) Swiss Re (2010), Aegon (2012)
  - Explanations? ⇒ Blake, Cairns & Dowd (2006, *BAJ*), Lin & Cox (2008, *IME*), Chen & Cummins (2010, *IME*), MacMinn & Brockett (2017, *JRI*)
- Future longevity market development:
  - Will insurers have different preferences for longevity products based on their structure?

⇒ Provide implication on the design of future longevity-linked securities.

Hedging Structure: Value Hedging vs. Cash-flow Hedging

In the finance literature (cf. Disatnik, Duchin & Schmidt, 2014, *RoF*), corporate hedging can take on different forms (when centered around the risk of future r):

- (Fair) value hedging: Issuing floating-rate debt—no fair value risk, but future (absolute) cash flow is uncertain
- Cash-flow hedging: Issuing fixed-rate debt—future cash flow is stable, but fair value risk (if r goes down, fair value of the debt goes up)

In the context of longevity market, different meanings/classifications (Biffis & Blake, 2009):

- Value hedging: "*implemented by using standardized hedging instruments written on transparent indices*"
- cash-flow hedging: "*the risk exposure is transferred to a counterparty which continues to pay the required cash flows*"

Hedging Structure: Value Hedging vs. Cash-flow Hedging

Here, we define:

- Value hedging: instruments with payment dependent (only) on the occurrence of some systematic longevity shock
  - (+) Medical advances; (-) Pandemic
- Cash-flow hedging: instruments with (series of) payment dependent on some mortality indices
  - Systematic longevity shock + other random effect

Objective:

- Comparison between the two choices based on a simple financial market model
  - Extend the model in MacMinn & Brockett (2017, *JRI*)
  - Managerial decision: maximizing (current) shareholder value
  - Value hedging dominates cash-flow hedging
- Numerical analysis (work in progress)

The Model

Notation

- Market is complete
- Two points of time: $t = 0$ (now) and $t = 1$ (then)
- ω: state of nature
- $p(\omega)$: standard Arrow-Debreu security price at time 0 in state ω (unit payoff in state ω and zero otherwise at $t = 1$)
- $\Psi(\omega)$: sum of A-D securities $\varepsilon \leq \omega$, $\Psi(\omega) = \int_0^\omega p(\varepsilon)d\varepsilon$
- $\Omega \equiv [0, \zeta]$: set of states of nature
  - Ω includes both economic and demographic (mortality) states

The Model

Demographic States

- In particular, we assume demographic states include:
  - Systematic longevity shock:
    - Identifiable events
    - Pandemic: Influenza, Zika, SARS, antibiotic resistance, etc.
    - Medical breakthrough: cancer cure, anti-aging pill (ELYSIUM), etc.
  - Un-identifiable (non-systematic) shock:
    - Even at the population level, the aggregate mortality rates are still affected with random shocks
- Assume N possible combinations of systematic longevity shock:
  - E.g., $N = 4$: {no shock; pandemic; medical breakthrough; both}
  - $\Omega = \{\Omega^{(1)}, \Omega^{(2)}, \ldots, \Omega^{(N)}\}$
  - $\Omega^{(i)}$ differs only from the perspective of systematic longevity shock

The Model

Reduced case: No Systematic Shock

- Assume there exists bijective function $f^{(i,j)}$, that maps two sets $\Omega^{(i)}$ and $\Omega^{(j)}$, $\forall i, j \in 1, \ldots, N$
  - Each element of $\Omega^{(i)}$ is paired with exactly one element of $\Omega^{(j)}$
  - Each element of $\Omega^{(j)}$ is paired with exactly one element of $\Omega^{(i)}$
  - No unpaired elements
- For states $\omega^{(i)} \in \Omega^{(i)}$ and $f^{(i,j)}(\omega^{(i)}) \in \Omega^{(j)}$, again, they only differ in the systematic longevity events, and are otherwise identical
- Define $\Omega^{(0)}$ as the set in which we do not consider the systematic longevity event at all
  - For $\omega^{(0)} \in \Omega^{(0)}$, similarly, there exist bijective function $f^{(0,i)}$, $i = 1, \ldots, N$
  - In particular, the updated A-D security prices in the reduced set would be

$$p^0(\omega) = \sum_1^N p(f^{(0,i)}(\omega^{(0)})), \quad \forall \omega^{(0)} \in \Omega^{(0)}$$

The Model

The Life Insurance Company

Setup is similar to MacMinn & Brockett (2017, *JRI*)

- Assumed to be in the annuity business $\Rightarrow$ More concerned with the longevity shock
- $\Gamma(\omega)$: premium income at time 1 on the book of business
- $\Delta(\omega)$: value at time 1 for assets held in reserve
- $A(\omega)$: payoff at time 1 on the annuity book
- $\Pi(\omega)$: total payoff on the business
  - $\Pi(\omega) = \Gamma(\omega) + \Delta(\omega) - A(\omega)$
- S: stock (shareholder) value at time 0
- L: liability value at time 0
- V: corporate value at time 0

The Model

Without Systematic Longevity Shock

In this case, our model setup reduces to the same as in MacMinn & Brockett (2017, *JRI*), where they show that due to the *moral hazard* problem, the stock value of the hedged firm is less than that of the unhedged firm

- Corporate management acts on behalf of shareholders and does not hedge

Here, we additionally consider the solvency requirement as imposed by the regulator:

- $P(\Pi < 0|\Omega^{(0)}) = P(\Gamma + \Delta < A|\Omega^{(0)}) = \alpha$

$\Rightarrow$ Still no hedging

We obtain below values:

- $S^u = \int_{\Omega^{(0)}} \Delta d\Psi^{(0)}$
- $L^u = \int_{\Omega^{(0)}} \Gamma d\Psi^{(0)}$
- $V^u = S^u + L^u = \int_{\Omega^{(0)}} (\Delta + \Gamma) d\Psi^{(0)}$

The Model

With Systematic Longevity Shock

- With considering systematic longevity shock, the total set increases from $\Omega^{(0)}$ to Ω
  - We assume the solvency requirement is no longer (automatically) met under Ω:
  $$P(\Pi < 0|\Omega) = P(\Gamma + \Delta < A|\Omega) > \alpha$$
  - Additional hedging strategies are needed to reduce the insolvency probability
    - ⋆ Value hedging instrument
    - ⋆ Cash-flow hedging instrument
- For fair comparison, we assume unchanged *average* level:
  - $\bar{A} = \mathbb{E}_{\Omega}(A) = E_{\Omega^{(0)}}(A)$, same for Γ and Δ

The Model

Value Hedging Instrument

Consider a value hedging instrument with a one-time payoff depending on the realization of the longevity shock.

- For simplicity, assume $N = 2$
  - $N = 1$: no systematic longevity shock, with probability q
  - $N = 2$: positive systematic longevity shock $\Rightarrow$ higher liability from the annuity book, probability $1 - q$
- Assume that the systematic shock is independent of all other factors (will be relaxed in the numerical analysis), then for each mapping $\omega^{(1)}$ and $\omega^{(2)} = f^{(1,2)}(\omega^{(1)})$, we define

$$A(\omega^{(1)}) = A(\omega^{(2)}) - K$$

$$\Rightarrow \mathbb{E}(A|\Omega^{(1)}) = \bar{A} - (1-q) \times K, \mathbb{E}(A|\Omega^{(2)}) = \bar{A} + q \times K$$

The Model

Value Hedging Instrument

For the value hedging instrument, it needs to compensate the insurer in the case when $N = 2$:

- Payoff:
  - $B(\omega) = -(1-q) \times K, \forall\, \omega \in \Omega^{(1)}$
  - $B(\omega) = q \times K, \forall\, \omega \in \Omega^{(2)}$
- Solvency requirement:

$$
\begin{aligned}
P(\Pi < 0) &= P(\Gamma + \Delta + B < A|\Omega^{(1)}) \times q + P(\Gamma + \Delta + B < A|\Omega^{(2)}) \times (1-q) \\
&= P(\Gamma + \Delta < \bar{A}|\Omega^{(1)}) \times q + P(\Gamma + \Delta < \bar{A}|\Omega^{(2)}) \times (1-q) \\
&= P_{\Omega}(\Gamma + \Delta < A) = \alpha.
\end{aligned}
$$

- $S^V = S^u = \int_{\Omega^{(0)}} \Delta d\Psi^{(0)}$
- $L^V = L^u = \int_{\Omega^{(0)}} \Gamma d\Psi^{(0)}$

The Model

Cash-flow Hedging Instrument

Generally, cash-flow hedging instruments provide a serial of cash payments that depend on future mortality realizations.

- Conceptually, we expect to see more variations of payout amounts from cash-flow hedges
- Here, we define a cash-flow hedging instrument as one that makes payoff $B(\omega) = A(\omega)$, $\forall \omega \in \Omega$
  - Same assumption as in MacMinn & Brockett (2017, *JRI*)
- Solvency condition: $P(\Pi < 0) = 0$
- Shareholder value: $S^{cf} = S^u - P^u$, where $P^u = \int_0^\delta (A - (\Gamma + \Delta))d\Psi$ is the implicit put option value
- Liability value: $L^{cf} = L^u + P^u$

The Model

Comparison of Hedging Instruments

Theorem

If the corporate executive officer has a fiduciary responsibility to act in the interest of shareholders and at the same time faces regulation pressure to maintain a sufficiently small bankruptcy probability, then they would prefer to use value hedging instruments over cash-flow hedging instruments.

- Value hedges:
  - Relieve the insolvency issue by offer protection to systematic longevity risk
  - Also maintains the implicit put option value that is enjoyed by the shareholders
- Cash-flow hedges:
  - Redistribute too much value from the shareholder to the policyholder
  - Unwelcome from the executive officer's perspective

Numerical Analysis

Numerical Analysis

- Relax the simple assumptions in the model section
  - $N > 2$, more periods, etc.
- Incomplete market $\Leftarrow$ General equilibrium theory
- Optimal hedging choices

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