Small Population Bias and Sampling Effects in Stochastic Mortality Modelling

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Background and Significance

Why analyse small population

- Experiencing faster mortality improvement, lower interest rate, more pressure on pension funds.
- Most pension schemes are less than 1% of national population.
- Significantly more variability exhibited for mortality rates of small population
- Stochastic models might poorly fit small populations

Motivation

For small population:

- Greater sampling variation of deaths causes increased uncertainty of parameter estimates and high levels of uncertainty on projected mortality rates.
- Diverge between future realized rates and projections, future sampling variation, uncertain projection.

Stochastic Model and Data

- Stochastic Model:

$$
\begin{aligned}
D(t,x)|\theta_1 &\sim \text{Pois}(m(\theta_1,t,x)E(t,x)) \\
m(\theta_1,t,x) &= -\log(1-q(\theta_1,t,x)) \\
\text{logit } q(\theta_1,t,x) &= \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \kappa_t^{(3)}((x-\bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)}
\end{aligned}
$$

- Data: Benchmark exposure $E_0(t,x)$ and corresponding deaths count $D_0(t,x)$ of the males in England and Wales (EW) in the HMD database, during year 1961 to 2011, aged 50-89 last birthday.

- Simulation Method
  - Estimate θ_1 for benchmark population, denoted as $\hat{\theta}_{1,0}$
  - Construct small population $E_w(t,x) = wE_0(t,x)$ for $w = 1, 0.1, 0.01, 0.001,$
  - (Re-) Simulate $D_w(t,x)|\hat{\theta}_{1,0} \sim \text{Pois}(m(\hat{\theta}_{1,0},t,x)wE_0(t,x))$
  - Estimate θ_1 for $D_w(t,x)$, denoted as $\hat{\theta}_1^w$.

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Two-Stage and Bayesian Approaches

- Two-stage approach leads to biased estimates of volatility for small populations
  - Large sampling variation affects latent parameter estimation, with significant noise obscuring the true signal (Cairns et al. 2011)
  - Result in non-negligible bias to the parameter estimation of the projecting model (Chen, Cairns and Kleinow 2016)
  - Over-fit the short cohorts (Cairns et al. 2019)
- Bayesian approach offers a way to avoid or reduce this bias by:
  - Combining the Poisson likelihood with the projecting time series models
  - The estimated latent parameters are restricted to be more like proposed time series models when projecting models dominate while modelling small populations.
  - Using more informative prior distribution with the knowledge of the larger benchmark population.
  - Better estimation for short cohorts.

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Prior Distributions

- $(\kappa^{(1)}_{t_1}, \kappa^{(2)}_{t_1}, \kappa^{(3)}_{t_1}) \propto 1$,

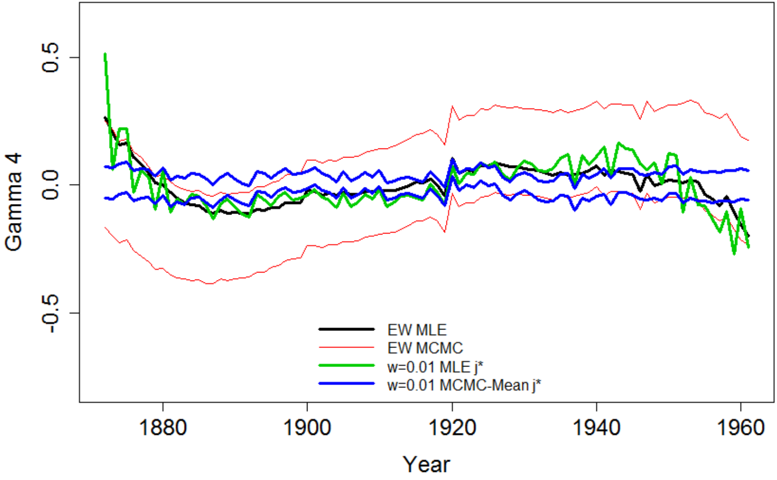
- $\boldsymbol{\kappa}_t = \boldsymbol{\kappa}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\epsilon}_t$ for $t \leq t_2$,
  - $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu) \propto 1$,
  - $\boldsymbol{\epsilon}_t \sim MVN(0, \boldsymbol{V}_\epsilon)$, i.i.d there dimensional multi-variate normal distribution independent of t,

- $\boldsymbol{V}_\epsilon \sim InverseWishart(\nu, \boldsymbol{\Sigma})$
  - MCMC-Mean: Fix the mean of the prior to $\hat{\boldsymbol{V}}_\epsilon^{\text{EW}}$
  - MCMC-Mode: Fix the mode of the prior to $\hat{\boldsymbol{V}}_\epsilon^{\text{EW}}$

- $\gamma^{(4)}_c = \alpha_\gamma \gamma^{(4)}_{c-1} + \epsilon_c$ for $c > t_1 - x_{n_a}$,
  - i.i.d $\epsilon_c \sim N(0, \sigma^2_\gamma)$,
  - $\alpha_\gamma \propto (1 - \alpha^2_\gamma)^g$ for $|\alpha_\gamma| < 1$,
  - $\sigma^2_\gamma \sim$ Inverse Gamma (a_γ, b_γ)

- $\gamma^{(4)}_{c_1} \sim N(0, \frac{\sigma^2_\gamma}{1-\alpha^2_\gamma})$

Results Cohort Effects



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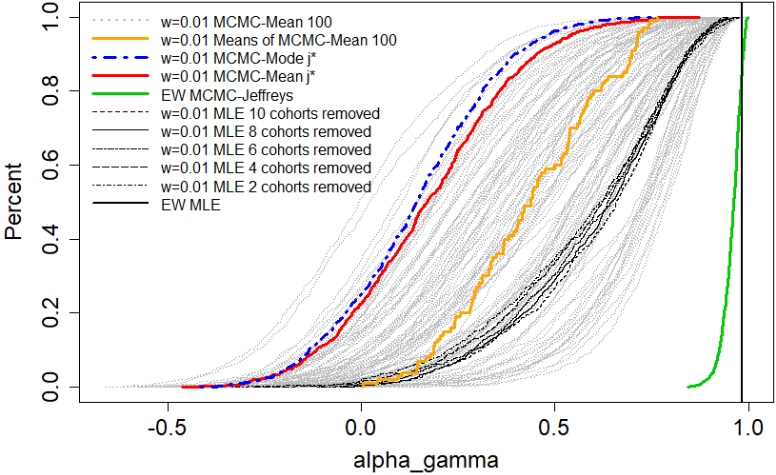
Results α_γ



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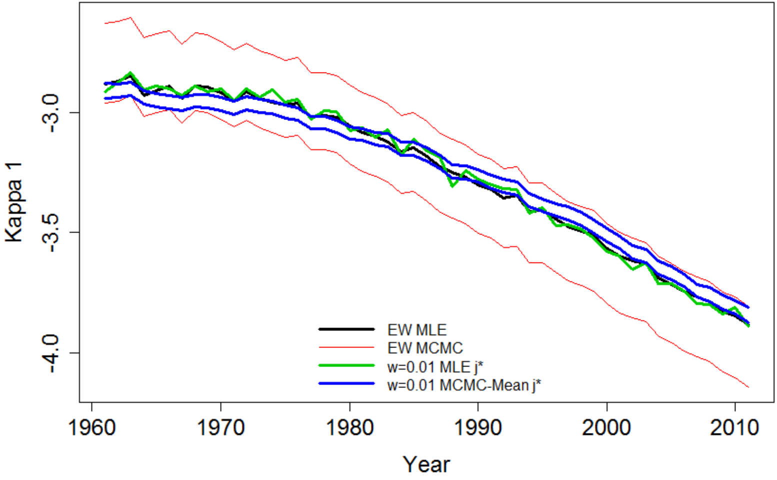
Results $\kappa^{(1)}$



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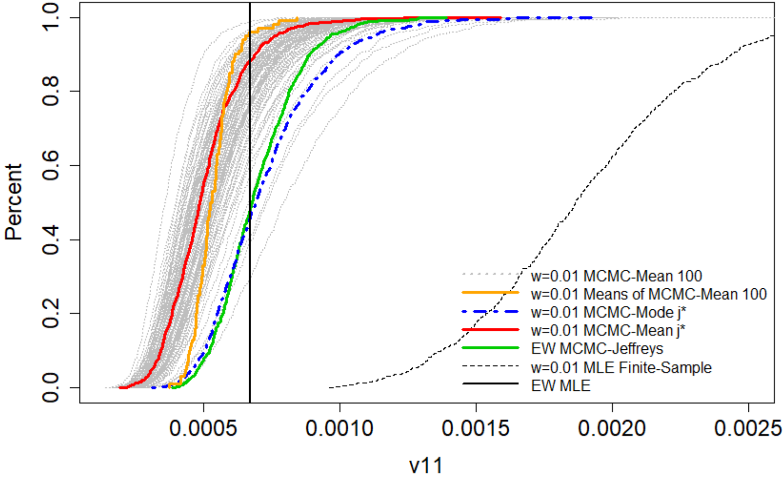
Results $\boldsymbol{V}_{\epsilon}(1,1)$



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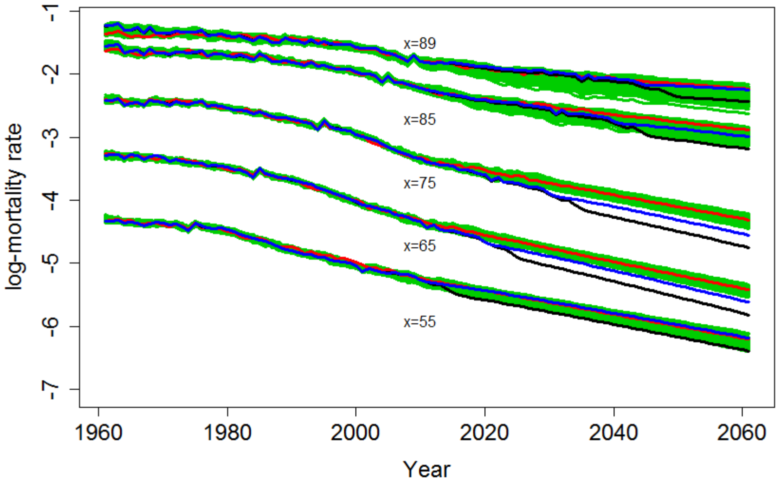
Results Projection



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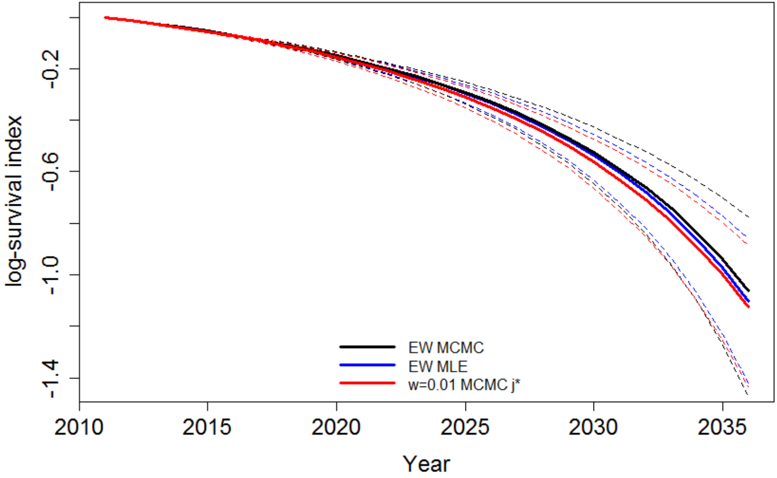
Results Survival Index



Results Longevity Risk of A Temporary Annuity

	$i = 4\%$	$i = 2\%$	$i = 1\%$	$i = 0.5\%$	$i = 0\%$
EW-MCMC	5.27	6.28	6.86	7.16	7.47
j^*-MCMC	4.95	5.83	6.33	6.59	6.84
EW-MLE	4.24	5.04	5.50	5.74	5.98
j^*-MLE	5.12	6.08	6.59	6.85	7.09

Table: The longevity risk (in percentage) of the temporary annuity.

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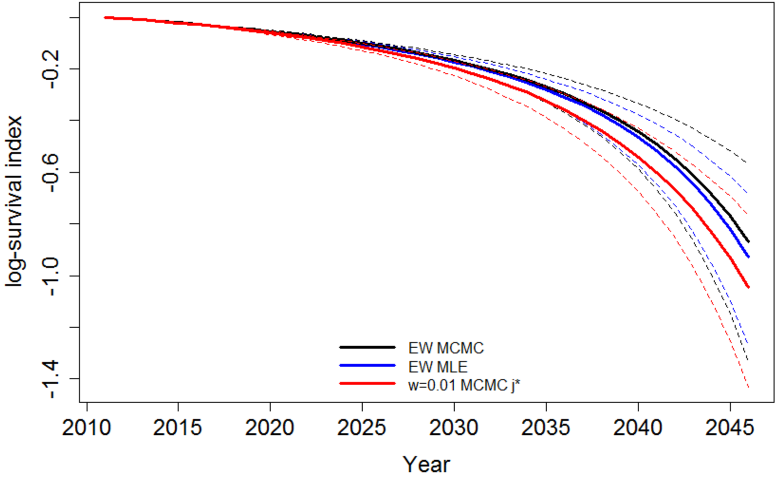
Results Survival Index



Results Longevity Risk of A Temporary Annuity Deferred by Ten Years

	$i = 4\%$	$i = 2\%$	$i = 1\%$	$i = 0.5\%$	$i = 0\%$
EW-MCMC	7.02	8.14	8.78	9.1	9.45
j^*-MCMC	7.23	8.3	8.9	9.23	9.56
EW-MLE	5.50	6.35	6.82	7.06	7.32
j^*-MLE	7.57	8.8	9.38	9.77	10.17

Table: The longevity risk (in percentage) of the deferred annuity.

Thank You!

Questions?