Bargaining for Over-The-Counter Risk Redistributions: The Case of Longevity Risk

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Introduction

Contribution

- Goal: redistributing stochastic variables (risk) Over-The-Counter in "fairest way"
- Setting:
  - Cooperative game-theoretic model
  - Redistribution obtained via swap-contracts
- Allow for all forms of redistributions
- Key issue:
  - No liquid market
  - Trade Over-The-Counter

Focus: Longevity risk; Why?

- Illiquid market, where there are no equilibrium prices
- Redistributions between annuities and death benefits (cf. Wang et al. (2010))
- Literature shows that longevity risk is prominent for pension funds and life insurers. See e.g. Hári et al.(2008) and Coughlan et al. (2007)
- Prices are heavily debatable (see Bauer et al (2010)). Two focusses:
  - equivalent utility pricing principle (Cui (2008) and Cox, Lin and Pedersen (2010))
  - Prices obtained directly from (scarce) longevity-linked bonds in the market (Lin and Cox (2005))

Conclusions

- We model the OTC bargaining problem as a Non-Transferable Utility (NTU) game.
- We allow for heterogeneous beliefs regarding the underlying probability distribution.
  Very relevant for applications with longevity risk.
- Calibrated example shows hedge benefit is large.

Longevity risk

Longevity risk

- Longevity risk: Risk that individuals live longer or shorter than expected
  - Micro longevity risk diminishes if pool size is sufficiently large (see Oliveiri and Pitacco (2001), Milevsky, Promislow and Young (2006) and Hári et al. (2008))
  - Macro longevity risk: Risk that the population as a whole lives longer or shorter
    - Systematic part of longevity risk

We focus on macro longevity risk.

Longevity model

- Key issue: large variety of longevity risk models
- Prominent examples:
  - Lee-Carter model (1992)
  - Cairns-Blake-Dowd model (2006, 2008)
  - P-spline model (Currie, Durban and Eilers (2004))
  - ⋮
- Different data used for obtaining longevity distribution
  - For instance, different horizon of data

The model

Firms redistribute risk in order to increase expected utility of the present value of the Net Asset Value at a future evaluation date T:

$$X_i(T) \equiv \frac{NAV_i(T)}{(1+r)^T} = \frac{A_i(T) - L_i(T)}{(1+r)^T},$$

where

- $A_i(T)$ is the asset value at time T
- $L_i(T)$ the value of the liabilities. Typically:

  $$L_i(T) = BEL_i(T) + MVM_i(T),$$

  where $BEL_i(T)$ is the best estimate of future liability payments and $MVM_i(T)$ the market value margin (e.g. according to Solvency II) (a risk loading)
- r is the risk-free rate

- Solvency II: set financial return equal to risk-free rate.

  We have

$$A_i(t) = (1+r)A_i(t-1) - \widetilde{L}_{i,t},$$

  where $\widetilde{L}_{i,t}$ is the liability payment at time t.
- Hence, we obtain

$$X_i(T) = A_i(0) - \sum_{\tau=1}^{T} \frac{\widetilde{L}_{i,\tau}}{(1+r)^\tau} - \frac{L_i(T)}{(1+r)^T}.$$

Important to note:

- In the current literature, redistributions have longer maturity and intermediate payment dates
- Then, every year there is a payment
- In our model, we allow for this, namely as $T = T^{\max}$:

$$X_i(T^{\max}) = A_i(0) - \sum_{\tau=1}^{T^{\max}} \frac{\widetilde{L}_{i,\tau}}{(1+r)^{\tau}}$$

- Rolling contract every year more dynamic as we can take into account that
  - The mortality model can be updated
  - There has been attrition
  - New participants have entered the fund
  - New regulations have been introduced
  - Poor asset returns increase need for hedging longevity
- Moreover, we obtain in a calibrated example that the standard deviation of $X_i(1)$ is approximately 50% of standard deviation of $X_i(T^{\max})$

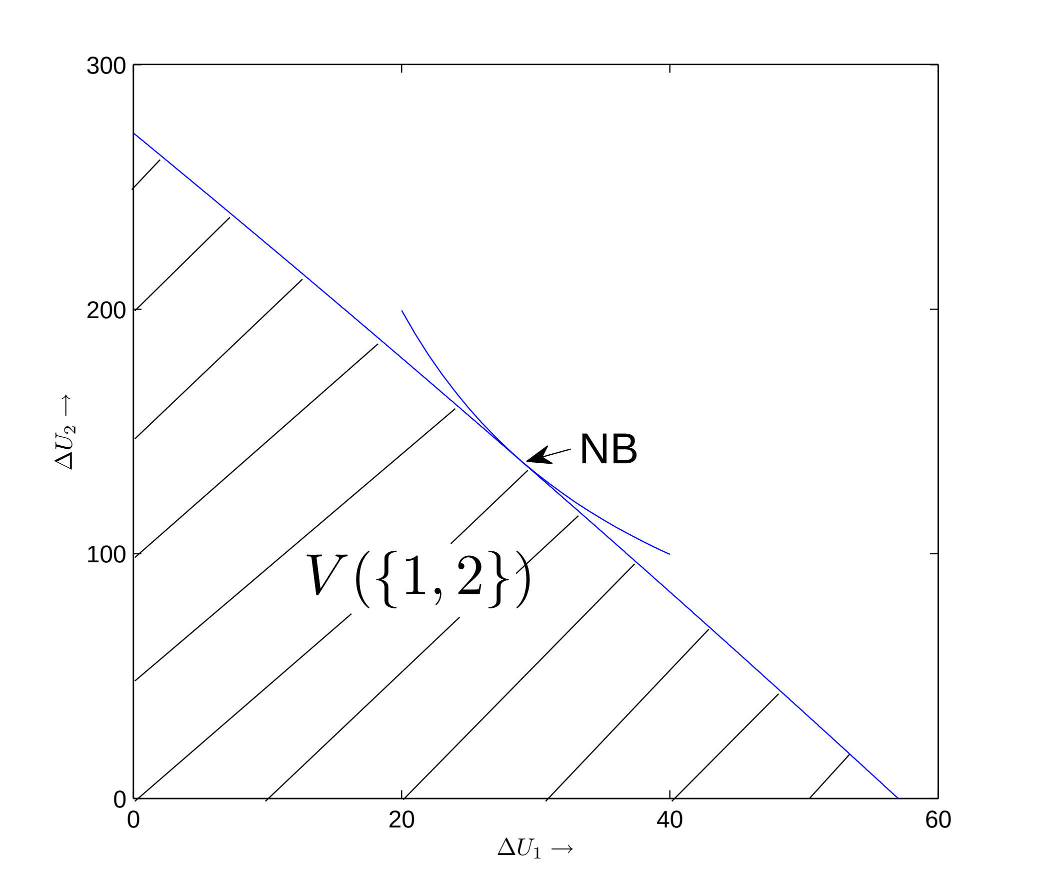
The Game

Assumptions

- Firms use a Von-Neuman-Morgenstern utility function u_i such that $u_i' > 0$, $u_i'' < 0$
- Let the risk profiles be given by $(X_i(T))_{i \in N}$ and the (heterogeneous) probability measures by $(\Omega, (\mathbb{P}_i)_{i \in N})$, where Ω finite
- There is complete information about the risk profiles , utility functions and beliefs regarding the underlying probability measures of all firms

What is a Non-Transferable Utility game (N, V)?

Nash-Bargaining problem (Nash (1950)) in case of 2 firms:



Bargain for $(X_i^{\text{post}})_{i \in N}$ such that $\sum_{i \in N} X_i^{\text{post}} = \sum_{i \in N} X_i$

- Firms valuate a risk using

$$\Delta U_i(X_i^{\text{post}}) = E^{\mathbb{P}_i}[u_i(X_i^{\text{post}}) - u_i(X_i)]$$

- Than, the we define the game:
$V(S) = \Big\{ a \in \mathbb{R}^S \Big| \exists (X_i^{\text{post}})_{i \in S} \in \mathbb{R}^{\Omega \times S}$
$: \sum_{j \in S} X_j^{\text{post}} = \sum_{i \in S} X_i, a \leq (\Delta U_i(X_i^{\text{post}}))_{i \in S} \Big\}$,
for all $S \subset N$.

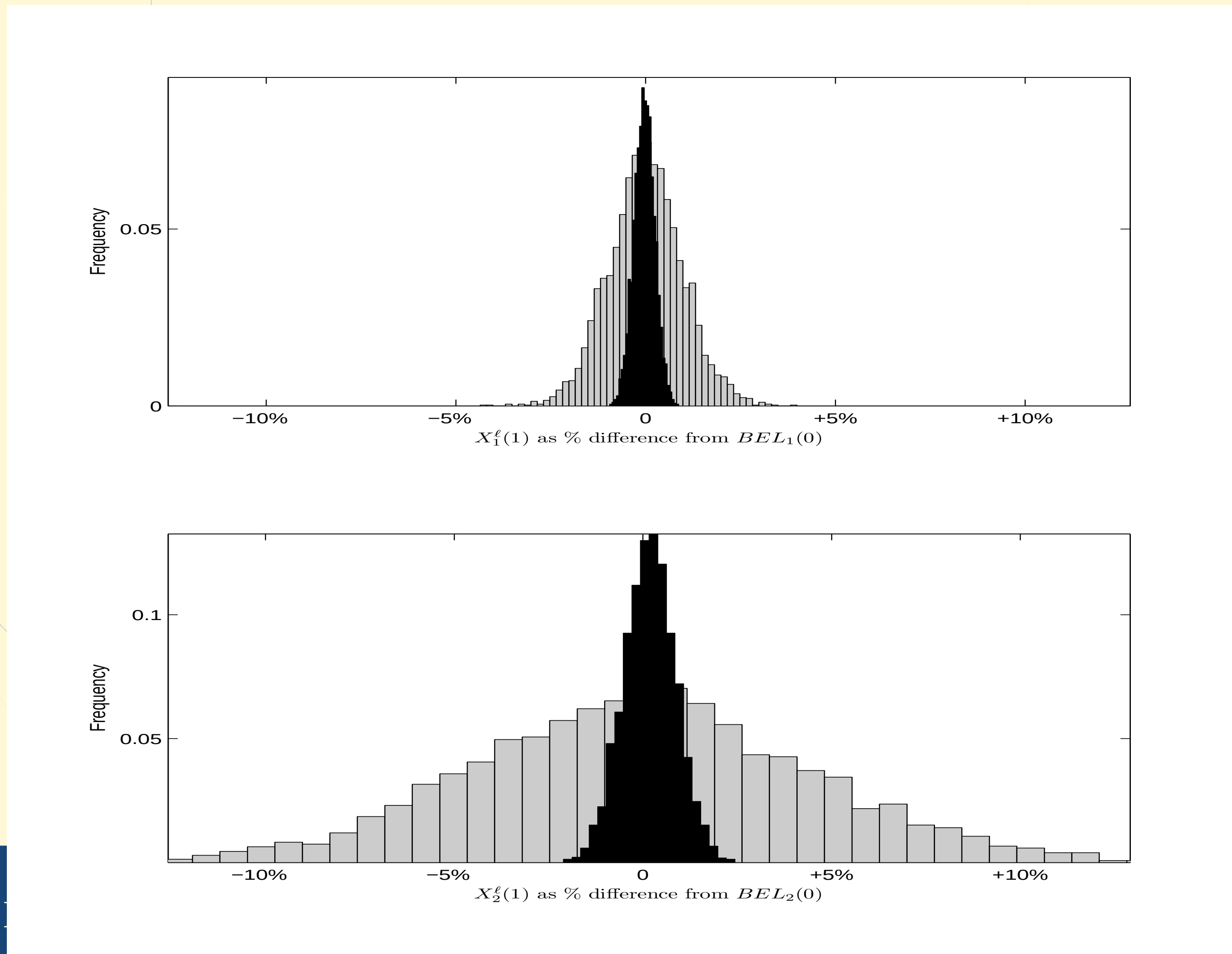
- Pareto optimal
  - Every $a \in V(N)$ such that there does not exist a redistribution $(X_i^{\text{post}})_{i \in N}$ such that $(\Delta U_i(X_i^{\text{post}}))_{i \in N} \gneqq a$
- Individually Rational
  - $\Delta U_i(X_i^{\text{post}}) \geq 0$ for all firms in N
- Core-element
  - For every element for the core, there does not exist a subset of firms that can form a redistribution that is weakly beneficial for all members of this set and strict for at least one firm
  - Core is non-empty

Numerical implementation

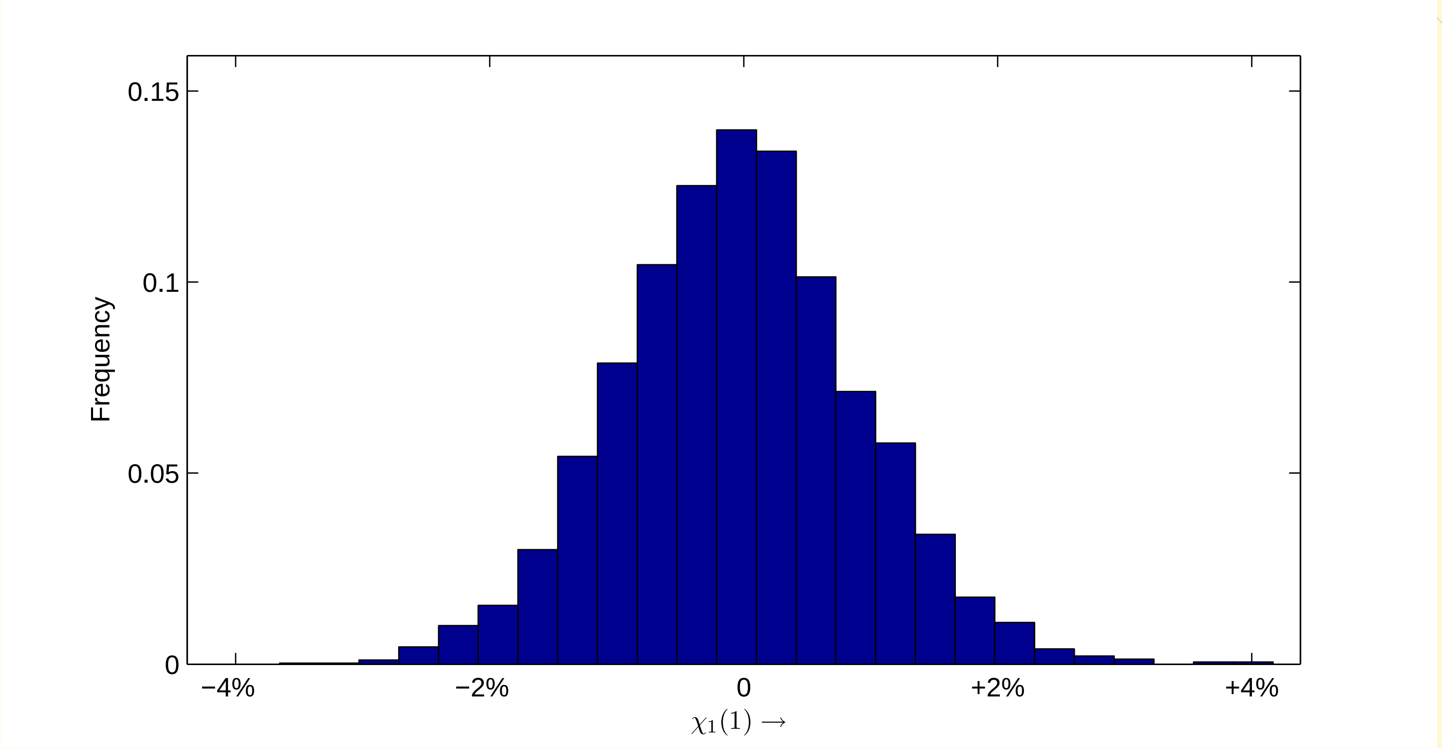
Calibration

- Interest rate is given by $r = 0.03$
- $MVM_i(T) = 0$ for all $i \in N$ and for all $T \geq 1$
- All firms use same Lee-Carter model and same data-set: $\mathbb{P}_i = \mathbb{P}$ (will be relaxed)
- We use data about a "realistic" liability portfolio of a pension fund
- We assume that the pension fund has 50,000 participants; each receive 1 unit a year after retirement;
- For the death benefit insurer, we assume:
  - Fixed pay-off of 10 units in case of death before retirement
  - young participants
  - varying size

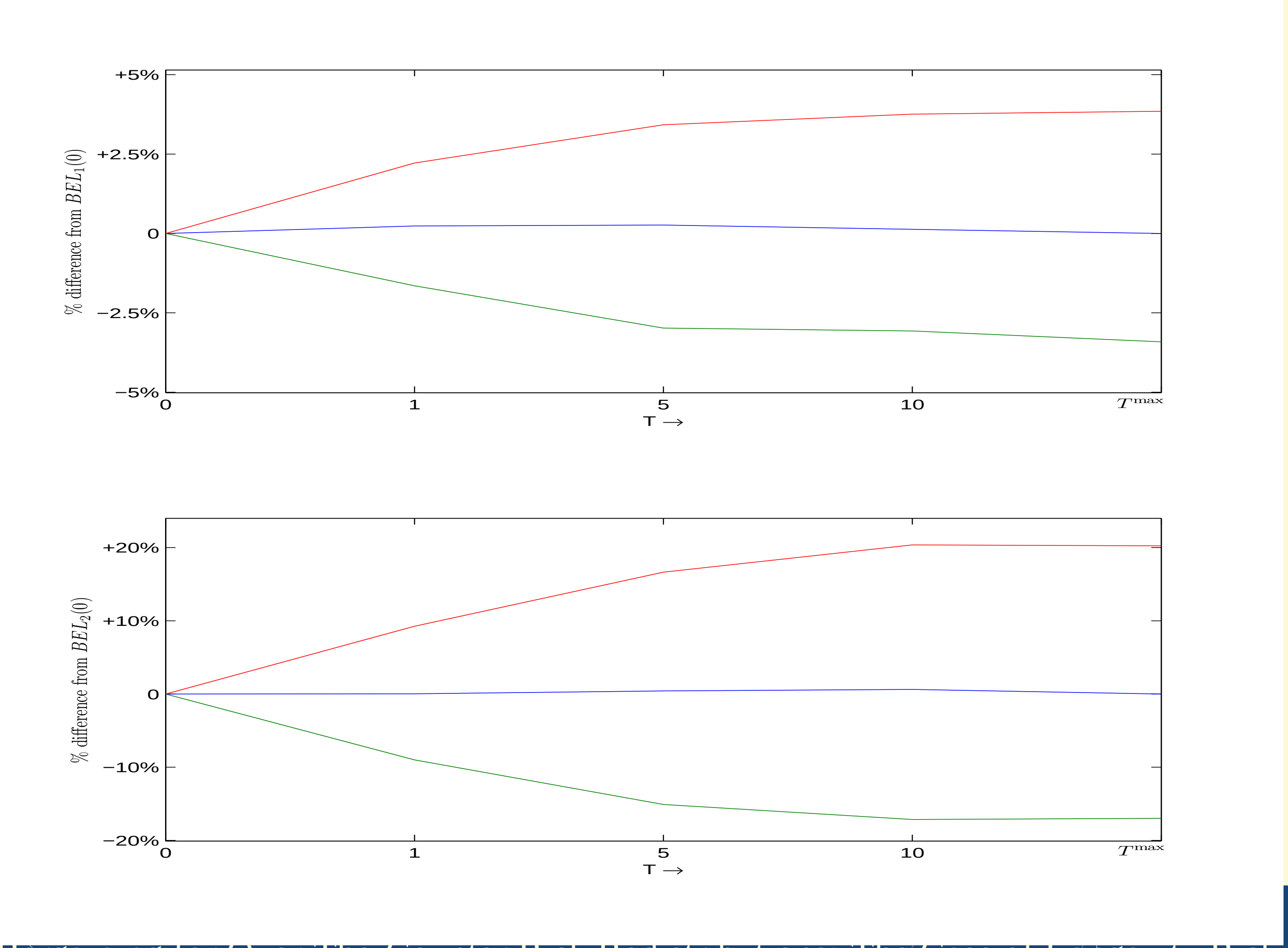
Let there be a average age pension fund and death benefit insurer. Value liabilities $X_i^\ell(1) = A_i(0) - X_i(1)$; prior and posterior:



Pay-off of swap only time $T = 1$:



$X_i(T)$ as function of T,
mean, 2.5%-quantile and 97.5-quantile:



Conclusion

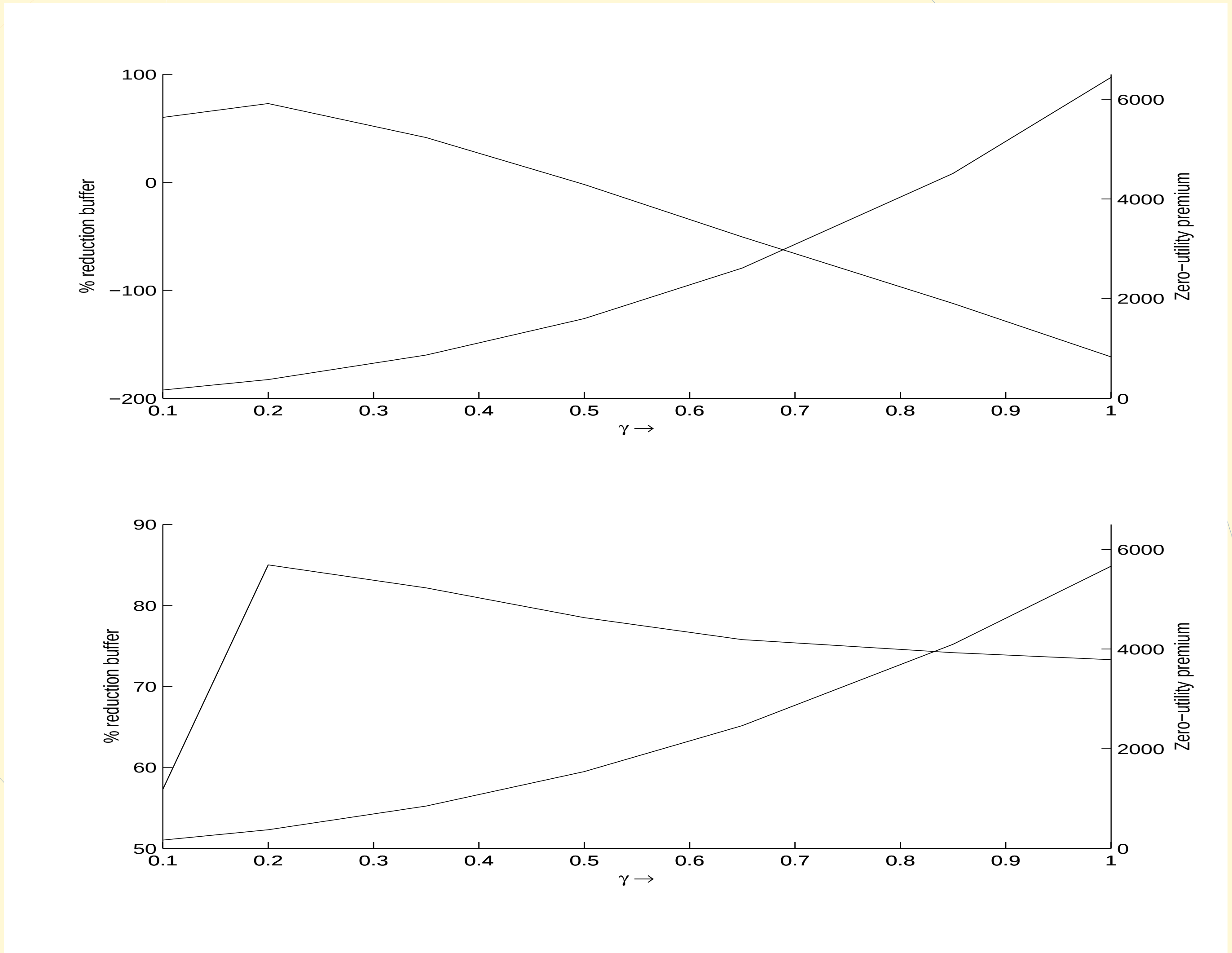
zero-utility premium $= p : \Delta U_i(X_i^{\text{post}} - p_i) = 0 \quad \forall i \in N$,

buffer $= \frac{Q_{0.975}(X_i^\ell(T)) - E[X_i^\ell(T)]}{E[X_i^\ell(T)]}$.

- We obtain:
  - Risk redistribution has worth approximately 375 for both firms (zero-utility principle), in case of an average age pension fund and a death benefit insurer
  - buffer reduces from 1.98% to 0.54% for pension fund and from 9.25% to 1.34% for the death benefit insurer:

T	Zero-Utility premium		% reduction buffer	
	Pension fund	Insurer	Pension fund	Insurer
1	377	374	73%	86%
5	1285	1246	85%	93%
10	1717	1650	84%	93%
$T^{\max}$	2322	2201	82%	93%

Gains as function of size γ of death benefit insurer:



Conclusion

- The case of two death benefit insurers and one pension fund.
- Let two death benefit insurers have size $\frac{\gamma}{2}$, so that total risk equals two-firm problem previously
- Then, for $T = 1$:

	Pension fund	Insurers ($i = 2, 3$)
zero-utility premium	168	168
$\%$ reduction buffer	80%	78%

Heterogeneous probability measures

- According to Borch (1962) and homogeneous probability measures, all Pareto optimal outcomes are obtained using $\sum_{j \in N} X_j(T)$ only
- Here, heterogeneous probability measures $(\mathbb{P}_i)_{i \in N}$ on $\sum_{j \in N} X_j(T)$only are relevant for determining Pareto set
- Therefore, we discretize $\sum_{j \in N} X_j(T)$ by a partition of the interval
- Every probability measure will result in different probabilities on “attaining” a part of the partition en