IMPORTANCE OF THE REFERENCE POPULATION FOR MULTI-POPULATION MODELS

Søren Kjærgaard, Jim Oeppen , Malene Kallestrup-Lamb, Christian M. Dahl , Marie-Pier Bergeron-Boucher, and Rune Lindahl-Jacobsen

Max-Planck Odense Center on the Biodemography of Aging

September 8, 2017

Introduction

- Sub-population mortality has mainly been forecast using death rates models. $m_{t,x} = \frac{D_{t,x}}{Exp_{t,x}}$

Introduction

- Sub-population mortality has mainly been forecast using death rates models. $m_{t,x} = \frac{D_{t,x}}{Exp_{t,x}}$
- Life table deaths provides an interesting alternative

Introduction

- Sub-population mortality has mainly been forecast using death rates models. $m_{t,x} = \frac{D_{t,x}}{Exp_{t,x}}$
- Life table deaths provides an interesting alternative
- This project extends and generalises models suggested by Oeppen and Bergeron-Boucher to sub-population mortality using data for socio-economic sub-groups.

Introduction

- Sub-population mortality has mainly been forecast using death rates models. $m_{t,x} = \frac{D_{t,x}}{Exp_{t,x}}$
- Life table deaths provides an interesting alternative
- This project extends and generalises models suggested by Oeppen and Bergeron-Boucher to sub-population mortality using data for socio-economic sub-groups.
- We present five models and compare forecasts from each model with the Lee-Carter model.

Compositional Data Analysis

- Life table deaths (d_x) measure the number of age-specific deaths in a initial population.

Compositional Data Analysis

- Life table deaths (d_x) measure the number of age-specific deaths in a initial population.
- Life table deaths is characterised by: $d_x \geq 0$ & $\sum d_x = 1$

Compositional Data Analysis

- Life table deaths (d_x) measure the number of age-specific deaths in a initial population.
- Life table deaths is characterised by: $d_x \geq 0$ & $\sum d_x = 1$
- Thus, life table deaths is compositional data

Compositional Data Analysis

- Life table deaths (d_x) measure the number of age-specific deaths in a initial population.
- Life table deaths is characterised by: $d_x \geq 0$ & $\sum d_x = 1$
- Thus, life table deaths is compositional data
- Only relative information is important $\rightarrow$ scale invariant

Compositional Data Analysis

- Aitchison (1986) describes compositional data analysis that transform compositional data to the real space

Compositional Data Analysis

- Aitchison (1986) describes compositional data analysis that transform compositional data to the real space
- Oeppen (2008) uses the clr transformation and SVD for the analysis of d_x

Compositional Data Analysis

- Aitchison (1986) describes compositional data analysis that transform compositional data to the real space
- Oeppen (2008) uses the clr transformation and SVD for the analysis of d_x

$$clr(d_{t,x}) = ln\left(\frac{d_{t,x}}{g_t}\right)$$

Compositional Data Analysis

- Aitchison (1986) describes compositional data analysis that transform compositional data to the real space
- Oeppen (2008) uses the clr transformation and SVD for the analysis of d_x

  $$clr(d_{t,x}) = ln\left(\frac{d_{t,x}}{g_t}\right)$$

  Can the framework suggested by Oeppen (2008) be used for mortality in socio-economic subgroups?

Five age constrained models

- Independent model (M1)

$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = b_{x,g}k_{t,g} + \epsilon_{t,x,g}, \tag{1}$$

- Vertical model (M2)

$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = b_{x}k_{t,g} + \epsilon_{t,x,g} \tag{2}$$

- Horizontal model (M3)

$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = b_{x,g}k_{t} + \epsilon_{t,x,g} \tag{3}$$

Models

- Relative model (M4)

$$clr(d_{t,x}^{N} \ominus \alpha_x) = B_x K_t + \epsilon_{t,x},$$

$$clr(d_{t,x,g} \ominus \alpha_{x,g} \ominus d_{t,x}^{N}) = b_{x,g} k_{t,g} + \epsilon_{t,x,g}, \qquad (4)$$

- Three dimensional model (M5)

$$clr(d_{t,x,g} \ominus \alpha_{x,g}) = k_t \beta_x \gamma_g + \epsilon_{t,x,g}, \qquad (5)$$

Including dependence among the mortality indexes

- M1, M3 and M5 estimate one k_t parameter vector
- k_t estimates in M2 (vertical model) are fitted to a cointegrated VAR model

$$\Delta k_t = \Pi k_{t-1} + \sum_{i=1}^{p-1} \Gamma \Delta k_{t-i} + B + \epsilon_t$$

- M5 (relative model) estimates k_t parameter vectors for each group and one national

Common trends among the socio-economic groups

$$\Delta k_t = \Pi k_{t-1} + \sum_{i=1}^{p-1} \Gamma \Delta k_{t-i} + B + \epsilon_t \tag{6}$$

- The cointegrated VAR model can be used to determine the number of common trends.

Common trends among the socio-economic groups

$$\Delta k_t = \Pi k_{t-1} + \sum_{i=1}^{p-1} \Gamma \Delta k_{t-i} + B + \epsilon_t \tag{6}$$

- The cointegrated VAR model can be used to determine the number of common trends.
- k_t can be decomposed into n-r common stochatic trends $\alpha'_{\perp} k_t$ and r cointegrating relations $\beta' k_t$

Common trends among the socio-economic groups

$$\Delta k_t = \Pi k_{t-1} + \sum_{i=1}^{p-1} \Gamma \Delta k_{t-i} + B + \epsilon_t \tag{6}$$

- The cointegrated VAR model can be used to determine the number of common trends.
- k_t can be decomposed into n-r common stochatic trends $\alpha'_{\perp} k_t$ and r cointegrating relations $\beta' k_t$
- As the system has to balance $\rightarrow$ the rank of Π can be used to determine r

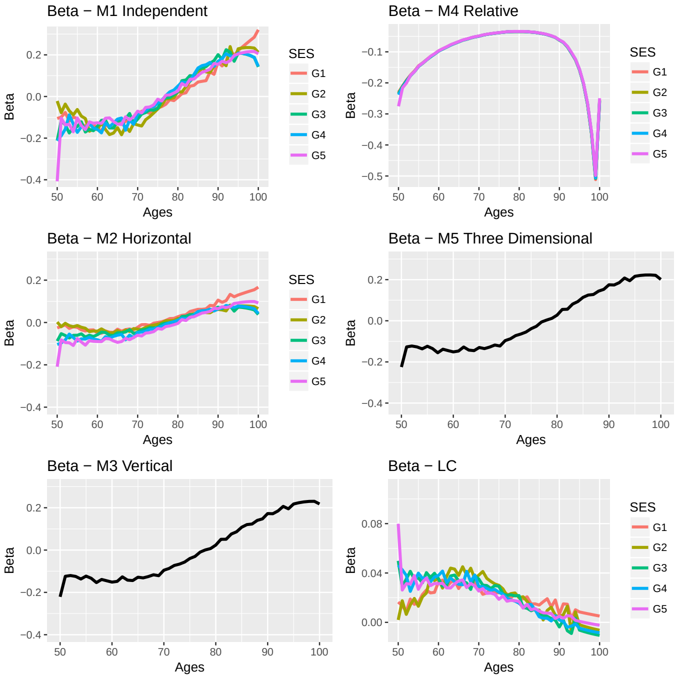
Common trends among the socio-economic groups

$$\Delta k_t = \Pi k_{t-1} + \sum_{i=1}^{p-1} \Gamma \Delta k_{t-i} + B + \epsilon_t \quad (6)$$

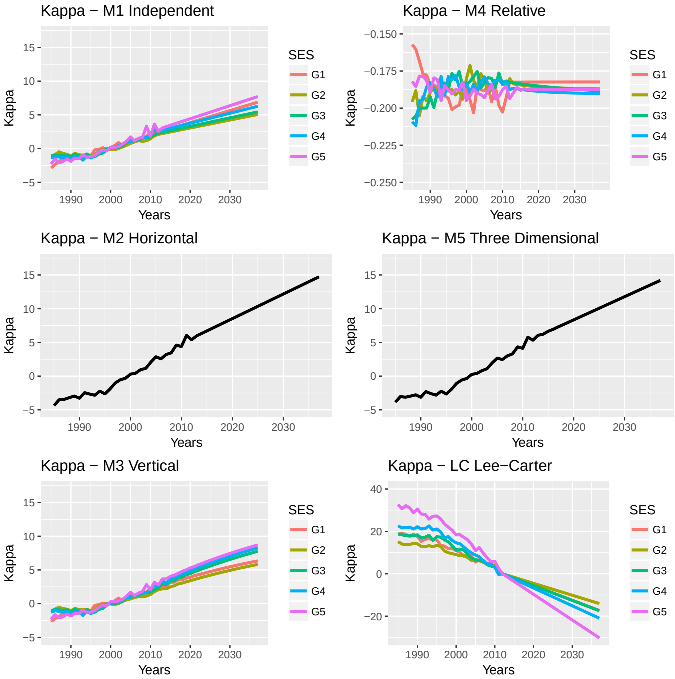
- The cointegrated VAR model can be used to determine the number of common trends.
- k_t can be decomposed into n-r common stochatic trends $\alpha'_{\perp} k_t$ and r cointegrating relations $\beta' k_t$
- As the system has to balance $\rightarrow$ the rank of Π can be used to determine r
- Using the Johansen test we find 2 common stochatic trends

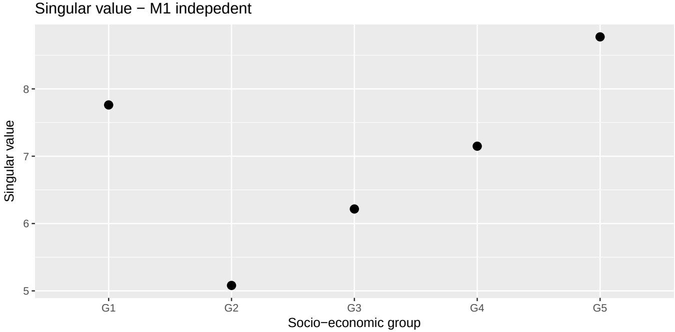
Data

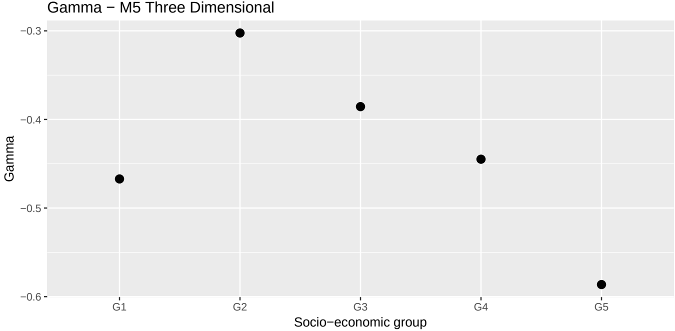
- Mortality data for Danish males are constructed using an affluence index weighting income and wealth
- The data publish in Cairns et al. (2016)

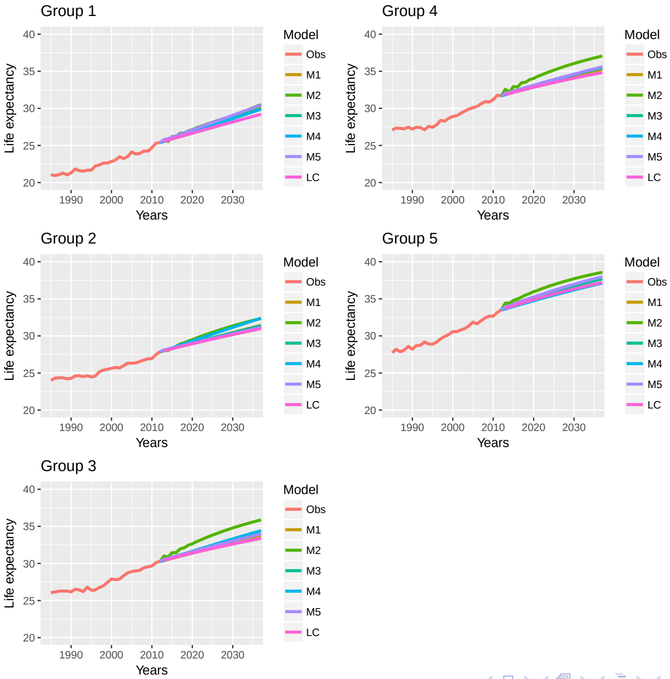


Beta − M1 Independent
Beta − M4 Relative
Beta − M2 Horizontal
Beta − M5 Three Dimensional
Beta − M3 Vertical
Beta − LC
Beta
Ages
SES
G1
G2
G3
G4
G5

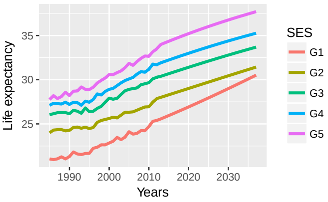

Kappa − M1 Independent
Kappa − M4 Relative
Kappa − M2 Horizontal
Kappa − M5 Three Dimensional
Kappa − M3 Vertical
Kappa − LC Lee−Carter
Kappa
Years
SES
G1
G2
G3
G4
G5


Singular value − M1 indepedent
Singular value
8
7
6
5
G1
G2
G3
G4
G5
Socio−economic group

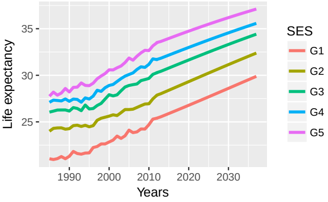

Gamma − M5 Three Dimensional
Gamma
−0.3
−0.4
−0.5
−0.6
G1
G2
G3
G4
G5
Socio−economic group


Group 1
Group 2
Group 3
Group 4
Group 5
Life expectancy
Years
40
35
30
25
20
1990
2000
2010
2020
2030
Model
Obs
M1
M2
M3
M4
M5
LC

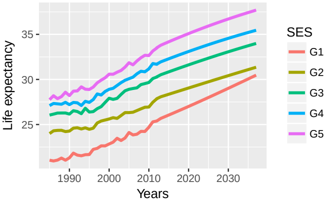
M1 − Independent



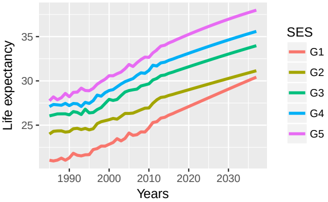
M4 − Relative



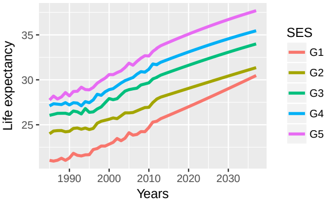
M2 − Horizontal



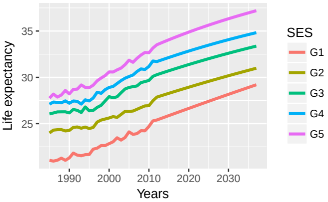
M5 − Three dimensional

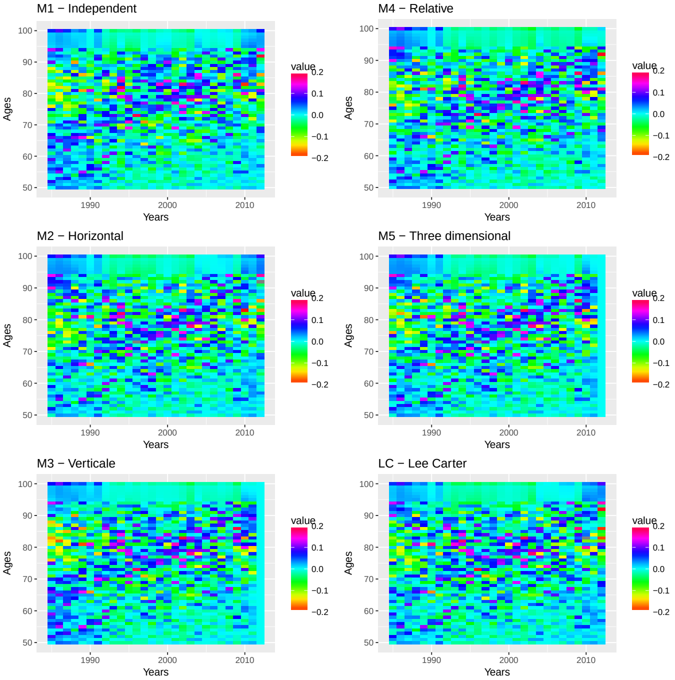


M3 − Vertical



LC − Lee−Carter




M1 − Independent
M4 − Relative
M2 − Horizontal
M5 − Three dimensional
M3 − Verticale
LC − Lee Carter
Ages
Years
value
0.2
0.1
0.0
−0.1
−0.2
100
90
80
70
60
50
1990
2000
2010

Residual analysis

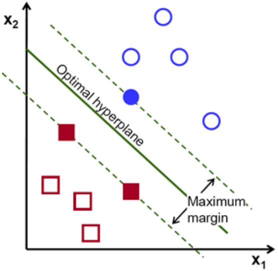
- Cause of death specific information is available for the Danish data
- Question: Is there any relation between misfits in the models and the causes people are dying from.
- Thus, we use support vector machine (SVM) to classify small and large residuals based on information about cause of death.
- The cause of death data is aggregated in 12 causes

Support Vector Machine (SVM)

- SVM separates observations with the largest possible linear margin
- We use a radical basis function kernel to find for non-linear solutions

Support Vector Machine (SVM)

- SVM separates observations with the largest possible linear margin
- We use a radical basis function kernel to find for non-linear solutions



Separations results - preliminary results

- We define large residuals such that we have two equally large groups (small and large)

Separations results - preliminary results

- We define large residuals such that we have two equally large groups (small and large)
- SVM is able to separate the residuals with a high accuracy

Separations results - preliminary results

- We define large residuals such that we have two equally large groups (small and large)
- SVM is able to separate the residuals with a high accuracy
- Weights in the SVM indicate the relative importance of the causes of death.

Separations results - preliminary results

- We define large residuals such that we have two equally large groups (small and large)
- SVM is able to separate the residuals with a high accuracy
- Weights in the SVM indicate the relative importance of the causes of death.

Table: SVM weights illustrated for the independent model

COD1	COD2	COD3	COD4	COD5	COD6	COD7	COD8	COD9	COD10	COD11	COD12
13.36	2.54	4.40	22.29	4.64	20.11	**45.12**	13.27	12.30	4.20	**54.33**	19.23

- COD1 = Infectious diseases
- COD2 = Cancer (not lung)
- COD3 = Blood diseases
- COD4 = Mental illness
- COD5 = Meningitis and diseases in the nervous system
- COD6 = Circulatory diseases
- COD7 = Lungs and breathing diseases
- COD8 = Digestive diseases and diseases urine, kidney
- COD9 = Senility without mental illness
- COD10 = Road and other accidents and suicide
- COD11 = Other
- COD12 = Lung cancer

Conclusions

- Coda models can successfully be used to forecast mortality in socio-economic sub-populations
- 2 trends are found to dominate mortality trends for Danish males by socio-economic status
- The relative model introduces the strongest coherence and forecasts are dominated by the national level
- Lungs and breathing diseases and other diseases are closest related to misfits of the independence models