Hedge Effectiveness of Index Based Transactions Across Socio-Economic Groups

Work in progress

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Motivation

- Blake et al. (2008): Insurers and reinsurers lacks the liquidity and capacity to offload enough longevity risk.
- A proposed solution to this has been to attract capital market investors.
- To attract capital market investors there is a need for standardization and transparency in the contracts.

Motivation

- The standardization often implies that aggregate population data is used as the reference population.
- However, this comes at a cost for pension funds and annuity providers wishing to offload risk as it leaves basis risk.
    - That is the risk that the mortality experience differs from that of the reference population.
- Hence, using an aggregate mortality index might lower the hegde effectiveness.

Motivation

- Following Coughlan et al. (2011): one important determinant of basis risk is that of socio-economic class, as indices typically account for gender and age.
- The idea of this paper is to investigate the implication on financial products of having different socio-economic groups
- Herein we use the data developed in Cairns et al. (2016).
- Investigate the basis risk and hedge effectiveness across different socio-economic subgroups.

The Data

- Individual specific data on a range of variables from 1980-2012 via Statistics Denmark.
- Including income, wealth, education among others on an individual specific level.
- Subdivide the population into ten groups, at each year and age based on income and wealth.
- Herein we use a new allocation procedure ensuring an equal share in the subgroups for each year and age.

Indexation into subgroups

We use the key covariates **income** and **wealth** to create the **Affluence** measure:

$$A_{i,t,x} = Wealth_{i,t-1,x-1} + 15 \cdot Income_{i,t-1,x-1}$$

We use the lagged to circumvent the missing variable problem in the year of death.

- Next we rank the individuals based on this affluence measure from 1 to N
- Allocate the 10% least affluent to group 1, and so on upto group 10 which contain the 10% most affluent.
- Giving ten groups of equal size at each year and age.
- Fix the group at age 67 (retirement age)

Life Expectancy across groups

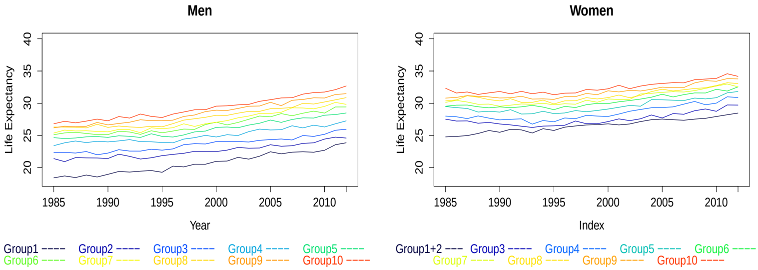

Figure: The remaining life expectancy at age 51 across the 10 (9) deciles for Danish men and women.

The Lee-Carter Model

To analyse the implications on 'mortality derivatives' from using socio-economic subgroups we need models for prediction and simulation of mortality rates.

The Lee and Carter (1992) is used as the Benchmark model for single populations.

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}$$

- α_x general death rates for different ages
- κ_t index variable capturing an overall trend
- β_x relative speed of change for different ages wrt. κ_t

The Li-Lee Coherent Model

The Li and Lee (2005) is a multi-population model (used to simulate mortality scenarios for evaluating hedge effectiveness)

$$\ln m^i_{x,t} = \alpha^i_x + \beta^i_x \kappa^i_t + B_x K_t + \varepsilon^i_{x,t}$$

- B_x is the LC estimate of β_x for the overall population
- K_t is the LC estimate of κ_t for the overall population
- α^i_x general death rates for different ages (Identical to LC)
- κ^i_t index variable capturing an overall trend
- β^i_x relative speed of improvements for different ages wrt. κ_t

q-forwards

- q-forward: An agreement to exchange at maturity date an amount equal to the realised mortality of a given population (floating) in return of a fixed mortality rate (fixed leg) agreed upon at the inception of the contract.
- Introduced in Coughlan et al. (2007b)

The Net Pay-off amount (NPA) at maturity T, is given by:

$$NPA_i(T) = \underbrace{H \cdot q_i^{realised}(T, x)}_{\text{Floating Payment}} - \underbrace{H \cdot \left\{ q_i^{fwd}(0, T, x) \cdot (1 - 0.01)^T \right\}}_{\text{Fixed Payment}} \tag{1}$$

S-forwards

- S-forward: An agreement between two counterparties to exchange at maturity an amount equal to the realised survival rate of a given population cohort in return for the fixed survival rate agreed upon at the beginning of the contract. See technical note LLMA (2010).
- Important building block for longevity swaps

The s year survival rate for an individual at age x_0 at t_0:

$$p(t_0, s, x_0) = \prod_{j=0}^{s-1} \left[1 - q_{x_0+j, t_0+j}\right] = \prod_{j=0}^{s-1} p_{t_0+j, t_0+j+1, x_0+j}$$

The Net Pay-off amount (NPA) at maturity T, is given by:

$$NPA_i(T) = \underbrace{H \cdot p_i^{realised}(0, T, x)}_{\text{Floating Payment}} - \underbrace{H \cdot \left\{ p_i^{fwd}(0, T, x) \cdot (1 - 0.01)^T \right\}}_{\text{Fixed Payment}}$$

Assumptions

To keep things simple we assume

- A constant discount rate of 3%
- A constant yearly risk premium of 1%
- A notional H of 100

q-forward results:

Table: Results for the 9 -year q-forward contracts (2004-2012) based on Lee-Carter projections of mortality rates for the groups 1-10 as well as the overall population. For age 60 at maturity.

	PV floating	PV fixed	PV RP	PV NPA
Grp 1	1.3563	1.3549	0.1283	0.0014
Grp 2	1.1809	1.0740	0.1017	0.1069
Grp 3	1.0864	0.9350	0.0885	0.1514
Grp 4	0.8294	0.6796	0.0643	0.1498
Grp 5	0.6346	0.5284	0.0500	0.1061
Grp 6	0.5141	0.3996	0.0378	0.1145
Grp 7	0.4673	0.3325	0.0315	0.1349
Grp 8	0.4110	0.3395	0.0321	0.0715
Grp 9	0.3701	0.2708	0.0256	0.0993
Grp 10	0.2856	0.2830	0.0268	0.0027
Total	0.7163	0.6169	0.0584	0.0994

q-forward results:

Table: Results for the 9 -year q-forward contracts (2004-2012) based on Lee-Carter projections of mortality rates for the groups 1-10 as well as the overall population. For age 74 at maturity.

	PV floating	PV fixed	PV RP	PV NPA
Grp 1	3.9741	4.0659	0.3849	-0.0918
Grp 2	3.6177	4.1350	0.3915	-0.5173
Grp 3	2.9070	3.7704	0.3569	-0.8633
Grp 4	2.7211	3.1213	0.2955	-0.4003
Grp 5	2.6016	2.8045	0.2655	-0.2028
Grp 6	2.3103	2.4424	0.2312	-0.1321
Grp 7	2.2110	2.4288	0.2299	-0.2178
Grp 8	1.8526	2.1068	0.1994	-0.2541
Grp 9	1.7405	1.8438	0.1746	-0.1033
Grp 10	1.4304	1.7964	0.1701	-0.3660
Total	2.4374	2.7237	0.2578	-0.2862

q-forward results:

Table: Results for the 9 -year q-forward contracts (2004-2012) based on Lee-Carter projections of mortality rates for the groups 1-10 as well as the overall population. For age 94 at maturity.

	PV floating	PV fixed	PV RP	PV NPA
Grp 1	18.8479	15.9228	1.5074	2.9251
Grp 2	20.9196	18.1576	1.7190	2.7620
Grp 3	19.1670	17.9740	1.7016	1.1930
Grp 4	19.3055	19.7590	1.8706	-0.4535
Grp 5	16.9820	21.1398	2.0013	-4.1578
Grp 6	18.7004	22.2863	2.1098	-3.5859
Grp 7	16.8209	18.1458	1.7179	-1.3249
Grp 8	16.2107	17.6663	1.6725	-1.4557
Grp 9	16.0189	15.6400	1.4806	0.3790
Grp 10	16.0050	16.0381	1.5183	-0.0331
Total	17.0092	16.8630	1.5964	0.1462

S-forward results:

Table: Results for the 9 -year S-forward contracts (2004-2012) based on Lee-Carter projections of mortality rates for the groups 1-10 as well as the overall population. For the cohort with age 60 at maturity, (i.e. age 51 at introduction).

	PV floating	PV fixed	PV RP	PV NPA
Grp 1	66.3317	67.2207	0.8295	-0.8890
Grp 2	68.2242	69.8829	0.6081	-1.6586
Grp 3	68.7024	70.4886	0.5563	-1.7862
Grp 4	71.1956	72.4996	0.3803	-1.3039
Grp 5	72.6422	73.4023	0.2995	-0.7600
Grp 6	73.4363	74.3034	0.2176	-0.8671
Grp 7	73.9191	74.5452	0.1955	-0.6261
Grp 8	73.9915	74.5483	0.1952	-0.5568
Grp 9	74.3532	74.9688	0.1565	-0.6157
Grp 10	74.4943	74.7964	0.1724	-0.3021
Total	71.6231	72.5699	0.3741	-0.9468

S-forward results:

Table: Results for the 9 -year S-forward contracts (2004-2012) based on Lee-Carter projections of mortality rates for the groups 1-10 as well as the overall population. For the cohort with age 74 at maturity, (i.e. age 65 at introduction).

	PV floating	PV fixed	PV RP	PV NPA
Grp 1	52.8047	52.4026	1.8525	0.4021
Grp 2	54.7680	53.4500	1.7929	1.3180
Grp 3	57.9814	56.2564	1.6230	1.7251
Grp 4	59.3741	59.2498	1.4262	0.1244
Grp 5	60.9929	61.1313	1.2947	-0.1384
Grp 6	62.5538	63.0645	1.1534	-0.5107
Grp 7	63.3669	63.4471	1.1248	-0.0802
Grp 8	64.9719	64.7616	1.0244	0.2103
Grp 9	66.1726	65.8561	0.9388	0.3165
Grp 10	67.3892	66.7722	0.8658	0.6171
Total	61.3034	60.7768	1.3199	0.5266

S-forward results:

Table: Results for the 9 -year S-forward contracts (2004-2012) based on Lee-Carter projections of mortality rates for the groups 1-10 as well as the overall population. For the cohort with age 94 at maturity, (i.e. age 85 at introduction).

	PV floating	PV fixed	PV RP	PV NPA
Grp 1	13.1485	16.3464	2.2245	-3.1978
Grp 2	13.0225	15.5010	2.1766	-2.4785
Grp 3	14.2579	14.1992	2.0947	0.0587
Grp 4	14.3630	13.9332	2.0767	0.4298
Grp 5	15.5990	13.0380	2.0128	2.5611
Grp 6	15.3565	12.1866	1.9471	3.1699
Grp 7	16.8373	16.0903	2.2104	0.7469
Grp 8	16.8546	17.3046	2.2740	-0.4500
Grp 9	18.3973	19.9534	2.3868	-1.5561
Grp 10	19.8343	19.8782	2.3841	-0.0439
Total	16.3129	17.1979	2.2687	-0.8850

Results for the ten groups

- We find that the sign of q-forward and S-forward payments tend to be more similar across groups for the younger age groups than older.
- This reflects more homogeneous improvements in mortality for younger ages across socio-economic groups.

q-Duration

- Following Coughlan et al. (2007a) the q-duration can be interpreted as the change in value of a portfolio due to a unit change in mortality, i.e. we use

$$q_{x,t}^{shock} = q_{x,t}\left(1 - 0.01\%\right)^{t} \tag{2}$$

- Life annuity model from Cairns et al. (2014) imposing the value of the pension as:

$$a_i\left(T, x\right) = \sum_{s=1}^{\infty} \left(1 + r\right)^{-s} p_i^{fwd}\left(T, s, x\right) \tag{3}$$

where $p_i^{fwd}\left(T, s, x\right)$ is the forward survival probability representing the best estimate at time T that an individual aged x in population i will survive for another s years.

q-Duration

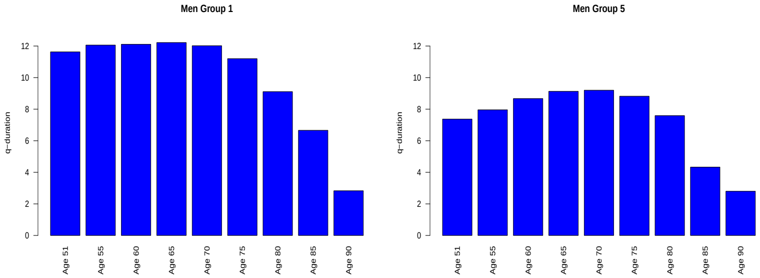

Figure: q-durations for different ages and affluence group 1 (left) and 5 (right), Danish men.

q-Duration

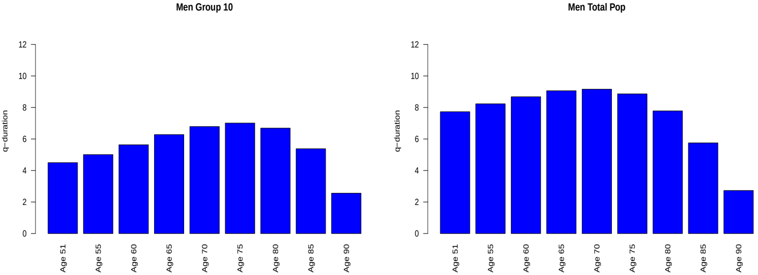

Figure: q-durations for different ages and affluence group 10 (left) and the total population (right), Danish men.

q-Duration

- The different longevity experience observed between groups make it clear that a larger number of different ages should be available for hedging, comparing with results from the aggregate population.
    - See e.g. Steur et al. (2009), and Coughlan et al. (2007b)

Hedge Effectiveness of index based transactions

- Evaluate the hedge effectiveness of the subgroups, using the Danish national data as the standardised index.
- That is we intuitively consider the groups $i = 1, 2, .., 10$ to be proxies for specific pension plans
- We consider a static hedging scenario for:
  - A deferred longevity swap
  - q-forward
- We take a prospective approach simulating data using the Li-Lee model
- Five step framework following Coughlan et al. (2011) and Cairns et al. (2014)

Evaluation of Hedge Effectiveness

(I) Hedging objective: Affluence group (i = 1,2,...,10), liability value, $L_i(T) = a_i(T;x)$ with T = 15 and t = 0 equals 2012

(II) Hedging instrument and hedged position:

  (i) Hedging instrument: Deferred longevity swap, value at T: $H_i(T) = a_{Tot}(T;x) - a_{Tot}^{fwd}(0,T,x)$, where *Tot* refers to the total population

  (ii) Hedged position: $P_i(h) = L_i(T) + h_i \cdot H_i(T)$

Evaluation of Hedge Effectiveness

(III) Method for assessment of hedge effectiveness: (Prospective)

  (i) Risk metric: $Var(P_i(h_i))$
  (ii) Basis for hedge effectiveness evaluation: Relative risk reduction: $RRR = 1 - \frac{Var(P_i(h_i))}{Var(L_i(T))} = 1 - \frac{Risk_{(Liability+Hedge)}}{Risk_{(Liability)}}$
  (iii) Scenario generator: Li-Lee model
  (iv) Valuation model: Lee-Carter model

Evaluation of Hedge Effectiveness

(IV) Hedge effectiveness calculation:
  (i) Simulate future mortality rates
  (ii) Evaluate position at time T
  (iii) Calibrate hedge ratio for population i, h_i^*:
  $$h_i^* = -\frac{cov(L_i, H_i)}{var(H_i)}$$
  (iv) Evaluate hedge effectiveness for pop. i: Search for the h_i^* that minimises $Var(P_i(h_i))$

(V) Analysis and interpretation of results

Evaluation of Hedge Effectiveness

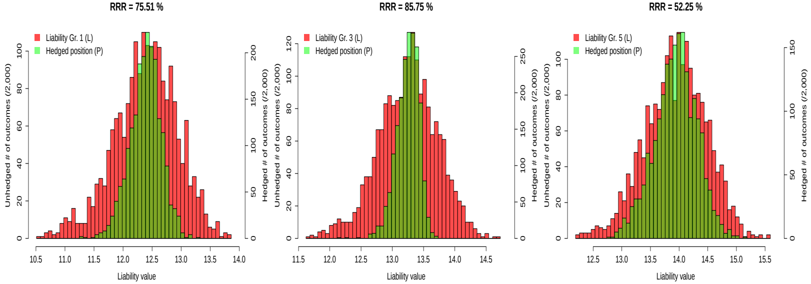

Figure: Hegde Effectiveness of a deferred longevity swap for affluence group 1 (left) and 3 (middle) and 5 (right), Danish men.

Evaluation of Hedge Effectiveness

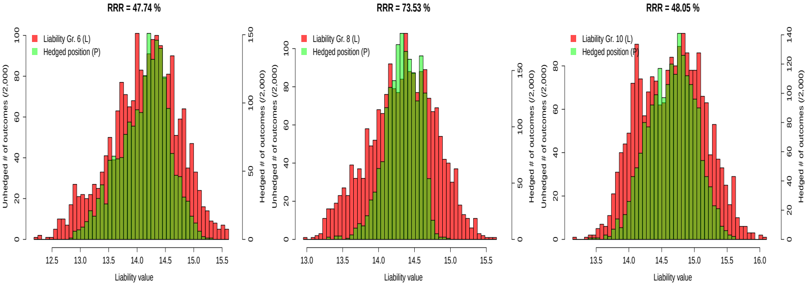

Figure: Hegde Effectiveness of a deferred longevity swap for affluence group 6 (left) and 8 (middle) and 10 (right), Danish men.

Table: Hedge effectiveness for a 15-year deferred longevity swap and 15-year q-forward

	Deferred Longevity Swap			q-Forward		
	Cor(L,H)	h*	RRR (%)	Cor(L,H)	h*	RRR (%)
Grp1	-0.8690	1.2522	75.5100	-0.8611	484.6111	74.1500
Grp2	-0.8078	1.1123	65.2500	-0.8206	462.8595	67.3400
Grp3	-0.9260	1.1390	85.7500	-0.9111	447.1495	83.0100
Grp4	-0.7226	1.0323	52.2100	-0.7300	413.3222	53.2800
Grp5	-0.7228	1.0340	52.2500	-0.7266	420.2895	52.8000
Grp6	-0.6909	1.1138	47.7400	-0.6805	438.6518	46.3000
Grp7	-0.8653	1.1039	74.8800	-0.8482	417.8894	71.9500
Grp8	-0.8575	1.0715	73.5300	-0.8515	417.3540	72.5100
Grp9	-0.7923	0.9983	62.7800	-0.7841	393.2490	61.4900
Grp10	-0.6932	0.9305	48.0500	-0.6867	350.8135	47.1500

Conclusion

- Larger similarity between subgroup improvements at younger ages
- The variation in q-duration across subggroups shows that a larger number of traded contracts is needed comparing with results using the aggregate population.
- Significant risk reduction from index based transactions.
- Similar risk reduction achieved from the q-forward and the deferred longevity swap.

Further Research

- A large number of potential extensions, herein:
- Customizing
    - Model specific adjustment factor
- Lumping subgroups together to increase the hedge effectiveness
    - Preliminary results indicate this improves the hedge effectiveness
- Different hedge effectiveness valuation measures
- Use the CBD-X multipopulation model Cairns et al. (2016)

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